INSTITUTE OF MATHEMATICS OF NAS ARMENIA

YEREVAN STATE UNIVERSITY

INTERNATIONAL CONFERENCE

HARMONIC ANALYSIS AND APPROXIMATIONS, III

20-27 SEPTEMBER, 2005

TSAHKADZOR, ARMENIA

ABSTRACTS

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C O N T E N T S

On convergence of double Fourier-Walsh series of functions of bounded Λ-variations

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We consider the convergence of double Fourier-Walsh series of continuous functions. For double series M. Dyachenko introduced the concept of $u(K)$-*convergence* (domains for which the partial sums are taken are not allowed to stretch too much). The special case of $u(K)$-*convergence* is convergence over spheres, squares triangles but not over rectangles. We prove that if an integrable function belongs to the Waterman class $\Lambda BV([0,1]^2)$ for $\Lambda = \left\{\overline{o}(\frac{\sqrt{n}}{\sqrt{\ln(n+1)}})\right\}_{n=1}^{\infty}$ and is continuous on a compact E, then its double Fourier-Walsh series is uniformly $u(K)$-*convergent* on E. The analog of this theorem for trigonometric system was proved by M. Dyachenko. The next results are analogs of Salhem's criterion for uniformly convergence over rectangles and for uniformly $u(K)$-*convergence* of double Fourier-Walsh series of continuous functions. For trigonometric systems these criteria was obtained by B. Golubov for convergence over rectangles and by A. Saakyan for $u(K)$-*convergence*. Then we show that for any compact subset $E \subset C([0,1]^2)$ there exists a homeomorphism τ of the interval $[0,1]$ such that Fourier-Walsh series of any function of the form $F(x,y) = f(\tau(x),\tau(y))$, $f \in E$ is simultaneously convergent over rectangles and $u(K)$-*convergent*. The same theorem for trigonometric series was proved by A. Saakyan.

Consistency Method in Reconstruction by Projection Curvatures

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Some results in convexity theory lead to the integral equations of type (1), which we call *generalized Radon equations*. The problem is to restore the unknown function $f(\Omega)$ defined on the unit sphere $\mathbf{S}^2 \subset \mathbb{R}^3$ by the integral

$$F(\omega, \psi) = \int_{\mathbf{S}_\omega} A(\omega, \psi, \varphi) f_\omega(\varphi)\, d\varphi. \qquad (1)$$

Here $\mathbf{S}_\omega \subset \mathbf{S}^2$ is the great circle with pole $\omega \in \mathbf{S}^2$, $\varphi \in \mathbf{S}_\omega$ and $\psi \in \mathbf{S}_\omega$, $d\varphi$ is the arc measure on $\mathbf{S}_\omega$, $f_\omega(\varphi)$ is the restriction of $f(\Omega)$ on $\mathbf{S}_\omega$. In case $A(\omega, \psi, \varphi) \equiv 1$ the problem becomes the classical Radon problem on the sphere.
In [1] we suggested what is called "consistency method" to solve these integral equations and test this method for the case of classical Radon equation. The method we apply to solve (1) is as follows. For every $\omega \in \mathbf{S}^2$, we reduce (1) to an integral equation on the circle $\mathbf{S}_\omega$, whose general solution $g(\omega, \varphi)$ we write as a Fourier series in the variable φ. Then we apply the consistency condition.
The consistency method is useful also for reconstruction of convex bodies. The existence and uniqueness of convex body $\mathbf{B} \subset \mathbb{R}^3$ for which the mean curvature radius coincides with given function was posed by Christoffel. The corresponding problem for Gauss curvature was posed by Minkovski. A. D. Aleksandrov and A. V. Pogorelov generalized these problems.
We consider a similar problem for the projection curvature radii of convex bodies (see [2]). We need some notation: $e(\Omega, \phi)$ is the plane containing the origin of $\mathbb{R}^3$ and the directions $\Omega \in \mathbf{S}^2$ and $\phi \in \mathbf{S}_\Omega$; $\mathbf{B}(\Omega, \phi)$ is the projection of $\mathbf{B}$ onto $e(\Omega, \phi)$ and $R(\Omega, \phi)$ is the curvature radius of $\partial\mathbf{B}(\Omega, \phi)$ at the point whose outer normal is Ω.
Let $F(\Omega, \phi)$ be a function defined on $\mathbf{S}^2 \times \mathbf{S}^1$ (see [1]). We pose the problem of existence and uniqueness of a convex body with $R(\Omega, \phi) = F(\Omega, \phi)$. We reduce the problem to a partial differential equation of second order for support function and using the consistency method, find a necessary and sufficient condition on $F(\Omega, \phi)$ that ensures a positive answer. Note, that the uniqueness (up to parallel shifts) follows from the classical uniqueness result on Christoffel problem. Also we find a representation of the support function of a convex body by projection curvature radii and suggest an

algorithm to construct that body.

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The inverse quantum scattering problem for Sturm-Liouville operator with certain behavior of the potential at infinity

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We consider the operator L in the space $L^2(R)$, generated by the differential operation $l(y) = -y'' + q(x)y$. We suppose that q (the potential) is a measurable real-valued function, defined on R, satisfying the condition

$$\int_{-\infty}^{0} (1-x)|q(x) - a^-|dx + \int_{0}^{\infty} (1+x)|q(x) - a^+|dx < \infty,$$

where $a^\pm$ are some real numbers. We assume, for simplicity, $a^+ < a^-$.

We prove that continuous spectrum of the operator L coincides with the semiaxis $[a^+, \infty)$ and the eigenvalues are simple, lie in the interval $(-\infty, a^+)$ and are finite in number.

For a $\mu \in (a^+, a^-)$ the equation $l(y) = \mu y$ has a unique non-trivial bounded solution within a constant factor. For a $\mu \in (a^-, \infty)$ the equation $l(y) = \mu y$ has two linearly independent bounded solutions. For $\mu \in$

$(a^+, a^-) \cup (a^-, \infty)$ using coefficients in the asymptotic behaviour of the bounded solutions of the equation $l(y) = \mu y$ we construct a system (called generalized eigenfunctions‘ system). Using also the eigenfunctions of the operator L we obtain the Fourier expansion.

Issuing the asymptotic behavior of the generalized eigenfunctions for the operator L we introduce the right scattering data

$$\{T, N^+(\mu)(\mu \in T), S^+(\mu)(\mu \in (a^+, a^-) \cup (a^-, \infty))\}, \tag{1}$$

where T is the set of all eigenvalues of the operator L, $N^+(\mu)$ are positive numbers and $S^+(\mu)$ is a continuously differentiable matrix-function, whose values are non-negative 2×2 matrices.

The inverse scattering problem is to find q if the right scattering data (1) is known. We prove the uniqueness of q and the problem of finding q is reduced to a linear integral equation.

A maximal theorem for a weighted space on the unit ball of $\mathbb{R}^n$

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Let $B = B_n$ be the open unit ball in the n-dimensional Euclidean space $\mathbb{R}^n$, and let $S = S_{n-1}$ be its boundary, the unit sphere. Given a function $f(x)$ on B and the nontangential approach region $\Gamma_\delta(\zeta)$, $\delta > 0$, $\zeta \in S$, the nontangential maximal function of $f(x)$ is defined by

$$(N_\delta f)(\zeta) = \sup_{x \in \Gamma_\delta(\zeta)} |f(x)|, \qquad \zeta \in S.$$

The (real) Hardy space $H^p(S)$, $0 < p < \infty$, is the set of harmonic functions $f(x)$ on B, for which $N_\delta f \in L^p(S)$, and the norm is defined by

$\|f\|_{H^p(S)} = \|N_\delta f\|_{L^p(S)}$. It is well known that H^p-functions have non-tangential boundary values and $H^p = L^p$ for $1 < p < \infty$, and $H^p \subset L^p$ for $0 < p \leq 1$, the inclusion is strict.

In contrast to Hardy spaces, the situation is quite different for volume integrable functions. In this talk we state a maximal theorem for harmonic functions weighted integrable on B. A measurable function $u(x)$ on B belongs to the weighted Bergman space L^p_α $(0 < p < \infty, \alpha > -1)$ iff

$$\|u\|_{p,\alpha} = \left(\int_B |u(x)|^p (1-|x|)^\alpha dx\right)^{1/p} < +\infty.$$

Let h^p_α be the subspace of L^p_α consisting of harmonic functions.

Given $0 < \delta < 1$, let $Q(x,\delta)$ be the closed ball with the center x and radius $\delta(1-|x|)$. We prove that for $\alpha > -1, 0 < p \leq 1, u(x) \in h^p_\alpha$, the maximal function

$$u^*_\delta(x) = \sup_{y \in Q(x,\delta)} |u(y)|, \qquad x \in B,$$

satisfies the Hardy–Littlewood type inequality

$$\|u^*_\delta\|_{p,\alpha} \leq C(p,\alpha,n,\delta)\|u\|_{p,\alpha}.$$

Integrals of multivalued mappings

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We define the Riemann integral for multivalued mappings and prove that the closure of the integral coincides with the Lebesgue integral [3]. Let $(E, \sum, P)$ be a probability space, where E is a compact in R^k.

Definition 1 *The Riemann Integral of multivalued mapping* $G : E \to 2^{R^m}$ *is called the following set*

$$(R) \int_E A(\theta) P(d\theta) = \left\{ \int_E g(\theta) p(d\theta) : g(\theta) \in G(\theta) \right\} \equiv M_R,$$

where $g(\theta)$ *is a continuous selection of the multivalued mapping* G.

Definition 2 *The Lebesgue Integral of multivalued mapping* $G : E \to 2^{R^n}$ *is called the following set*

$$(L) \int_E A(\theta) P(d\theta) = \left\{ \int_E g(\theta) p(d\theta) : g(\theta) \in G(\theta) \right\} \equiv M_L,$$

where $g(\theta)$ *is Lebesgue integrable.*

Theorem 1 *Let the multivalued mapping* $G : E \to 2^{R^n}$ *be continuous and* $G(\theta)$ *is convex-compact for all* θ *in some compact* $E \subset R^m$. *Then* $\overline{M}_R = M_L$.

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On a Boundary Value Problem for Elliptic Equations with Nonsymmetric Roots

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Let $D = \{z||z| < 1\}$ – be a unit disk of a complex plane and $\Gamma = \partial D$. We consider an elliptic equation

$$\sum_{k=0}^{n} A_k \frac{\partial^n u}{\partial x^k \partial y^{n-k}} = 0, \quad (x,y) \in D, \tag{1}$$

where A_k - are complex constants, such that the roots λ_k of the corresponding characteristic equation $\sum\limits_{k=0}^{n} A_k \lambda^{n-k} = 0$ satisfy the condition $\lambda_k \neq \overline{\lambda_j}$, $\Im\lambda_k \neq 0$, $k, j = 1, \ldots, n$. The solution u of the equation (1) belongs to the class $C^n(D) \bigcap C^{(n-1,\alpha)}(D \bigcup \Gamma)$ and satisfies the Riemann type boundary value condition on the boundary Γ

$$\Re \frac{\partial^k u}{\partial r^k} = f_k(x,y). \quad (x,y) \in \Gamma, \quad k = 0, \ldots, n-1. \tag{2}$$

Here $\frac{\partial}{\partial r}$ – is a derivative in the direction of the radius-vector of $(x,y) \in \Gamma$. We prove the following

Theorem 1 *The problem (1) – (2) is Noetherian. This problem has a solution for arbitrary functions* $f_k \in C^{(n-k-1,\alpha)}(\Gamma)$ $(k = 0, \ldots, n-1)$, *and the corresponding homogeneous problem (when* $f_k \equiv 0$*) has* n^2 *linearly independent solutions, which are pure imaginary polynomials of the order* $2n-2$.

On the "B-Products" of Spaces of Besov Type

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Denote by G^+ the set of positive, infinitely differentiable functions $\mu(\xi)$ on R_n, such that for all $\xi \in R_n$ and $\alpha = (\alpha_1, ..., \alpha_n)$, $\alpha_i = 0, 1$ $(i = 1, ..., n)$ there exists a positive constant c satisfying $|\xi^\alpha D^\alpha \mu(\xi)| \leq c\mu(\xi)$.

For $\mu \in G^+$, $1 < p < \infty$, $1 \leq q \leq \infty$, $-\infty < s < \infty$, we put

$$B^s_{p,q}(\mu; R_n) = B^s_{p,q}(\mu) = \{f \in S' :$$

$$||f|| = \left(\int_0^\infty \left|\left|F^{-1}\frac{t^{1/2}\mu^{1+s}}{\mu^2+t}Ff\right|\right|^q_{L_p(R_n)}\frac{dt}{t}\right)^{1/q} < \infty\right\}.$$

For $\mu(\xi) = (1+|\xi|^2)^{1/2}$ the defined B-space coincide with the classical Besov space. For the description of the real interpolation spaces of the pairs of spaces of Besov type with different anisotropies we need to consider the following spaces with mixed norms.

Let $\mu, \nu \in G^+$, $1 < p < \infty$, $1 \le q \le \infty$, $-\infty < s, m < \infty$. The following space

$$B^s_{p,q}(\mu)\cdot B^m_{p,q}(\nu) = \left\{f \in S'(R_n) : ||f||_{B^s_{p,q}(\mu)\cdot B^m_{p,q}(\nu)} = \right.$$

$$= \left(\int_0^\infty\int_0^\infty \left|\left|F^{-1}\frac{t^{1/2}\mu^{1+s}}{\mu^2+t}\cdot\frac{u^{1/2}\nu^{1+m}}{\nu^2+u}Ff\right|\right|^q_{L_p(R_n)}\frac{dt}{t}\frac{du}{u}\right)^{1/q} < \infty\right\}.$$

we call "B-product" of the spaces $B^s_{p,q}(\mu)$ and $B^m_{p,q}(\nu)$.

Many interesting properties of the "B-products" are based on the following theorem.

Theorem 1 *Let $\mu, \nu \in G^+$, $1 < p < \infty$, $1 \le q \le \infty$. Then*

$$B^0_{p,q}(\mu)\cdot B^0_{p,q}(\nu) = B^0_{p,q}(\mu\nu)\cdot B^0_{p,q}(\frac{\mu}{\nu}).$$

Next theorem describes the real interpolation spaces of the pairs of spaces of Besov type with different anisotropies in terms of the "B-products".

Theorem 2 *Let $\mu, \nu \in G^+$, $1 < p < \infty$, $1 \le q \le \infty$, $0 < \theta < 1$. Then*

$$\left(B^1_{p,q}(\mu), B^1_{p,q}(\nu)\right)_{\theta,q} = B^{1-\theta}_{p,q}(\mu)\cdot B^\theta_{p,q}(\nu)$$

An application of Laguerre polynomials to convolution type integral equations

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The following integral equation is considered

$$f(x) = g(x) + \int_0^r K(x-t)f(t)dt, \qquad (1)$$

where $K \in L_1(-r, r)$ is even function. In case of polar kernel and other cases of non-smooth kernel functions , the numerical solution of (1) by direct discretization of the integral is non-effective and leads to greater uncontrollable mistakes even in the case of the big number of nodes of a quadrature. Besides the choice of a non-uniform net damages the toeplitz structure of the equation. Therefore the solution of (1) in some cases was combined with development of special analytical and numerical methods. These methods for solution of the equations arising in the theory of radiative transfer and the kinetic theory of gases have most far advanced. These methods are effective for a narrow class of the kernels, admitting effective approximation in L_1 by linear aggregates of exponentials:

$$K(x) \approx \sum a_m \exp(-s_m|x|) \qquad (2)$$

In N.B. Yengibaryan's recent work the method for the solution of (1) on the half line based on decomposition of the kernel K by Laguerre polynomials in a combination to the new version of V. Ambartsumyan non-linear equation is suggested. This method has allowed widening essentially a class of Wiener-Hopf equations, permitting the effective solution.

In the present work the above-mentioned method is developed for case of the equation (1) on a finite interval (which, as a rule, is more difficult than the equation on a half line). Suggested approach is based on the approximation of the kernel by more flexible aggregates than (2):

$$K(x) \approx \exp(-\gamma x) P_n(x), \quad x > 0,$$

where $P_n(x) = \sum_{1}^{n} a_k x^k, \;\; a_n \neq 0$, in a combination to a number of new analytical constructions of theory of convolution type equations. In particular, the factorization method of an establishment of relation between solutions of the equation (1) on a half line and corresponding equation on a finite interval is suggested. This work is supported by ISTC (grant A-823).

Inequivalence of wavelet systems in $L_1(\mathbb{R}^d)$ and $BV(\mathbb{R}^d)$

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Sufficient conditions for the inequivalence of the d-variate Haar wavelet system and another wavelet system in the spaces $L_1(\mathbb{R}^d)$ and $BV(\mathbb{R}^d)$ are presented:

Assume that H is the (isotropic or anisotropic) Haar system on $\mathbb{R}^d$ and Ψ is another (resp. isotropic or anisotropic) d-variate wavelet system derived from the univariate wavelet ψ.

If

$$\int_{[0,+\infty)} \psi(t)dt \neq 0,$$

then H and Ψ are not equivalent in $L_1(\mathbb{R}^d)$.

If

$$\int_{[\frac{1}{3},+\infty)} \psi'(t)dt + \int_{[\frac{2}{3},+\infty)} \psi'(t)dt \neq 0,$$

then H and Ψ are not equivalent in $BV(\mathbb{R}^d)$.

These results are then applied to the Strömberg wavelet and compactly supported Daubechies wavelets.

The Interaction of Alternation Points and Poles in Rational Approximation

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The interrelation of alternation points for the minimal error function and poles of best Chebyshev approximants is investigated if uniform approximation on the interval $[-1, 1]$ by rational functions of degree $(n(s), m(s))$ is considered, $s \in \mathbb{N}$. In general, the alternation points need not to be uniformly distributed with respect to the equilibrium measure on $[-1, 1]$, even not to be dense on the interval. We show that, at least for a subsequence $\Lambda \subset \mathbb{N}$, the asymptotic behavior of the alternation points to the degrees $(n(s), m(s)), s \in \Lambda$, is completely determined by the location of the poles of the best approximants, and vice versa, if $m(s) \leq n(s)$ or $m(s) - n(s) = o(s/\log s)$ as $s \to \infty$.

The results extend and sharpen results of *Borwein, Braess, Grothmann, Kovacheva, Kroó, Lubinsky, Peherstorfer, Saff* and the author. All proofs of these previous results essentially used the fact that the degrees of the denominators of the rational approximants were (up to a fixed additive constant) not bigger than the degrees of the numerators.

Interpolation by bivariate polynomials based on Radon projections

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We consider the problem of characterization of configurations of $N := \binom{n+2}{2}$ chords $\{I_k\}_{k=1}^{N}$ in the unit disk such that the interpolation problem

$$\int_{I_k} P = f_k, \quad k = 1, \ldots, N,$$

is unisolvent in the space of bivariate algebraic polynomials P of total degree n. The interpolation formula of Hakop Hakopian is a famous example of this type. We discuss some recent results which provide other examples and lead to algorithms for reconstruction of functions f on the disk from their Radon projections $\{\int_{I_k} f\}$.

What are the limits of Lagrange projectors?

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By definition, a Lagrange projector associates with each function in its domain the unique function in its range that agrees with the former at a certain (finite) set. The talk explores the (pointwise) limit of such projectors on polynomials in several variables as some of the interpolation sites coalesce.

A method for the Poisson equation

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We present a method for solving the Poisson equation with the Dirichlet condition, which employes polyharmonic splines. The numerical solution is a sum of the solution of a Dirichlet problem for the Laplace equation and the solution of the Poisson equation on $\mathbb{R}^n$. The method provides the numerical solution with good accuracy and low computational cost, in all dimensions.

B-splines and wavelets

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The necessary and sufficient conditions for the (nonorthogonal) wavelet multiresolution analysis with arbitrary (for example B-spline) scaling function are established.

The following results are obtained:

1) the general theorem which declares necessary and sufficient conditions for the possibility of multiresolution analysis in the case of arbitrary scaling function;

2) the reformulation of this theorem for the case of B-spline scaling function from W_2^m;

3) the complete description of the family of wavelet bases generated by B-spline scaling function;

4) the concrete construction of the unconditional wavelet basis (with minimal support of wavelet) generated by B-spline scaling function which belongs to W_2^m.

These wavelet bases are simple and convenient for applications. In spite of their nonorthogonality, these bases possess the following advantages: 1) compactness of the set $\operatorname{supp} \psi$ and minimality of its measure; 2) simple explicit formulas for the change of level. These advantages compensate the nonorthogonality of described bases.

This work is partly supported by Russian Foundation for Basic Research (Grants 03-01-00801 and 04-01-00590) and the program "Universities of Russia" (Grant 04.01.453).

Metric Approximation of Univariate Set-Valued Functions

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This talk reviews known approximation operators for univariate set-valued functions, and presents a new effective method for adapting linear approximation operators for real-valued functions to set-valued functions with general compact images. This adaptation is done by replacing linear combinations of numbers with new "metric linear combinations" of finite sequences of compact sets. The new "metric linear combination" extends the binary metric average of Artstein. The resulting operators are termed "metric analogue" operators for set-valued functions. Approximation estimates for the metric analogue operators are presented. As examples, metric Bernstein operators, metric Shoenberg operators and metric polynomial interpolants are discussed.

This talk reports on a joint work with E. Farkhi and A. Mokhov.

On the divergence of Fourier-Walsh series in $L^1(E)$ metric

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In 1932 F. Riesz proved that there exists a function $f_0(x) \in L^1[0, 2\pi]$ so that its Fourier series with respect to the trigonometric system does not converge in $L^1[0, 2\pi]$. Similar result was obtained for the Walsh system.

In [1] we proved the following:

Theorem 1. Let the sequences $\{\beta_k\}_{k=1}^{\infty}$ and $\{M_n\}_{n=1}^{\infty}$ satisfy

$$\lim_{k\to\infty} (M_{2k} - M_{2k-1}) = +\infty, \quad \beta_k > 0, \quad \lim_{k\to\infty} \beta_k = 0.$$

Then for any $\varepsilon > 0$ there exists function $f_0(x) \in L^1[0,1]$, $f_0(x) = 0$ for $x \notin [0, \varepsilon]$ such that the Fourier-Walsh series of $f_0(x)$ divergence in $L^1[0,1]$ metric and

$$\sum_{n=1}^{\infty} |a_n(f_0)| \, \beta_n < \infty$$

$$a_n(f_0) = 0, \quad n \notin \bigcup_{s=1} \infty \left[M_{2s-1}, M_{2s}\right].$$

The main result of the talk is the following:

Theorem 2. Let the sequence $\{\beta_k\}_{k=1}^{\infty}$ with $\beta_k > 0$, $\lim\limits_{k\to\infty} \beta_k = 0$ be given. Then for any set $E \subset [0,1]$ with positive measure there exists a function $f_0(x) \in L^1[0,1]$, such that the Fourier-Walsh series of $f_0(x)$ divergence in $L^1(E)$ metric and

$$\sum_{n=1}^{\infty} |a_n(f_0)| \, \beta_n < \infty,$$

where $\{a_n(f_0)\}$ is Fourier-Walsh coefficients of $f_0(x)$.

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On a topology on the space of functions without second kind of discontinuity

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Let $\mathbb{L}^d(\mathbb{R}_+)$ be the space of $\mathbb{R}^d$ -valued functions f on $\mathbb{R}_+ = [0, \infty)$ having one-sided finite limits $f(t-0)$ and $f(t+0)$ at the each point $t \in \mathbb{R}_+$. We define the set of "symbols" $\Theta = \{\theta = t - 0 (t > 0) \text{ or } \theta = t + 0 (t \geqslant 0)\}$ and consider the space $\mathbb{L}^d(\Theta) = \left\{f(\theta) : \Theta \to \mathbb{R}^d\right\}$.

Let $\Lambda^0 = \{\lambda^o : \mathbb{R}_+ \to \mathbb{R}_+\}$ be the set of all strictly increasing continuous functions on $\mathbb{R}_+$ such that $\lambda^0(0) = 0$ and $\lambda^0(t) \uparrow \infty$ when $t \uparrow \infty$ (λ^0 is a "change of time"). We define by $\Lambda = \{\lambda : \Theta \to \Theta\}$ the set of functions on Θ such that $\lambda(\theta) = \lambda^0(t) \pm 0$ for all $t \in \mathbb{R}_+$, where $\theta = t \pm 0 \in \Theta$.

Theorem 1. (cf. [3]) There is *a metrizable topology* on $\mathbb{L}^d(\Theta)$ (the Skorokhod topology) for which the space $\mathbb{L}^d(\Theta)$ is *a complete separable* (á Polish) *space*. This topology is characterized as follows:

a sequence of functions $\{f_n\} \subset \mathbb{L}^d(\Theta)$ converges to $f \in \mathbb{L}^d(\Theta)$ if and only if there is a sequence of functions $\{\lambda_n\} \subset \Lambda$ such that as $n \to \infty$

a) $\sup\limits_{t} \left|\lambda_n^0(t) - t\right| \to 0$;

b) $\sup\limits_{t \leqslant N} |f_n(\lambda_n(\theta)) - f(\theta)| \to 0$ for all $N \in \mathbb{N}$, where $\theta = t \pm 0 \in \Theta$.

Theorem 2. (The Ascoli-Arzelas theorem for $\mathbb{L}^d(\theta)$).

A subset $A \subset \mathbb{L}^d(\Theta)$ is *relatively compact* for the Skorokhod topology if and only if

a) $\sup\limits_{f \in A} \sup\limits_{t \leqslant N} |f(\theta)| < \infty$ for all $N \in \mathbb{N}\,(\theta = t \pm 0)$;

b) $\lim\limits_{\tau \downarrow 0} \sup\limits_{f \in A} w'_N(f, \tau) = 0$ for all $N \in \mathbb{N}$,

where

$$w'_N(f,\tau) = \inf \left\{ \max_{1 \leqslant k \leqslant n} w_f\,[t_{k-1} + 0, t_k - 0] : 0 = t_0 < \ldots < t_n = N, \right.$$

$$\left. \min_{1 \leqslant k < n} (t_k - t_{k-1}) \geqslant \tau \right\},$$

$$w_f\,[t_{k-1} + 0, t_k - 0] = \sup\left\{|f(\theta) - f(\theta')| : \theta, \theta' \in [t_{k-1} + 0,\, t_k - 0]\right\}.$$

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Approximation of and by the Riemann zeta function

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It is possible to approximate the Riemann zeta function by meromorphic functions having only a simple pole at $s = 1$ and for which the Riemann hypothesis fails. On the other hand, any meromorphic function having only simple poles can be approximated by linear combinations of translates of the Riemann zeta function.

Asymptotics for Hermite type polynomials

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These polynomials $H_n^{(q)}$ have been defined by Polya through the Rodrigues type formula

$$H_n^{(q)}(z) := (-1)^n e^{z^q} \left(\frac{d}{dz}\right)^n e^{-z^q},$$

$q \geq 2$ being an integer. Generalizing standard asymptotics for the classical Hermite polynomials $H_n^{(2)}$ formulae of Plancherel-Rotach type are proved for $H_n^{(q)}$. In addition the asymptotic distribution function of the zeros of $H_n^{(q)}$ is derived as $n \to \infty$.

On the Estimates of Monomials through the Polynomials and classification of extremum points of the multivariable Functions

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For the real multivariable polynomials with real coefficients the following closely related problems are considered:

1. To describe the set of points $\nu \in \mathbb{R}^n_+ = \{\nu \in \mathbb{R}^n; \nu_i \geq 0, i = 1, \cdots, n\}$ satisfying the following inequality

$$|\xi^\nu| \leq C \sum_{i=1}^{M} |P_i(\xi)|, \qquad \forall \xi \in \mathbb{R}^n \tag{1}$$

where C is a constant depending only on given polynomials $P_1, \cdots, P_M$.

2. To find conditions for a prescribed analytic function such that a given stationary point is a point of local extremum.

In this talk we are going to present the necessary and sufficient conditions for each of this cases, which solve the above prescribed problems.

On some properties of the normalized numerical range

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We investigate some properties of the numerical range $W(A)$ and the normalized numerical range $W_n(A)$ of a Hilbert space operator A. Particular attention is paid to the investigation of the question when the origin of the coordinate system belongs to $W_n(A)$. It is shown that 0 does not belong to $W_n(A)$ if and only if A is an orthogonal sum of the zero operator and an operator A_0 such that $0 \notin W(A_0)$. The necessary and sufficient algebraic conditions for both inclusions $0 \in W(A)$ and $0 \in W_n(A)$ are mentioned. Some relations between the different parts of the operator spectrum and $W_n(A)$ are established.

Lacunary summability

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In this talk summability methods of weighted mean type are considered which are generated by lacunary sequences. A result on the summability of the geometric series is proved which has far-reaching consequences with respect to general analytic continuation of power series into their α-Mittag-Leffler-star, to overconvergence as well as to the universal behavior of trigonometric series in the sense of Menšov.

Asymptotic Behavior of the Finite Predictor for Stationary Processes[1]

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Let $X(t)$, $t \in \mathbb{Z} = \{0, \pm 1, \ldots\}$, be a wide sense stationary stochastic process with mean zero and spectral density (s.d.) $f(\lambda)$, $\lambda \in [-\pi, \pi]$. Denote by $H = H(X)$ the Hilbert space generated by the process $X(t)$, and let $H_s^t(X)$ be the subspace of H spanned by the random variables $X(u)$, $s \leq u \leq t$.

[1] The work was supported in part by the NFSAT/CRDF Grant No. MA 070-02/12011

The best linear finite predictor $\hat{X}(0)$ for the random variable $X(0)$ by the past of length n, that is, by the random variables $X(-n), \ldots, X(-1)$, is the orthogonal projection of the element $X(0) \in H$ onto the subspace $H_{-n}^{-1}(X)$. The length of the corresponding perpendicular, which we denote by $\sigma_n^2(f)$, is called one-step prediction error by the past of length n. Let $\sigma^2(f) = \sigma_\infty^2(f)$ denote the prediction error by the entire past. The following classical result is known as **Kolmogorov-Szegö alternative:** *Either*

$$\log f \notin L_1[-\pi, \pi] \quad \Leftrightarrow \quad H = H_{-\infty}^0 \quad \Leftrightarrow \quad \sigma^2(f) = 0, \tag{1}$$

or else

$$\log f \in L_1[-\pi, \pi] \quad \Leftrightarrow \quad H \neq H_{-\infty}^0 \quad \Leftrightarrow \quad \sigma^2(f) > 0. \tag{2}$$

Moreover, in both cases the one-step prediction error by the entire past $\sigma^2(f)$ *is given by* $\sigma^2(f) = \exp\left\{\frac{1}{2\pi}\int_{-\pi}^{\pi} \log f(\lambda)\, d\lambda\right\}$.
The processes with spectral densities $f(\lambda)$ satisfying (1) are called deterministic (or singular), while the processes with spectral densities $f(\lambda)$ satisfying (2) are called nondeterministic (or regular).
We set $\delta_n(f) = \sigma_n^2(f) - \sigma^2(f)$. It is clear that $\delta_n(f) \geq 0$ and $\delta_n(f) \to 0$ as $n \to \infty$.

In the talk we present results describing the rate of decreasing of quantity $\delta_n(f)$ to zero as $n \to \infty$, depending on the properties of s.d. $f(\lambda)$, both for deterministic and nondeterministic processes.

It turns out that for nondeterministic processes the asymptotic behavior of $\delta_n(f)$ is determined by the differential properties of s.d. $f(\lambda)$, while for deterministic processes the asymptotic behavior of $\delta_n(f)$ is determined by the geometric properties of s.d. $f(\lambda)$.

We also discuss the inverse problem: the asymptotic behavior of prediction error $\delta_n(f)$ is known, what can be said about the properties of s.d. $f(\lambda)$?

On the divergence of greedy approximation in $L^1(0,1)$ with respect to general Haar systems

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Let $\Psi = \{\psi_k\}_{k=1}^{\infty}$ be a normalized basis in a Banach space X. For any $x \in X$ we have

$$x = \sum_{k=1}^{\infty} c_k(x, \Psi)\psi_k.$$

For each $x \in X$ and $m = 1, 2, \dots$ we define

$$G_m(x) = \sum_{n \in A} c_n(x, \Psi)\psi_n$$

where A is a set of cardinality m such that $min\{|c_n(x, \Psi)| : n \in A\} \geq max\{|c_n(x, \Psi)| : n \notin A\}$. The set A may not be uniquely defined and if this happens we take any such set. The operator $G_m(x)$ is non-linear and discontinuous.

A basis Ψ is called *quasi-greedy* in X if for each $x \in X$ the sequence $G_m(x)$ converges to x in the norm of X. It is known (see [1]) that the Haar system is not quasi-greedy in $L^1(0,1)$.

Theorem 1. *No general Haar system is quasi-greedy basis for $L^1(0,1)$.*

For any $m \in N$ and $x \in X$ we define the X*-greedy* m*-term algorithm* of $x \in X$ with respect to $\{x_n\}_{n=1}^{\infty}$ (the sequence $G_k^{(m)}(x)$) by induction on k (see [2]). We put $G_1^{(m)}(x) = \sum_{i=1}^{m} \hat{\alpha}_i x_{\hat{k}_i}$, where $\hat{\alpha}_i$ and $\hat{k}_i$ $(i = 1, 2, \dots, m)$ are determined to satisfy

$$\|x - \sum_{i=1}^{m} \hat{\alpha}_i x_{\hat{k}_i}\|_X = \inf_{\substack{\alpha_1,\dots,\alpha_m \\ k_1,\dots,k_m}} \|x - \sum_{i=1}^{m} \alpha_i x_{k_i}\|_X.$$

For any $k \geq 1$ we define

$$G_{k+1}^{(m)}(x) = G_k^{(m)}(x) + G_1^{(m)}(x - G_k^{(m)}(x)).$$

The problem of convergence of L^p-greedy 1-term algorithm with respect to the Haar system is posed in [2].

Theorem 2. *For any $m \in N$ there exists a function $f_m \in L^1(0,1)$ such that the L^1-greedy m-term algorithm of f_m with respect to the Haar system does not converge to f_m.*

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Greedy algorithm with respect to the Haar system and modification of functions

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Let $\chi = \{\chi_n\}$ be a Haar system, normalized in $L^1(0,1)$ and let $G_m(x)$ be a m-th greedy approximant of element $x \in L^1(0,1)$ with respect to the χ. It is known that the Haar system is not quasi-greedy basis in $L^1(0,1)$ i. e. for any $C > 0$ there exists $m \in N$ and $x \in L^1(0,1)$ such that

$$\|G_m(x)\|_1 > C\|x\|_1.$$

The following result holds

Theorem 1. For each $0 < \epsilon < 1$ there exists a measurable set $E \subset [0,1]$ with $| E |> 1-\epsilon$ such that to every $f \in L^1(0,1)$ there corresponds a function $\tilde{f} \in L^1(0,1)$, that coincides with f on E such that the elements of expansion

$\sum_{n=1}^{\infty} c_n(\tilde{f})\chi_n$ can be rearranged so that for any integer m

$$\|\sum_{k=1}^{m} c_{\varrho(k)}(\tilde{f})\chi_{\varrho(k)}\| \ \leq \ 4\|\tilde{f}\| \leq 12\|f\| \qquad c_{\varrho(m)}(\tilde{f}) \ > \ c_{\varrho(m+1)}(\tilde{f}).$$

Theorem 2. For each subset $E \subset [0,1]$ with $0 <\mid E \mid< 1$ there exists a function $f_0 \in L^1(0,1)$ such that if the function $f \in L^1(0,1)$ coincides with f_0 on E, then the sequence $\{\mid c_n(f) \mid\}_{n=1}^{\infty}$ may not be decreasing.

Theorem 3. For each $0 < \epsilon < 1$ there exists a subset $E \subset [0,1]$ with $\mid E \mid> 1-\epsilon$ such that for each function $f \in L^1(0,1)$ one can find a function $\tilde{f} \in L^1(0,1)$ coincides with f on E such that all non-zero components in the sequence $\{c_n(\tilde{f})\}_{n=1}^{\infty}$ are possed in the decreasing order.

Up to the boundary upper semi-continuity of $|\nabla u|$ for solutions to parabolic equations

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Let D be a given bounded domain in $\mathbb{R}^{n+1}$ and Q be a parabolic cylinder with center at $(x^0,t^0) \in \partial D$. Suppose $u(x,t)$ is a solution to the following boundary value problem

$$\begin{cases} \mathcal{L}u = f & \text{in } D, \\ u \geq 0 & \text{in } D \cap Q, \\ u = 0 & \text{on } \partial D \cap Q, \end{cases}$$

where $\mathcal{L}$ is a parabolic operator, and f a non-negative bounded function D. In this lecture we will prove, that if $\partial D \cap Q$ has the "exterior $C_x^{1,\mathrm{Dini}} \cap C_t^{1/2,\mathrm{Dini}}$ condition" then $|\nabla u|$ is upper semi-continuous at (x^0,t^0).

On Bezout's Theorem and Nullstellensatz

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We consider the fundamental theorem of algebra (MFTA) presented in [1,2]. Two important consequences of MFTA are derived (announced in [3]). The first one is the Bezout theorem for n polynomials from $k[x_1, \dots, x_n]$, where k is an algebraically closed field. Notably the intersection multiplicities are very transparent and simple. They are characterized by means of partial differential operators given by polynomials from D-invariant linear spaces. The second consequence is a membership characterization for polynomial ideals in $k[x_1, \dots, x_n]$ based on above mentioned multiplicities. One gets Nullstellensatz, with an additional statement concerning the power, from here. The bivariate case of these results is presented in [4].

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On the Ambarzumyan's Theorem for Dirac Operator

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If we denote by $\lambda_n(q,\alpha,\beta), n = 0,1,2,...$ the eigenvalues of Sturm-Liouville problem

$$\begin{array}{ll} -y'' + q(x)y = \lambda y, & x \in (0,\pi), q \in L^1[0,\pi], \lambda \in \mathbb{C} \\ y(0)\cos\alpha + y'(0)\sin\alpha = 0, & \alpha \in (0,\pi], \\ y(\pi)\cos\beta + y'(\pi)\sin\beta = 0, & \beta \in (0,\pi], \end{array}$$

then Ambarzumyan's theorem states that

$$\text{if} \quad \lambda_n(q, \frac{\pi}{2}, \frac{\pi}{2}) = n^2, \ n = 0,1,2,..., \quad \text{then} \quad q(x) = 0 \quad a.e.$$

The question is: is there the analog of Ambarzumyan's theorem for Dirac boundary value problem ($p, q \in L^1[0,\pi], \lambda \in \mathbb{C}$)

$$\begin{cases} \left\{ \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \frac{d}{dx} + \begin{pmatrix} p(x) & q(x) \\ q(x) & -p(x) \end{pmatrix} \right\} y = \lambda y, & x \in (0,\pi), y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \\ y_1(0)\cos\alpha + y_2(0)\sin\alpha = 0, & \alpha \in (-\frac{\pi}{2}, \frac{\pi}{2}], \\ y_1(\pi)\cos\beta + y_2(\pi)\sin\beta = 0, & \beta \in (-\frac{\pi}{2}, \frac{\pi}{2}]? \end{cases}$$

We have some particular answers (by $\lambda_n(p,q,\alpha,\beta), n \in \mathbb{Z}$ we denote the eigenvalues of problem):

1. If $\lambda_n(0,q,\alpha,0) = n - \frac{\alpha}{\pi}, n \in \mathbb{Z}, \quad \alpha \in (-\frac{\pi}{2}, 0) \cup (0, \frac{\pi}{2})$, then $q(x) = 0$ a.e.

2. If $\lambda_n(p,0,\alpha,\frac{\pi}{4}) = n - \frac{\alpha}{\pi} + \frac{1}{4}, n \in \mathbb{Z}, \alpha \neq \frac{\pi}{4}$, then $p(x) = 0$ a.e.

Hilbert's Boundary Problem in the Weighted Spaces

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Let $\rho(t)$ be a measurable nonnegative function on the unite circle $T = \{t : |t| = 1\}$ which will be referred as weight function. We consider the following Hilbert's type problem in the class $L^1(\rho)$: find analytic functions in $D^+ = \{z; |z| < 1\}$ and $D^- = \{z; |z| > 1\}$ satisfying the following boundary condition:

$$\lim_{r\to 1-0} \left\|\Phi^+(rt) - a(t)\Phi^-(r^{-1}t) - f(t)\right\|_{L^1(\rho)} = 0, \qquad (1)$$

where $f(t) \in L^1(\rho)$, $a(t) \in C^\delta(T)$ $(\delta > 0)$, $a(t) \neq 0$.

We say that the point $t_0 \in T$ is singular for the weight function $\rho(t)$, if in every neighborhood $T_0 \subset T$ of t_0 either $\rho(t) \notin L^\infty(T_0)$ or $(\rho(t))^{-1} \notin L^\infty(T_0)$.

Let $t = 1$ be the only singular point of $\rho(t)$. We'll say that the function $\rho(t)$ is regularly varying at the point $t = 1$ from the right, if it can be expressed in the form

$$\rho(e^{i\theta}) = \exp\left(g_1(\theta) + \int_\theta^{\theta_0} \frac{g_2(\lambda)}{\lambda} d\lambda\right), \theta \in (0, \pi],$$

where θ_0 is some point from $(0, \pi]$, g_1 is measurable bounded function on $(0, \pi]$, and $g_2(\phi)$ is continuous function on $[0, \pi]$.

In the same way we can define the class of regularly varying functions at $t = 0$ from the left. Denote

$$\alpha = \sup\left\{\beta; \rho(t)\, |1 - t|^{-\beta} \in L^\infty(T)\right\}.$$

We have proved the following: Let $\alpha \in (-1, 0)$, $\kappa = inda(t) = 0$ and the function $\rho(t)$ is regularly varying from the left and from the right at $t = 1$, then the condition

$$M(\rho)(t) < C\rho(t),$$

where $M(\rho)(t)$ is Hardy-Littlewood maximal function and C is a constant, is necessary and sufficient for the problem (1) to have a solution for any $f(t) \in L^1(\rho)$.

We have considered the problem (1) also in the case of arbitrary α and κ.

On a problem of P.L. Ul'yanov

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In the connection to the fact that in the known theorems of uniqueness of trigonometric series (Valleé-Poussent ets.) the convergence to the finite sum is considered, P.L. Ulýanov posed the following problem: what can be said in the case when convergence to infinity is also allowed (see [1], p. 28).

We have obtained the following

Theorem 1. Let $\alpha_n \downarrow 0$, $(\alpha_n) \notin l^2$ and $\varphi(x)$ is a positive decreasing unbounded function on $(0, 2\pi]$. Then there exists a series

$$a_0 + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$$

such that

1) $\rho_n \equiv \sqrt{a_n^2 + b_n^2} \leq \alpha_n \quad \forall n$

2) the series converges to $f(x)$ everywhere and all its partial sums $S_n(x)$ are everywhere positive

3) $\mu\{x : f(x) > y\} \leq \mu\{x : \varphi(x) > y\} \quad \forall y > 0$, where μ is the Lebesgue measure (in particular $f \in \cap_{p<\infty} L^p$), but the series is not a Fourier series.

Note, that Theorem 1 is true for every rearrangement of the trigonometric series, if the condition 1) is replaced by

1$^{\text{o}}$) $\sum_{k=1}^{n} k \cdot \rho_k = o(n)$.

Theorem 2. If the trigonometric series satisfies to the condition 1^o) and if for every x (maybe except for a countable subset) there exists a subsequence of integers $n_k(x)$ with the lower density not more than 3/4 such that

$$S_{n_k(x)}(x) \to f(x) \in L^\infty,$$

then it is a Fourier series.

Note that in the case when $f \in L^\infty$ is everywhere finite the requirement on the density can be omitted.

Theorem 3. There exist essentally different continuous increasing functions F and Φ, such that their Fourier-Stieltes series (null-series) in every fixed point either both are convergent to 0 or both are $(C, 1)$- summable to $+\infty$.

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Generalized Sobolev–Poincaré Inequality on Metric Measure Spaces

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Let (X, d, μ) be a Hausdorff space with a regular Borel measure μ and a quasimetric d. Suppose that the quasimetric and the measure are connected by the doubling condition of order $\gamma > 0$

$$\mu B(x,s) \le c \left(\frac{s}{r}\right)^{\gamma} \mu B(x,r), \quad x \in X, \ 0 < r \le s.$$

Set

$$S_\eta f(x) = \sup_B \frac{1}{\eta(r)} \fint_B |f - f_B| \, d\mu, \quad f_B = \fint_B f \, d\mu = \frac{1}{\mu B} \int_E f \, d\mu,$$

where sup is taken over all balls B containing the point $x \in X$ (r is the radius of B).

Theorem. *Let $p > 0$, $0 < \alpha < \gamma/p$, $\eta(t)t^{-\alpha} \uparrow$, $\eta(t)t^{-\gamma/p} \downarrow$. If x_0 is the Lebesgue point of the function $f \in L^1_{\text{loc}}(X)$, then*

$$|f(x_0) - f_{B(x_0,r)}| \le c\eta(r) \left[S_\eta f(x_0)\right]^{1-\frac{\alpha p}{\gamma}} \left(\fint_{B(x_0,r)} (S_\eta f)^p \, d\mu \right)^{\alpha/\gamma}, \quad r > 0.$$

Furthermore, for $\frac{1}{q} = \frac{1}{p} - \frac{\alpha}{\gamma}$ and for any ball $B = B(x,r)$

$$\left(\fint_B |f - f_B|^q \, d\mu \right)^{1/q} \le c\eta(r) \left(\fint_{2a_d B} (S_\eta f)^p \, d\mu \right)^{1/p}$$

and for any Lebesgue point $x_0 \in B$ of the function f

$$\left(\fint_B |f - f(x_0)|^q \, d\mu \right)^{1/q} \le c\eta(r) \left[S_\eta f(x_0) + \left(\fint_{2a_d B} (S_\eta f)^p \, d\mu \right)^{1/p} \right].$$

In particular, here we can take $\eta(t) = t^\alpha$, $0 < \alpha < \gamma/p$.

We use such inequalities to obtain generalizations of Sobolev type embedding theorems and "selfimproving" property of Poincaré inequality.

On the conjugate to the general Franklin system

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The classical Franklin system has been used to solve various problems in functional analysis. In particular, S.V.Bochkarev has used it to construct a basis in disc algebra (1974). He has also proved that the trigonometric conjugate to the (periodic) Franklin system is a basis in $C(T)$ (1985).

By a general Franklin system corresponding to a given sequence of knots we mean an orthonormal system consisting of piecewise linear functions with these knots. We have been interested in the following question: how the properties of a general Franklin system depend on the regularity of the corresponding sequence of knots. Here we discuss some necessary and some sufficient conditions for the sequence of knots under which the trigonometric conjugate to the (periodic) general Franklin system is a basis in $C(T)$, or under which Bochkarev's construction applied to a general Franklin system gives a basis in $C(T)$.

Basis of Rectangles and Differentiation of Integrals

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For any number $s \in [0, \frac{\pi}{2})$ we define $\mathcal{R}_s$ to be the set of all rectangles R in $\mathbb{R}^2$ having slope s, i.e. R has a side forming angle s with the vector $(1, 0)$. We say a basis $\mathcal{R}_s$ differentiates the integral of the function $f \in L(\mathbb{R}^2)$, if

$$\lim_{d(R)\to 0, x\in R\in\mathcal{R}_s} \frac{1}{|R|}\int_R f = f(x) \tag{1}$$

almost everywhere in $\mathbb{R}^2$. By the classical theorem of Jessen-Marcinkiewicz-Zygmund $\mathcal{R}_s$ differentiates the integrals of the functions from the class

$L \log L(\mathbb{R}^2)$. On the other hand S. Saks proved that the same is not true for the class $L(\mathbb{R}^2)$. In view of this A. Zygmund posed the following problem: given $f \in L(\mathbb{R}^2)$, is it possible to find a direction s such that $\mathcal{R}_s$ differentiates $\int f$? J. Marstrand in [1] gave a negative answer to this question.
We say a set $S \subset [0, \frac{\pi}{2})$ is differentiation set (DF-set), if for some $f \in L(\mathbb{R}^2)$ $\mathcal{R}_s$ differentiates the integral $\int f$ as $s \in S$, and (1) doesn't hold a.e. as $s \notin S$. So J. Marstrand's theorem says that the empty set is DF-set. In G.Lepsveridze [2] it is proved, that any finite set is DF-set. No other DF-sets were known. The following theorem gives a complete characterization of DF-sets.

Theorem 1 *A set $S \subset [0, \pi/2)$ is differentiation set if and only if it is F_σ-set.*

We study also the maximal operators

$$M_\Omega f(x) = \sup_{\delta>0,\, \omega\in\Omega} \frac{1}{2\delta} \int_{-\delta}^{\delta} |f(x+t\omega)| dt$$

where $\Omega \subset [0, \pi/2)$ is a set of slopes. Developing the method used in G.A.Karagulyan and M.Lacey [3] we proved

Theorem 2 *If $\Omega \subset [0, \pi/2)$ is μ-lacunary, then $\|M_\Omega f(x)\|_2 \lesssim \sqrt{\mu}\|f\|_2$.*

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Weighted $\overline{\partial}$–Integral Representations of Smooth Functions in the Unit Ball

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For C^1–functions f given on the closure of the unit ball B_n of the space C^n the family of integral representations is obtained:

$$f(z) = \int_{B_n} f(w)K(z,w)dm(w) - \int_{B_n} < (\overline{\partial} f)(w), w - z > T(z,w)dm(w),$$

where kernels K and T depend on several parameters . For special values of these parameters well-known integral representations are obtained.

Analytic weighted spaces of functions of bounded mean oscillation on the unit disc

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We introduce the weighted space $\mathrm{BMO}^1_\lambda(\mathbb{D})$ on the unit disc $\mathbb{D}$ in $\mathbb{C}$ as a set of measurable on $\mathbb{D}$ functions satisfying $\sup_{z\in\mathbb{D}} \|\varphi\circ\alpha_z(\cdot) - \widetilde{\varphi}_\lambda(z)\|_{L^1_\lambda(\mathbb{D})} < \infty$. Here $w \to \alpha_z(w)$ denotes the Moebius transform of the unit disc to itself that maps $w = 0$ to $w = z$, $\widetilde{\varphi}_\lambda(z)$ - stands for the Beresin transform of the function φ, $L^1_\lambda(\mathbb{D}) = \{f : \|f\|_{L^1_\lambda(\mathbb{D})} = \int_{\mathbb{D}} |f(w)|d\mu_\lambda(w)\}$, where $d\mu_\lambda(z) = (\lambda+1)(1-|z|^2)^\lambda d\mu(z)$, $d\mu(z) = \frac{1}{\pi}dxdy$, $\lambda > -1$.

We give the description of the spaces $\mathrm{BMO}^1_\lambda(\mathbb{D})$ in terms of the Beresin transform. The usage of the Beresin transform in the definition and the description of $\mathrm{BMO}^1_\lambda(\mathbb{D})$ is quite natural from the viewpoint of the complex analysis. However, we also provide description of these spaces in terms of means over the discs $D(z,r) = \{w \in \mathbb{D} : \beta(z,w) < r\} \subset \mathbb{D}$ in the hyperbolic Bergman metric: $\beta(z,w) = \frac{1}{2}\ln\frac{1+|\alpha_z(w)|}{1-|\alpha_z(w)|}$. Finally, we apply the mentioned descriptions to the study of compactness of Toeplitz operators with $\mathrm{BMO}^1_\lambda(\mathbb{D})$ - symbols acting on weighted Bergman spaces on the unit disc.

Boundary Behavior Estimates for Differentiable Functions

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Let $p > 0$, $m \in \mathbb{N}$, $mp \geq n-1$,$n \geq 2$, $l = \left[m - \frac{n-1}{p}\right]$ ($[a]$ is the integer part of a).

Let also $\Omega \subset \mathbb{R}^n$ be a bounded domain of the class C^1 and

$$\mathcal{N}v(x) = \sup\{|v(y)| : y \in \Omega,\ |x-y| < \mathrm{dist}(y, \partial\Omega)\}$$

be the nontangential maximal function.

For the function $u \in C^m(\Omega)$ with $\mathcal{N}(\nabla^m u) \in L^p(\partial\Omega)$ we have:

1) if $l = m - \frac{n-1}{p}$, then

$$\|\mathcal{M}(\nabla^l u)\|_{L^p(\partial\Omega)} \leq c\left(\|\mathcal{N}(\nabla^m u)\|_{L^p(\partial\Omega)} + \sum_{k=l}^{m} |\nabla^k u(x_0)|\right),$$

where

$$\mathcal{M}v(x) = \sup\left\{|v(y)| : y \in \Omega,\ |x-y| < a\left(\ln\frac{2\mathrm{diam}\Omega}{\mathrm{dist}(y,\partial\Omega)}\right)^{-\frac{n-2}{n-1}}\right\}$$

($x \in \partial\Omega$);

2) if $l < m - \frac{n-1}{p}$, then for every multiindex $\nu = (\nu_1, \ldots, \nu_n)$ with $\sum_{k=1}^n \nu_k = l$

$$D^\nu u = \frac{\partial^l u}{\partial x_1^{\nu_1} \ldots \partial x_n^{\nu_n}} \in \mathrm{Lip}\left(m - l - \frac{n-1}{p}\right)$$

and

$$|D^\nu u(x) - D^\nu u(y)| \leq c \left(\|\mathcal{N}(\nabla^m u)\|_{L^p(\partial\Omega)} + \sum_{k=l+1}^{m} |\nabla^k u(x_0)| \right) |x-y|^{m-l-\frac{n-1}{p}}$$

($x_0 \in \Omega$ is a fixed point).

These propositions improve some results from [1–2]: in the case $m = 1$ (then $l = 0$) they were obtained in [1], while the case $mp < n - 1$ for any m was considered in [2].

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Some applications of Approximation Theory in non-parametric statistic

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These last years, lot of progress in non linear statistic (denoising of a signal stochastically observed, image restoration, density estimation...) have been done, using results in non linear approximation theory. In this talk, I will try to explain how the representation of a function in a suitable basis, with respect to which a good theory of approximation is available, could help to solve non parametric statistical problems.

On general periodic Franklin systems

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By general periodic Franklin system corresponding to a dense sequence of knots $\mathcal{T} = (t_n, n \geq 0)$ from $[0,1]$ we mean a sequence of orthonormal piecewise linear,continuous and periodic functions with knots $\mathcal{T}$, that is, n-th function f_n from the system has knots $t_0, \ldots, t_n$ and $f_n(0) = f_n(1)$. The main results are

Theorem 1 *Let $\mathcal{T}$ be an admissible sequence of knots and $\{f_n, n \geq 1\}$ be the corresponding general periodic Franklin system. Then there exists a constant $C > 0$, such that for every $f \in L^1[0,1]$*

$$S_n f(x) \leq C\tilde{\mathcal{M}} f(x), \textit{for all } x \in [0,1] \textit{ and } n = 1, 2, ...,$$

where $S_n f(x)$ is the n-th partial sum of Fourier-Franklin expansion of f. Here $\tilde{\mathcal{M}} f(x)$ is the maximal function defined by

$$\tilde{\mathcal{M}} f(x) = \sup \frac{1}{|I|} \int_I |f(t)| dt,$$

where sup *is taken over all generalized intervals containing x.*

Theorem 2 *Let $\mathcal{T}$ be an admissible sequence of knots. Then the corresponding general periodic Franklin system is an unconditional basis in $L^p[0,1]$, $1 < p < \infty$.*

Theorem 3 *Let $\mathcal{T}$ be an admissible sequence of knots. Then the corresponding general periodic Franklin system is a Greedy basis in $L^p[0,1]$, $1 < p < \infty$.*

On Structure of Solution of one Integral-Differential Equation with Completely Monotonic Kernel

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The following integral-differential equation

$$\frac{df}{dx} + c\int_0^{+\infty} K(x-t)f(t)dt = 0, \quad c>0, \quad x\in(0,+\infty), \tag{1}$$

with condition

$$f(+\infty) \equiv \lim_{x\to+\infty} f(x) = 0 \tag{2}$$

is considered, where

$$K(x) = \int_a^b e^{-|x|s} d\sigma(s) \tag{3}$$

Here $\sigma(s)$-is monotonic non-decreasing function on $[a,b)$ $(+\infty \geq b > a > 0)$, *s.t.*

$$\int_{-\infty}^{+\infty} K(x)dx = 2\int_a^b \frac{d\sigma(s)}{s} = 1 \tag{4}$$

We denote $c_0 = \sup\limits_{\alpha\in(0,a)} \frac{\alpha}{\int_a^b \frac{2s}{s^2-\alpha^2} d\sigma(s)}$.

Theorem 1 *For $0 < c \leq c_0$ the problem (1)-(4) has a non trivial solution in the Sobolev's space $W_1^1(0,+\infty)$. Moreover,*

a) if $0 < c < c_0$, then $f(x) = e^{-\alpha_1 x}[1 + \int_0^{+\infty} \frac{1-e^{-sx}}{s} d\rho(s)]$, where $\alpha_1 \in (0,a)$, $\rho -$ is non negative measure on $(0,+\infty)$ s.t. $\int_0^{+\infty} \frac{d\rho(s)}{s} < +\infty$,

b) if $c = c_0$, then $f(x) = e^{-\alpha_2 x}[1 + mx + \int_0^{+\infty} \frac{1-e^{-sx}}{s} d\rho(s)]$, where $\alpha_2 \in (0,a)$, $m \geq 0$.

Theorem 2 *If $c > c_0$ and there exists $\alpha \in (0, a)$, for which the function*

$$u(z) = 1 - c(2(z+\alpha)\int\limits_a^b \frac{d\sigma(s)}{s^2(s^2-(z+\alpha)^2)} + \frac{1}{z+\alpha})$$

has a complex root z_0, s.t. $a-\alpha \geq Rez_0 > 0$, $Imz_0 > 0$, then the problem (1)-(4) in Sobolev's space $W_1^1(0, +\infty)$ has non trivial solution

$$f(x) = e^{-\alpha x}[2Ae^{-\beta x}\cos(\omega x - \theta) - \int\limits_0^{+\infty} e^{-xs} d\mu(s)],$$

where $\beta = Rez_0$, $\omega = Imz_0$, $Ae^{i\theta} = (u'(z_0))^{-1}$, $\mu(s)$ - is the finite measure and continuous at 0.

Fourier expansion of differential operator by means of eigenfunction

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We consider an $m \geqslant 2$ order ordinary linear self-adjoint differential operator $\mathcal{L}$ in $L^2(\mathbb{R})$, generated by the differential operation ℓ:

$$\ell(y) = \frac{1}{i^m} y^{(m)} + \sum_{k=0}^{n-1} \frac{1}{i^{2k}} \left(p_{2k} y^{(k)}\right)^{(k)} +$$

$$+ \sum_{k=0}^{n-1} \frac{1}{2i^{2k+1}} \left\{ \left(p_{2k+1} y^{(k)}\right)^{(k+1)} + \left(p_{2k+1} y^{(k+1)}\right)^{(k)} \right\},$$

where y is a function defined on $\mathbb{R}$, i is the imaginary unit, $n = \left[\frac{m}{2}\right]$, $n' = \left[\frac{m-1}{2}\right]$ and p_k is a real measurable function on $\mathbb{R}$, satisfying

$$\int\limits_{-\infty}^{0} |p_k(x) - a_k^-|\, dx + \int\limits_0^{+\infty} |p_k(x) - a_k^+|\, dx < \infty, \qquad k = 0, 1, \ldots, m-2,$$

with some real numbers $a_k^{\pm}$. We consider polynomials $Q^{\pm}(\lambda) = \lambda^m + \sum_{k=0}^{m-2} a_k^{\pm}\lambda^k$, $\lambda \in C$. We denote by $\varkappa$ the number of such values $\mu \in \mathbb{R}$ for which one of the equations $Q^{+}(\lambda) = \mu$ and $Q^{-}(\lambda) = \mu$ has a multiple real root, and if $\varkappa \neq 0$ we denote these values by $\mu_1 < \mu_2 < \ldots < \mu_\varkappa$. Let also $\mu_0 = -\infty$, $\mu_{\varkappa+1} = \infty$ and $\delta = 0$ for odd m and $\delta = 1$ for even m. For each $k = \delta, \delta+1, \ldots, \varkappa$ for $\mu \in (\mu_k, \mu_{k+1})$ The number of real roots of equations $Q^{\pm}(\lambda) = \mu$ is constant and denoted by $2r_k^{\pm} + 1 - \delta$. We prove that if $k = \delta, \delta+1, \ldots, \varkappa$ is not an eigenvalue of $\mathcal{L}$, then the differential equation $\ell(y) = \mu_y$ has $r_k^{+} + r_k^{-} + 1 - \delta$ linear independent bounded solutions $\varphi_i(x,\mu)$, $j = \delta, \delta+1, \ldots, r_k^{+} + r_k^{-}$, and under some normalization the following theorem holds.

Theorem. *For every function $f \in L^2(\mathbb{R})$ the Fourier expansion is true*

$$f(x) = \sum_j \psi_j(x) \int_{-\infty}^{\infty} f(t)\overline{\psi_j(t)}\,dt + \sum_{k=\delta}^{\varkappa} \sum_{j=\delta}^{r_k^{+}+r_k^{-}} \int_{\mu_k}^{\mu_{k+1}} \Phi_j(\mu)\varphi_j(x,\mu)\,d\mu, \quad x \in \mathbb{R}, \tag{1}$$

and the Parseval's equality holds

$$\int_{-\infty}^{\infty} |f(x)|^2\,dx = \sum_j \left| \int_{-\infty}^{\infty} f(t)\overline{\psi_j(t)}\,dt \right|^2 + \sum_{k=\delta}^{\varkappa} \sum_{j=\delta}^{r_k^{+}+r_k^{-}} \int_{\mu_k}^{\mu_{k+1}} |\Phi_j(\mu)|^2\,d\mu, \tag{2}$$

where $\{\psi_j(x)\}$ is the orthonormal system of all eigenfunctions of the operator $\mathcal{L}$ and

$$\Phi_j(\mu) = \int_{-\infty}^{\infty} f(t)\overline{\psi_j(t)}\,dt, \quad \mu_k < \mu < \mu_{k+1}, \quad \delta \leqslant j \leqslant r_k^{+} + r_k^{-} + 1 - \delta. \tag{3}$$

Moreover, the second integral in (1) and the integral (3) converge by norm of the space $L^2(\mathbb{R})$ and $L^2(\mu_k, \mu_{k+1})$ respectively.

Mixed norms and Sobolev-type inequalities

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We study mixed norm spaces that arise in connection with embeddings of Sobolev and Besov spaces. We prove Sobolev type inequalities in terms of these mixed norms. Applying these results, we obtain optimal constants in embedding theorems for anisotropic Besov spaces. This gives an extension of the estimate proved by Bourgain, Brezis and Mironescu for isotropic Besov spaces.

Convergent subsequences of partial sums of Fourier series of integrable functions

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Let $\mathbf{T} = \mathbf{R}/\mathbf{2}\pi\mathbf{Z}$, $L(\mathbf{T})$ be the set of all integrable functions $f : \mathbf{T} \to \mathbf{C}$. We associate with a function $f \in L(\mathbf{T})$ its trigonometric Fourier series

$$f \sim \sum_{k=-\infty}^{\infty} \hat{f}(k) e^{ikx}, \quad \hat{f}(k) = \frac{1}{2\pi} \int_{\mathbf{T}} f(x) e^{-ikx} dx.$$

For $n \in \mathbf{N}$ define the n-th partial sum of f as

$$S_n(f;x) = \sum_{k=-n}^{n} \hat{f}_k e^{ikx}.$$

Let $\varphi : [0,\infty) \to [0,\infty)$ be a nondecreasing function. Denote

$$\varphi(L) = \left\{ f \in L(\mathbf{T}) : \int_{\mathbf{T}} \varphi(|\mathbf{f}(\mathbf{x})|) \mathbf{dx} < \infty \right\}.$$

If the Fourier series of f is a series of power type, that is $\hat{f}(k) = 0$ for $k < 0$ and $\inf_j m_{j+1}/m_j > 1$, then the partial sums $\{S_{m_j}(f, x)\}$ converge to $f(x)$ almost everywhere. Hence, for these partial sums and for any integrable function f with

$$\int_{\mathbf{T}} |f(x)| \log(1 + |f(x)|) dx < \infty$$

the partial sums $\{S_{m_j}(f, x)\}$ converge to $f(x)$ almost everywhere [1 , chapter 15]. V. Totik [2] proved that for any increasing sequence of positive integers $\{m_j\}$ there is an integrable function f such that

$$\forall x \in [0, 2\pi] \quad \sup_j |S_{m_j}(f; x)| = \infty. \qquad (1)$$

Theorem. *For any increasing sequence of positive integers* $\{m_j\}$ *and for any function* φ such that $\varphi(u) = o(u \log \log u)$ *as* $u \to \infty$ *there exists a function* $f \in \varphi(L)$ *satisfying (1).*

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Stability Principles and Approximation Problems in Variational Calculus

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An important field for the application of stability principles of optimization theory is the calculus of variations. In the context of control theory, the problem of establishing the continuity of the control by using Pontrjagin's maximum principle - leading to a value of the control for fixed t - reduces to a stability question for finite-dimensional optimization. In our approach of pointwise minimization of the Lagrangian, we employ a simultaneous optimization w.r.t. both state and control variables. If in addition the Legendre - Riccati condition is satisfied, a condition that allows us to identify an equivalent convexified variational problem, we can apply corresponding stability principles. This approach also provides an elementary access to the fundamental theorems of variational calculus.

As applications we consider a problem of modular approximation and a parameter-free approximation of time-series data by monotone functions. For given time-series data we present a method, based on variational calculus, to determine a smooth monotone function that approximates these data (together with their derivative) in the least squares sense.

Inequalities for the Sharp-Maximal Function in Metric Spaces

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Let (X, d, μ) be a Hausdorff space with a regular Borel measure μ and a quasimetric d (the triangle inequality is replaced by the condition: there exists a constant $a_d \geq 1$ such that $d(x,y) \leq a_d[d(x,z) + d(z,y)]$).

Suppose that for any ball $B \subset X$ the map $P_B : L^1_{\mu,\mathrm{loc}}(X) \mapsto L^1_{\mu,\mathrm{loc}}(X)$ is given, and set

$$\mathcal{S}_\eta f(x) = \sup_B \frac{1}{\eta(r)} \fint_B |f - P_B f| \, d\mu, \quad f_B = \fint_B f \, d\mu = \frac{1}{\mu B} \int_E f \, d\mu,$$

where sup is taken over all balls B containing the point $x \in X$ (r is the radius of B).

Theorem. *Let* $q \geq p > 0$, $\gamma \geq \delta > 0$, *a measure* μ *and an outer measure* ν *satisfy conditions*

$$\mu(B(x,t)) \geq ct^{\gamma}, \ \ \nu(B(x,At)) \leq ct^{\delta-\gamma}\mu(B(x,t))$$

for some $A > 4a_d^2$. *Let also* $\sigma : \mathbb{R}_+ \mapsto \mathbb{R}_+$ *and* $\eta(t) = t^{\frac{\gamma}{p}-\frac{\delta}{q}}\sigma(t)$. *Then the following inequality holds*

$$\|\mathcal{S}_\sigma f\|_{L^q_\nu(X)} \leq c\|\mathcal{S}_\eta f\|_{L^p_\mu(X)}, \ \ f \in L^1_{\mu,\mathrm{loc}}(X).$$

This result have the series of applications to generalized Sobolev classes

$$W^p_\alpha(X) = \{f \in L^p(X) : \mathcal{S}_\eta f \in L^p(X)\},$$

(here $\eta(t) = t^\alpha$ and $P_B f = f_B$ in the definition of $\mathcal{S}_\eta$) on a metric-measure space and the corresponding capacity $\mathrm{Cap}_{\alpha,p}$.

Corollary. *If* $f \in W^p_\alpha(X)$, $1 < p \leq q$, $0 < \beta < \alpha \leq 1$, $\delta = \alpha - \beta - \gamma\left(\frac{1}{p} - \frac{1}{q}\right) > 0$, *then*

1) for $\mathrm{Cap}_{\delta,q}$*-almost all* $x \in X$

$$\lim_{t\to+0} t^{-\beta}\left[f(x) - \fint_{B(x,t)} f\,d\mu\right] = 0,$$

2) for any $\varepsilon > 0$ *there exists a constant* $C_\varepsilon > 0$ *and a set* $E_\varepsilon \subset X$ *such that* $\mathrm{Cap}_{\delta,q}(E_\varepsilon) < \varepsilon$ *and*

$$|f(x) - f(y)| \leq C_\varepsilon d^\beta(x,y), \ \ x,y \notin E_\varepsilon.$$

Entropy numbers of embeddings of weighted Besov spaces

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Let X, Y be quasi-Banach spaces, and let $T : X \to Y$ be a bounded linear operator. Then the k^{th} (dyadic) entropy number of T is defined as

$$e_k(T) = \inf\{\varepsilon > 0 : T(B_X) \text{ can be covered by } 2^{k-1} \text{ balls in } Y \text{ of radius } \varepsilon\},$$

where B_X denotes the closed unit ball in X. Clearly,

$$T \text{ is compact} \quad \text{if and only if} \quad \lim_{k\to\infty} e_k(T) = 0\,.$$

Thus the sequence $(e_k(T))_{k=1}^{\infty}$ can be used to quantify the notion of compactness. Moreover, entropy numbers are closely related to eigenvalues; this is the basis for applications to spectral theory.

In the talk we consider weighted Besov spaces

$$B^s_{p,q}(\mathbb{R}^d, w) := \{f \in \mathcal{S}'(\mathbb{R}^d) : fw \in B^s_{p,q}(\mathbb{R}^d)\}\,,$$

where $B^s_{p,q}(\mathbb{R}^d)$ stands for the usual Besov spaces (with $0 < p,q \le \infty$ and $s \in \mathbb{R}$), and the weights satisfy certain regularity conditions. We give necessary and sufficient conditions for the compactness of the embedding

$$B^{s_1}_{p_1,\,q_1}(\mathbb{R}^d, w_1) \hookrightarrow B^{s_2}_{p_2,\,q_2}(\mathbb{R}^d, w_2)$$

in the cases where the ratio $w_1(x)/w_2(x)$ is of polynomial or logarithmic type. Moreover, asymptotically sharp bounds for the entropy numbers of these embeddings will be established. Compared to the unweighted case, some new phenomena occur. This is joint work with H.-G. Leopold (Jena), W. Sickel (Jena) and L. Skrzypczak (Poznań).

Nehari Theorem on the Polydisk

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The classical Nehari theorem characterizing the bounded Hankel operators, is extended to the polydisk, where it is a much more difficult result. The classical Nehari theorem is related to factorization, commutators, BMO, and the div curl lemma. Some of these relationships extend to the polydisk. Others are being investigated. I will recall the Nehari theorem, outline a proof, and outline the proof in the polydisk. This is joint work with Sarah Ferguson and Erin Terwilleger.

On the Fourier transform of the indicator of a domain with C^1 -smooth boundary

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Suppose D is a bounded domain in $\mathbb{R}^n$ and 1_D the indicator function of D, i.e. the function whose value on D is 1, and whose value on the complement of D is 0. Consider the Fourier transform $\widehat{1_D}$ of 1_D. It is well known that if the boundary ∂D of D is C^2 -smooth, then $\widehat{1_D} \in L^p(\mathbb{R}^n)$ if and only if $p > \frac{2n}{n+1}$. We study the case of C^1 -smooth boundary. Note that generally there is no critical value of p for this case, namely, it is not difficult to construct a domain $D \subset \mathbb{R}^2$ with $\partial D \in C^1$, satisfying $\widehat{1_D} \in L^p(\mathbb{R}^2)$ for all $p > 1$. At the same time we show that if D is a domain in $\mathbb{R}^n$ with $\partial D \in C^{1+\alpha}$, where $0 < \alpha \leq 1$, then $\widehat{1_D} \notin L^p(\mathbb{R}^n)$ for $p \leq 1 + \frac{\alpha(n-1)}{n+\alpha}$. For $n = 2$ this result is sharp, namely, for any α, $0 < \alpha \leq 1$, there is a domain $D \subset \mathbb{R}^2$, with $\partial D \in C^{1+\alpha}$, such that $\widehat{1_D} \in L^p(\mathbb{R}^2)$ for all $p > 1 + \frac{\alpha}{2+\alpha}$.

Actually we consider the general case of $C^{1,\,\omega}$ -smooth boundary, where ω is a given modulus of continuity. The above results are the particular cases of $\omega(\delta) = \delta^\alpha$. Let $\nu_D(x)$ be the outer unit vector, normal to ∂D at a point $x \in \partial D$. Thus we have the mapping ν_D of ∂D onto the unit sphere S^{n-1}. We write $\partial D \in C^{1,\,\omega}$ if the modulus of continuity of ν_D is $O(\omega(\delta))$ as $\delta \to +0$. We show that if $\partial D \in C^{1,\,\omega}$ and

$$\int_0^1 \frac{\delta^{n(p-1)-1}}{\omega(\delta)^{n-p}} d\delta = \infty,$$

then $\widehat{1_D} \notin L^p(\mathbb{R}^n)$. For $n = 2$ this result is sharp, namely, given a modulus of continuity ω, there is a domain $D \in \mathbb{R}^2$, with $\partial D \in C^{1,\,\omega}$, such that for any p, satisfying

$$\int_0^1 \frac{\delta^{2p-3}}{\omega(\delta)^{2-p}} d\delta < \infty,$$

we have $\widehat{1_D} \in L^p(\mathbb{R}^2)$.

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Multidimensional Hausdorff operators on the real Hardy space

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A wide range of multidimensional operators of Hausdorff type is introduced as continuations of the preceding study, joint with F. Moricz, of one-dimensional Hausdorff operators. A sufficient condition is found for belonging of these operators to the classical real Hardy space. Further extensions are discussed as well as open problems. This is a joint work with A. Lerner.

Boundary-regular functions which are not analytically continuable

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Suppose that G is a domain in the complex plane. We deal with the problem whether there exist holomorphic functions f on G satisfying several properties simultaneously:

- The boundary of G is the natural boundary for the function f.
- All derivatives of f have a continuous extension up to the boundary of G.
- The Taylor expansion of f around a fixed center $\zeta \in G$ is lacunary with gaps of a prescribed density.
- The function f satisfies several universal properties.

(joint paper with M. C. Calderon-Moreno and L. Bernal-Gonzalez).

Approximation by Neural Networks and Learning Theory

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We consider the problem of Learning Neural Networks from samples. The sample size which is sufficient for obtaining the almost-optimal stochastic approximation of function classes is obtained. In the terms of the accuracy confidence function we show that the least square estimator is optimal for the problem. These results can be used to solve Smale's network problem.

q-Bounded Systems: Common Approach to Fisher-Micchelli's and Bernstein-Walsh's Type Problems

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We consider two types of fast approximation problems named Fisher-Micchelli's and Bernstein-Walsh's type problems respectively (F-M and B-

W type problems):

1) For a given space H to find a system $\{e_k\}_{k=1}^{\infty}$ such that each element of H admits the fast approximation by linear combinations $\sum_{k=1}^{n} a_k^{(n)} e_k$.

2) For a given system $\{e_k\}_{k=1}^{\infty}$ to find the elements permitting the fast approximation by linear combinations $\sum_{k=1}^{n} a_k^{(n)} e_k$.

We have developed a method to investigate above problems using the systems named q-bounded:

Definition 1 *Let X be a Banach space, $\left(e_k^{(n)}\right)_{k\leqslant n} \subset X,\ n = 1, 2, \dots$ be a triangle matrix and L_n be the linear span of the finite system $\left\{e_k^{(n)}\right\}_{k=1}^{n}$ in X. Let q be a positive number. The matrix $\left(e_k^{(n)}\right)_{k\leqslant n}$ is called*

i. *q-lower bounded in X, if for each sequence $P_n = \sum_{k=1}^{n} a_k^{(n)} e_k^{(n)} \in L_n$ holds*

$$\limsup_{n\to\infty} \sqrt[n]{\|P_n\|_X} \geqslant q \limsup_{n\to\infty} \sqrt[n]{\max_{1\leqslant k\leqslant n} \left|a_k^{(n)}\right|}.$$

ii. *q-upper bounded in X, if*

$$\limsup_{n\to\infty} \sqrt[n]{\max_{1\leqslant k\leqslant n} \left\|e_k^{(n)}\right\|_X} \leqslant q.$$

The system $\{e_k\}_{k=1}^{\infty} \subset X$ is called q-lower(upper) bounded, if the matrix $\left(e_k^{(n)}\right)_{k\leqslant n}$ is q-lower(upper) bounded for $e_k^{(n)} := e_k,\ k \leqslant n$. The system is called 0-lower bounded (∞-upper bounded), if it is q-lower(upper) bounded for some $q \in (0, \infty)$.

One can use the biorthogonal systems as a technique to check the q-boundedness. The following lemmas show the close relation between q-bounded systems and F-M and B-W type problems.

Lemma 1 *Let H, X be Banach spaces, $H \subset X$, B_H be the unit ball of H. If there is a matrix $\left(\varphi_k^{(n)}\right)_{k\leqslant n} \subset H$, which is q_1-lower bounded in X and*

q_2-upper bounded in H then

$$\limsup_{n\to\infty} \sqrt[n]{d_n\left(B_H, X\right)} \geqslant \frac{q_1}{q_2},$$

where $d_n\left(B_H, X\right)$ is the Kolmogorov n-width.

Lemma 2 *Let $\{e_k\}_{k=1}^{\infty}$ be a 0-lower and ∞-upper bounded system in a Banach space X and $x \in X$. There are polynomials $P_n = \sum_{k=1}^{n} a_k^{(n)} e_k$ satisfying $\sqrt[n]{\|x - P_n\|} \underset{n\to\infty}{\longrightarrow} 0$ if and only if $x = \sum_{k=1}^{\infty} x_k e_k$, where $\sqrt[k]{|x_k|} \underset{k\to\infty}{\longrightarrow} 0$.*

By verifying q-boundedness of $e_k(z) = z^k, k = 0, 1, \ldots$ in view of Lemmas 1 and 2 we get the results of Fisher-Micchelli and Bernstein-Walsh.

Taking exponential or rational functions as a q-bounded systems in Lebesgue and Hardy's spaces we obtain new F-M and B-W type results.

The tangential touch between fixed and free boundaries for the two-phase obstacle-like problem

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We consider the following two-phase obstacle-problem-like equation in the unit half-ball

$$\Delta u = \lambda_+ \chi_{\{u>0\}} - \lambda_- \chi_{\{u<0\}}, \; \lambda_\pm > 0.$$

We prove that the free boundary touches the fixed boundary (uniformly) tangentially if the boundary data f and its first and second derivatives vanish at the touch-point.

An algorithm for calculation of the prediction error for some stationary stochastic processes

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In this talk we will describe an algorithm for exact calculation of the prediction error for stationary stochastic processes with discrete time and with spectral density of the form $\left|\frac{p(z)}{q(z)}\right|^2$ where $p(z)$ and $q(z)$ are arbitrary polynomials.

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Lacunary polynomial approximation on compact sets in the complex plane

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For a compact plane set K let $A(K)$ denote the Banach space of all functions continuous on K and holomorphic in the interior of K, endowed with the uniform norm. The famous Mergelian's theorem states that the polynomials are dense in $A(K)$ if and only if K has connected complement.
If 0 is not on interior point of K, one may ask for subsets Λ of the non-negative integers having the property that the linear span of the powers z^ν ($\nu \in \Lambda$) is still dense in $A(K)$. The question has a long tradition and many contributions to various aspects have been given. We add one more. In particular, we characterize the sets Λ having the property that the linear span is dense for all K with $0 \notin K$ (and connected complement).

On Stability for Families of Nonlinear Equations

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A stability principle concerning solutions of sequences of nonlinear equations for continuous operators on $\mathbb{R}^n$ is described that can be applied to a wide class of operators for which point-wise convergence already implies continuous convergence, in particular to sequences of monotone operators.

For sequences of convex functions, equicontinuity and hence continuous convergence already follows from pointwise convergence. In this treatise we will show that a similar statement holds for sequences of monotone operators. Continuous convergence in turn implies stability of solutions under certain conditions on the limiting problem.

Both, solutions of equations and approximation problems, can be treated in the framework of variational inequalities. In fact, sequences of variational inequalities show a similar stability behaviour, where again continuous convergence is of central significance. We employ this scheme in the context of two-stage solutions.

Stability questions for minimal solutions of point-wise convergent sequences of convex functions have been treated in a number of publications. It turns out that stability can be guaranteed if the set of minimal solutions of the limit problem is bounded. As an application, ill posed optimization problems are replaced by sequences of (numerically) well posed problems. The question arises, whether a corresponding statement holds on the equation level for certain classes of mappings that are not necessarily potential operators. Questions of this type arise e.g. in the context of smooth projection methods for semi-infinite optimization.

Fast Approximation and Generated Biorthogonal Systems

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Let $\{e_k\}_{k=1}^n$ and $\{\varphi_k\}_{k=1}^n$ be biorthogonal systems in the Hilbert space H with the same linear span E. Then they are called generated biorthogonal systems (GBS). We consider GBS as an analogue of orthonormal systems: the projection $P_E x$ of each element $x \in H$ on E is $\sum\limits_{k=1}^{n} (x, \varphi_k)\, e_k$. As $x - P_E x$ has a nice integral representation for some classical spaces of functions, we investigate the subspaces $\Im_1, \Im_2 \subset H$ of functions permitting approximation by $\{e_k\}_{k=1}^{\infty}$ as fast as a geometrical progression and faster respectively.

If $\sup\limits_n \sqrt[n]{\|e_n\|} < \infty$, $\sup\limits_n \sqrt[n]{\max\limits_{1 \leqslant k \leqslant n} \left\|\varphi_k^{(n)}\right\|} < \infty$, where $\{e_k\}_{k=1}^n$ and $\left\{\varphi_k^{(n)}\right\}_{k=1}^n$ are GBS, we prove that $\Im_2$ is just the set of elements $x = \sum\limits_{k=1}^{\infty} x_k e_k$, $\sqrt[k]{|x_k|} \underset{k \to \infty}{\longrightarrow} 0$.

The case $H = L^2(0, \infty)$, $e_k(x) = e^{-\lambda_k x}$, $k = 1, 2, \ldots$ for λ_k chosen from a compact subset of the open right half-plane presented as an application.

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Porosity in best approximation theory

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In this lecture we present a review of results concerning porous sets in best approximation theory. In particular a porous version of Kuratowski-Ulam Theorem and its application to best approximation theory are considered.

New results on entire universal functions

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First, there exists an entire function φ and a sequence $b := \{b_n\}_{n\in\mathbb{N}}$ of complex numbers with $b_n \to \infty$ such that φ and every derivative of φ are bounded on any line and universal under translations with respect to b, i. e. a suitable sequence $\{\varphi(z+b_{n_k})\}$ of additive translates of φ converges to any preassigned entire function locally uniformly in the whole plane.

It is also shown that if a sequence b of complex numbers satisfies a certain necessary and sufficient property, then the set $\mathfrak{U}_b(\mathbb{C})$ of all functions with the properties as above is a dense but not a G_δ-subset in $H(\mathbb{C})$ endowed with the compact-open topology.

Second, if a sequence $\{w_n\}_{n\in\mathbb{N}}$ satisfies a certain condition, then there exists an entire function φ universal under translations whose zeros are all known and every w_n is a zero of φ.

Third, let $\{z_n\}_{n\in\mathbb{N}}$ be any sequence in $\mathbb{C}$ which has no cluster point in $\mathbb{C}$. Then there exists an entire function φ with $\varphi(z_n) = 0$ for all $n \in \mathbb{N}$ and φ is universal in terms of MacLane, i. e. a suitable sequence $\{\varphi^{(n_k)}\}$ of the derivatives of φ converges to any preassigned entire function locally uniformly in the whole plane.

On the Integral Properties of the Strong Maximal Operator

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From our results it follows that:

1) For every measurable function f on $\mathbb{R}^n$ and $\lambda > 0$ there exists a measure preserving and invertible mapping $\omega : \mathbb{R}^n \to \mathbb{R}^n$ such that for the values of a variable greater than λ the distributions of strong maximal functions of $f \circ \omega$ corresponding to arbitrary orientations of orthogonal coordinate axes are uniformly estimated from above by the distribution of the Hardy–Littlewood maximal function of $f \circ \omega$;

2)The distribution of a strong maximal function of arbitrary $f \in L(\mathbb{R}^n)$ is either finite everywhere or infinite everywhere.

Shift generated Haar spaces on track fields

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The question which functions F generate Haar spaces V by shifts in the form

$$V := < F(z - z_1), F(z - z_2), \ldots, F(z - z_n) > \qquad (1)$$

was a topic of some papers, written together with Walter Hengartner (deceased 2003) [2002,2005]. In order to make the description complete, one has to specify a non empty compact domain of definition $D \subset \mathbb{C}$ for the functions $\varphi_j(z) := F(z - z_j), j = 1, 2, \ldots, n$ and in addition one requires that F is defined on $\mathbb{C}\backslash\{0\}$ with values in $\mathbb{C}$. For the points z_j one requires that they are mutually distinct with $z_j \in \mathbb{C}\backslash D$, $j = 1, 2, \ldots, n$. The following definition was used.

Definition. 1. Let $n \in \mathbb{N}$ be fixed and $D \subset \mathbb{C}$ be compact. A function $F : \mathbb{C}\backslash\{0\} \to \mathbb{C}$ will be called an n-dimensional Haar space generator for D if V defined in (1) is an n-dimensional Haar space for all mutually distinct points $z_j \in \mathbb{C}\backslash D,\ j = 1, 2, \ldots, n$.

2. The function F will be called a universal Haar space generator if it is an n-dimensional Haar space generator for all $n \in \mathbb{N}$.

Our first result was as follows.

Theorem 1. Let D be the compact unit disk in $\mathbb{C}$ and F analytic on $\mathbb{C}\backslash\{0\}$. Then F is a universal Haar space generator for D if and only if F has the form

$$F(z) := \frac{\mathrm{e}^{a+bz}}{z} \text{ for arbitrary } a, b \in \mathbb{C}. \tag{2}$$

For the general case of an arbitrary compact set D we found the following result.

Theorem 2. Let D be an arbitrary compact set in $\mathbb{C}$ and F analytic on $\mathbb{C}\backslash\{0\}$. Then F is a universal Haar space generator for D if and only if F has the form (2) or the form

$$F(z) := \frac{\mathrm{e}^{a+bz}}{z^2} \text{ for arbitrary } a, b \in \mathbb{C}, \tag{3}$$

where this form can possibly occur only in the following exceptional cases:

(a) $D = \overline{D^\circ}$ and

(b) D is smooth (no corners) and convex but different from ellipses including disks.

It can be shown that the question whether F in (3) is a universal Haar space generator for D can be reduced to the question whether F defined by

$$F(z) := \frac{1}{z^2} \tag{4}$$

is a universal Haar space generator for D. One such exceptional compact set has the form of a track field: a rectangle with two semi-circles attached at opposite sides. We will treat some of the exceptional cases including the track field and show that F of case (3) is in these cases not a universal Haar space generator.

Wavelet Representations of Coorbit Spaces Invariant Under Symmetry Group

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We study the atomic decompositions of coorbit spaces invariant under symmetry groups using the wavelet transforms associated with an irreducible unitary square-integrable representation of a locally compact group on a Hilbert space. Our theorem provides an extension to the corresponding results of Feichtinger and Gröchenig (1989) and Rauhut (2003).

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The uniform approximation by polynomials on real nondegenerate Weyl's polyhedron

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Let D be a Weyl's polyhedron in $\mathbb{C}^n$, i. e. a bounded domain

$$D = \{z \in \mathbb{C}^n : |\chi_i(z)| < 1, \quad i = 1, 2, \ldots, N\}, \quad N \geq n,$$

where $\chi_1, \chi_2, \ldots, \chi_n$ are some holomorphic polynomials.

The sets $\sigma_k = \{z \in \overline{D} : |\chi_k(z)| = 1\}$, $k = 1, 2, \ldots N$, form the edges of a polyhedron D, and a collection of n-dimensional ribs $\sigma_{k_1 \ldots k_n} = \sigma_{k_1} \cap \ldots \cap \sigma_{k_n}$ $(1 \leq k_i \leq N)$ forms its distinguished boundary.

Further, let $A(D)$ be the uniform algebra of functions holomorphic in the domain D and continuous on $\overline{D}$ and let $P(D)$ be the uniform closure of polynomials on $\overline{D}$.

It is proved the following

Theorem 1 *If the polyhedron D has a real nondegenerate distinguished boundary and $N = n$, then $P(D) = A(D)$.*

The real nondegeneracy of a distinguished boundary means, that in points of distinguished boundary the edges intersect in a common position, i. e. $d|\chi_{k_1}| \wedge \ldots \wedge d|\chi_{k_n}| \neq 0$ on $\sigma_{k_1 \ldots k_n}$.

In the proof of the theorem we use the existence of solutions of $\overline{\partial}$-equation, which allow the uniform estimate. The conclusion of a theorem is true also in the case, if we omit a condition $N = n$, but instead require a real nondegeneracy on all k-dimensional ($k = 1, \ldots, n$) edges, and not just on a distinguished boundary.

On n-Convexity

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In this talk we will review the topic of n-convex functions. In particular we will present definitions, characterizations and properties of n-convex functions, and also talk about the problem of interpolation and extension.

Entire holomophic mappings on Stein manifolds

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We will consider a sequence of distinct points on a Stein manifold and by applying an approximation theorem, the existence of an entire holomorphic mapping which has schlicht balls with prescribe radii and centers will be proved.

Almost everywhere convergence of multiple Fourier series and integrals

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The talk will concern one (or both) of the following topics:

1) Extension to star-shaped domains of a result by Carbery and Soria proving convergence almost everywhere of partial integrals, related to spherical domains, of inverse Fourier transforms for functions in L^2 with logaritmic Sobolev properties.

2) Progress on singular integrals related to convergence almost everywhere of square partial sums for double Fourier series.

Mapping properties for oscillatory integrals in d-dimensions

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For $a_j, b_j \geq 1, j = 1, 2, \cdots, d$, we prove that the operator

$$Kf(x) = \int_{\mathbb{R}^d_+} k(x,y) f(y) dy$$

maps $L^p(\mathbb{R}^d_+)$ into itself for $p = 1 + \frac{1}{r}$, where $r = \frac{a_1}{b_1} = \cdots = \frac{a_d}{b_d}$, and the kernel $k(x,y) = \varphi(x,y) e^{ig(x,y)}, \varphi(x,y)$ satisfies for $|x-y| > 0$

$$|\partial_x^\alpha \partial_y^\beta \varphi(x,y)| \leq C_{\alpha\beta} |x-y|^{-|\alpha|-|\beta|}, \forall \alpha, \beta \in \mathbb{N}^d, \mathbb{N} = \{0, 1, \cdots\}.$$

(e.g. $\varphi(x,y) = |x-y|^{i\tau}, \tau$ real) and the phase $g(x,y) = x^a \cdot y^b = x_1^{a_1} y_1^{b_1} + \cdots x_d^{a_d} y_d^{b_d}$.

We also obtain $L^p(\mathbb{R}_+^d)$ mapping properties for the operators with more general real-valued phases

$$g(x,y) = x^a \cdot y^b + \mu_{\bar{1}}(x)\mu_{\bar{1}}(y)\Phi(x^a, y^b),$$

where

$$|\partial_x^\alpha \partial_y^\beta \Phi(x,y)| \leq C_{\alpha\beta} \text{ for } x, y \geq \bar{1}, \forall \alpha, \beta \in \mathbb{N}^d, \sum_{j=1}^{d} (\alpha_j + \beta_j) \geq 1.$$

but in case $a_j \geq b_j$ for $j = 1, \cdots, d$, we need to assume that $b_1 \cdot b_2 \cdots b_d > 1$. Also we set $\mu_{\bar{1}}(x) = \mu_1(x_1) \cdots \mu_1(x_d)$ with $\mu_1(t) = 1$ for $t \geq 2, \mu_1(t) = 0$ for $0 \leq t \leq 1, 0 \leq \mu_1(t) \leq 1$ and $\mu_1(t) \in C^\infty(\mathbb{R}_+)$. Examples of $\Phi's$ are $\log(\sum_{j=1}^d x_j + y_j)$ or $(\sum_{j=1}^d x_j + y_j)^l, 0 \leq l \leq 1$.

On the convergence of Fourier series for the spherical functions in the norm of $L^1(S^3)$

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For $f(x) \in L(S^3)$ we set

$$a_n^{(m)}(f) = \frac{2n+1}{2\pi} \cdot \frac{(n-m)!}{(n+m)!} \int_0^{2\pi} \int_0^{\pi} f(\theta,\varphi) p_n^m(\cos\theta) \cos m\varphi \sin\theta \, d\theta \, d\varphi,$$

$$b_n^{(m)}(f) = \frac{2n+1}{2\pi} \cdot \frac{(n-m)!}{(n+m)!} \int_0^{2\pi} \int_0^{\pi} f(\theta,\varphi) p_n^m(\cos\theta) \sin m\varphi \sin\theta \, d\theta \, d\varphi,$$

$$Y_n[f,(\theta,\varphi)] = \frac{1}{2} a_n^{(0)}(f) p_n(\cos\theta) +$$

$$+ \sum_{m=1}^{n} p_n^m(\cos\theta) \left[a_n^{(m)}(f) \cos m\varphi + b_n^{(m)}(f) \sin m\varphi \right],$$

where $p_n(t)$, $n = 0, 1, 2, ...$ are the standard, and $p_n^m(t)$, $m = 1, \ldots, n$, $n = 0, 1, 2, \ldots$ are the adjoint Legandre polynomials.

The series

$$\sum_{n=0}^{\infty} Y_n[f, (\theta, \varphi)]$$

is called Fourier-Laplace series of function $f(\theta, \varphi)$.

Let

$$S_N(f, (\theta, \varphi)) = \sum_{n=0}^{N} Y_n[f, (\theta, \varphi)]$$

In 1973 A. Bonami and J. Clerc [5] have proved, that for each number $p \neq 2$ there exists a function $f_p(\theta, \varphi) \in L^p(S^3)$, such that

$$\overline{\lim}_{N\to\infty} ||S_N(f_p, (\theta, \varphi))||_{L^p(S^3)} = +\infty \ .$$

The following theorem holds

Theorem. For any $\epsilon > 0$ there exists a measurable set $E \subset S^3$, with measure $|E| > 4\pi - \epsilon$ such that for each $f(\theta, \varphi) \in L^1(S^3)$ there is a function $g(\theta, \varphi) \in L^1(S^3)$ coinciding with $f(\theta, \varphi)$ on E, such that for any natural number N, we have

$$||S_N(g, (\theta, \varphi))||_{L^1(S^3)} \leq B||g||_{L^1(S^3)} \ ,$$

with a constant $B > 0$ independent of g and N.

Greedy algorithm with regard to Faber-Schauder subsystems

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Let $\Psi = \{\psi_n\}_{n=1}^{\infty}$ be a normalized basis in Banach space X. Then for each element $f \in X$ there exists a unique series by system $\{\psi_n\}_{n=1}^{\infty}$ converging to f in the norm of X:

$$f = \sum_{n=1}^{\infty} c_n(f)\psi_n \quad ,$$

Let an element $f \in X$ be given. We call a permutation $\sigma = \{\sigma(n)\}_{n=1}^{\infty}$ of natural numbers decreasing and write $\sigma \in D(f, \Psi)$, if

$$|c_{\sigma(n)}(f)| \geq |c_{\sigma(n+1)}(f)|, \quad n = 1, 2, ... \ .$$

We define the m-th greedy approximant of f with regard to the basis Ψ corresponding to a permutation $\sigma \in D(f, \Psi)$ by formula

$$G_m(f) = G_m(f, \Psi, \sigma) = \sum_{n=1}^{m} c_{\sigma(n)}(f)\psi_{\sigma(n)} \quad .$$

This nonlinear method of approximation is known as Greedy algorithm .

Definition 1 *A system $\Psi = \{\psi_n\}_{n=1}^{\infty}$ is called quazi-greedy in X if for each element $f \in \overline{span}(\Psi)$ the sequence $G_m(f, \Psi, \sigma)$ converges to f in the norm of X for any $\sigma \in D(f, \Psi)$.*

It is known that there is no quazi-greedy basis in $C_{[0,1]}$, i.e. the Faber-Schauder system is not quazi-greedy in $C_{[0,1]}$. Let $\Phi = \{\varphi_n(x)\}_{n=0}^{\infty}$ be Schauder system and let Δ_n be the support of $\varphi_n(x)$, $n = 2, 3, ...$. We define the classes $\Phi_{(1)}$ and $\Phi_{(2)}$ of subsystems as follows:
$\Phi_R = \{\varphi_{n_k}(x)\}_{k=1}^{\infty} = \{\bar{\varphi}_k(x)\}_{k=1}^{\infty} \in \Phi_{(1)}$ if and only if

$$mes\bar{\Delta}_k = \frac{1}{2^{k-1}} \ ; \ \bar{\Delta}_{k+1} \subset \bar{\Delta}_k \ ,$$

$\Phi_{R'} = \{\varphi_{n_{k_j}}(x)\}_{j=1}^{\infty} = \{\bar{\varphi}_{k_j}(x)\}_{j=1}^{\infty} \in \Phi_{(2)}$ if and only if $\bigcap_{j=1}^{\infty} \bar{\Delta}'_{k_j} = x_0 \in Q_{[0,1]}$ ($\bar{\Delta}'_{k_j}$ is the closure of $\bar{\Delta}_{k_j}$), where $Q_{[0,1]}$ is the set of rational quantities of the section $[0, 1]$.

Theorem 1 *The subsystem $\Phi_{R'} = \{\bar{\varphi}_{k_j}(x)\}_{j=1}^{\infty} \in \Phi_{(1)} \bigcup \Phi_{(2)}$ is quazi-greedy in $C_{[0,1]}$ if and only if*

$$\bigcap_{j=1}^{\infty} \bar{\Delta}'_{k_j} = x_0 \in \left\{ \frac{i}{2^k} \ , \ k = 0, 1, ... \ ; \ i = 0, 1, ..., 2^k \right\} \ .$$

Fusion of Univalent Functions

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In the first part of this talk we give a result on fusion of univalent functions: Let G_1, G_2 be Jordan domains in the complex plane such that the intersection of their closures consists of only one point. If f_1 resp. f_2 is a function univalent on a neighborhood of $\overline{G_1}$ resp. of $\overline{G_2}$, then, for each $\varepsilon > 0$, there exists a function, which is univalent on some neighborhood of $\overline{G_1} \cup \overline{G_2}$ and fulfills $\|f - f_j\|_{G_j} < \varepsilon$ simultaneously for $j = 1, 2$. The proof bases on a generalization of Alice Roth's Fusion Lemma (G. Schmieder: Fusion Lemma and boundary structure. J. Approx. Theory 71 (1992), 305-311).

In the second part we give an application of this result. Let some closed and nowhere dense subset C of the unit circle ∂E be given. Then there is some univalent function $g : E \to \mathbb{C}$ with the property that the fixed point cluster of g is exactly C. Note that C may have Lebesgue measure as close to 2π as we wish to have, In a former article (T. Gharibyan, G. Schmieder: Fixed points of univalent functions. CMFT 3 (2003), 299–304) it is proved that the fixed point cluster of each univalent function $E \to \mathbb{C}$, which is not the identity, is nowhere dense. Thus the mentioned result is best possible.

Haar-Fourier multipliers

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Let $\{\chi_n^k, k = 0, 1 \text{ for } n = 0, k = 1, 2, ..., 2^n \text{ for } n \geq 1\}$ be the Haar system normed in L_∞. Denote by Ω the set of indices (n, k). Each sequence

$\mu_{n,k}, (n,k) \in \Omega$ generates the multiplier

$$M\left(\sum_{(n,k)\in\Omega} c_{n,k}\chi_n^k\right) = \sum_{(n,k)\in\Omega} \mu_{n,k}c_{n,k}\chi_n^k.$$

We study the space of multipliers acting in a pair of rearrangement invariant (r. i.) spaces E and F endowed with the norm

$$||\mu||_{(E,F)} = \sup_{||x||_E \le 1} ||Mx||_F.$$

A new criterion for the unconditionality of Haar system in a separable r. i. space is obtained. Denote by M_0 the multiplier corresponding to the sequence $\mu_{n,k} = (-1)^n$.

Theorem 1 *The Haar system forms an unconditional basis in a separable r. i. space E iff M_0 is bounded in E.*

It is well known that the Haar system is not unconditional in L_1 and L_∞ and $||M||_{L_1} = ||M||_{L_\infty}$ for any multiplier M. The norm $||M||_{L_1}$ has been found up to absolute constant. This statement leads to the criterion of boundedness of M in any r. i. space because $(L_1, L_1) \subset (E, E)$ for any r. i. space E.

Denote by $\Lambda(\varphi)$ the Lorentz space endowed with the norm

$$||x||_{\Lambda(\varphi)} = \int_0^1 x^*(t)d\varphi(t)$$

where $x^*(t)$ is the decreasing rearrangement of $|x(t)|$ and $\varphi(t)$ is a concave increasing function on $[0,1]$, $\varphi(0) = \varphi(+0) = \lim_{t\to 0} t/\varphi(t) = 0$.

Theorem 2 *For any function φ there exists a multiplier M s. t. $M \in (\Lambda(\varphi), \Lambda(\varphi)) \setminus (L_1, L_1)$.*

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Bounded projections and representation of linear continuous functionals in Lizorkin–Triebel type spaces of holomorphic functions in the polydisk

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In the talk we deal with a scheme which was first given in the papers [1], [2] and goes back to investigations of E. Stein for studying holomorphic Lizorkin–Triebel type spaces in the polydisk. We consider four scales of spaces with quasi-norms

$$\left\| \left(\int_X |S^\alpha f(z)|^q \omega(|z|)\, d|z| \right)^{1/q} \right\|_{L^p(T^n)},$$

where $p, q \in (1, \infty)$, T^n is n-dimensional torus of the polydisk $\mathbb{U}^n = \mathbb{U} \times \cdots \times \mathbb{U}$; $\mathbb{U} = \{z : |z| < 1\}$; the couple (X, ω) consists of $X = [0,1]$ (or $[0,1]^n$) and $\omega(|z|) = (1-|z|)^\alpha$, $z \in \mathbb{U}$ (or $\omega(|z|) = \prod_{k=1}^n (1-|z_k|)^\alpha$); $\alpha > -1$, S^α is a differential operator of $H(\mathbb{U}^n)$ onto $H(\mathbb{U}^n)$

$$(S^\alpha f)(z) = \sum_{k_1,\ldots,k_n \geq 0} (k_1 + \cdots + k_n + 1)^\alpha a_k z^k,$$

or

$$(S^\alpha f)(z) = \sum_{k_1,\ldots,k_n \geq 0} (k_1 + 1)^\alpha \cdots (k_n + 1)^\alpha a_k z^k.$$

We find equivalent norms for these scales by means of L^p norms (specific for each scale) of Lusin integrals, and prove theorems on boundedness for Bergman type projections and on representation of functionals with $\min(p, q) > 1$. This generalizes results known previously for $p = q$.

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On Dirichlet problem for elliptic systems on the plane

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The Dirichlet problem for the elliptic system of second order with only leading constant coefficients is considered in a piecewise smooth domain on the plane. The Fredholm criterion and index formula for this problem in Holder spaces are given. In particular examples of the elliptic systems are found such that the index of Dirichlet problem is not 0.

On representation of measurable functions by general Franklin series

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Let $\{f_n\}_{n=0}^{\infty}$ be the general Franklin system corresponding to a quasi-dyadic weak regular partition of $[0,1]$. The following theorems are proved.

Theorem 1 *For any almost everywhere finite measurable function* $f(x)$ *there exist a series* $\sum\limits_{n=0}^{\infty} a_n f_n(x)$, *which absolutely converges to* $f(x)$ *almost everywhere.*

Theorem 2 *For any almost everywhere finite measurable function* $f(x)$ *and any* $\varepsilon > 0$ *there exists a function* $g(x)$ *such, that*

1) $\mu\{x \in [0,1] : f(x) \neq g(x)\} < \varepsilon$,

2) the Franklin-Fourier series of $g(x)$ *is absolutely uniformly convergent.*

Wavelet approximation and lattice packing

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Problems of approximation in a class of Hörmander spaces, including Sobolev spaces, by subspaces of wavelet type generated by lattice translations of given function are considered.

Widths that describe the approximation properties of such subspaces are defined, and their exact values are enumerated. Necessary and sufficient conditions are obtained for the optimality of subspaces on which these widths are realized. Criteria for the optimality of lattices in terms of the density of lattice packing of certain Lebesgue sets (for Sobolev spaces, of densities of packing by identical spheres) are established. Problems of comparison of the wavelet widths with the Kolmogorov widths of the same average dimension are discussed.

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On Greedy Algorithms with restricted depth search

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We continue to study efficiency of approximation and convergence of greedy type algorithms in uniformly smooth Banach spaces. This talk is based on a development of two my recent papers in the direction of making practical algorithms out of theoretical approximation methods. The Weak Chebyshev Greedy Algorithm (WCGA) is a general approximation method that works well in an arbitrary uniformly smooth Banach space X for any dictionary $\mathcal{D}$. It is an inductive procedure with each step of implementation consisting of several substeps. We describe the first substep of a particular case of the WCGA. Let $t \in (0,1]$. Then at the first substep of the mth step we are looking for an element φ_m from a given symmetric dictionary $\mathcal{D}$ satisfying

$$F_{f_{m-1}}(\varphi_m) \geq t \sup_{g\in\mathcal{D}} F_{f_{m-1}}(g) \tag{1}$$

where f_{m-1} is a residual after $(m-1)$th step and $F_{f_{m-1}}$ is a norming functional of f_{m-1}. It is a greedy step of the WCGA. It is clear that in the case of infinite dictionary $\mathcal{D}$ there is no direct computationally feasible way of evaluating $\sup_{g\in\mathcal{D}} F_{f_{m-1}}(g)$. This is the main issue that we address in the paper. We consider countable dictionaries $\mathcal{D} = \{\pm\psi_j\}_{j=1}^{\infty}$ and replace (1) by

$$F_{f_{m-1}}(\varphi_m) \geq t \sup_{1\leq j\leq N_m} |F_{f_{m-1}}(\psi_j)|, \quad \varphi_m \in \{\pm\psi_j\}_{j=1}^{N_m}.$$

The restriction $j \leq N_m$ is known in the literature as the depth search condition. We prove convergence and rate of convergence results for such a modification of the WCGA.

Trigonometric series of Nikol'skii classes

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We study when sums of trigonometric series belong to given function classes. For this purpose we describe the Nikolskii class of functions and, in particular, the generalized Lipschitz class. Results for the series with positive and general monotone coefficients are presented. Also, the convergence results for series with general monotone coefficients in the different L_p-metrics are obtained.

On Differential Equations with Singularities in a Class of Analytic Functions

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Let D be a simply connected bounded domain in the complex plane with smooth boundary $\Gamma = \partial D$. We consider the equation

$$\sum_{k=0}^{n} P_k(z)\varphi^{(k)}(z) = f(z), \quad z = x + iy, \quad (x, y) \in D, \tag{1}$$

where P_n - is a polynomial of order n with roots from D, P_k $(k = 1, \ldots, n-1)$ – are polynomials of order $\leq k$, f – is a given analytic in D function, $f \in C^{(\alpha)}(D \bigcup \Gamma)$.

The solution φ of the equation (1) is analytic in D and belongs to $C^{(n,\alpha)}(D \bigcup \Gamma)$. In the domain $D^- = \mathbf{C} \backslash (D \bigcup \Gamma)$ we consider a conjugate to (1) equation

$$\sum_{k=0}^{n} (-1)^k (P_k(z)\psi(z))^{(k)} = 0, \quad z = x + iy, \quad (x, y) \in D^-, \tag{2}$$

Here ψ is unknown analytic in D^- function, such that $\psi(\infty) = 0$. The later equation may be represented in the form $\sum_{k=0}^{n} Q_k(z)\psi^{(k)}(z) = 0$, where $Q_n(z) = (-1)^n P_n(z)$, and $Q_k(z) = a_k z^k + \ldots$ $(k = 0, \ldots, n)$ – are polynomials of order $\leq k$ with leading coefficients a_k. We introduce a polynomial $T(\lambda) = a_0 + \sum_{k=1}^{n} (-1)^k \lambda(\lambda + 1) \ldots (\lambda + k - 1)$ and prove the following

Theorem 1 *The equation (1) has a unique solution for arbitrary analytic function f if and only if $T(\lambda) \neq 0$ for $\lambda = 1, 2, \ldots$.*

Theorem 2 *If $T(\lambda_0) = 0$ for some positive integer λ_0, then the equation (1) is fredholmian and has a solution if and only if the function f satisfies to equalities $\int_\Gamma f(z)\psi_j(z)dz = 0$, for $j = 1, \ldots, k_0$. Here ψ_j – are linearly independent solutions of (2).*

We apply the obtained results to pose and solve the correct boundary value problems for elliptic equations.

Norm smoothness and a.e. rate of convergence of approximations of convolution type

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Consider an approximation process of convolution type, $T_t f := k_t * f$, on $L^p(\mathbb{R}^n)$, $p \geq 1$. It is discussed how smoothness in norm of a particular element $f \in L^p(\mathbb{R}^n)$ implies a certain pointwise rate of convergence almost everywhere of $T_t f(x)$ towards $f(x)$ when $t \to 0+$.

Sampling of Bandlimited Signals

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In this lecture we will discuss the sampling problem when the bandwidth of the signals are unknown. We will present a solution based on the Gelfand-Levitan inverse spectral theory and the Kramer irregular sampling formula.

On the best coapproximation

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Buck in 1965 introduced the elements ε-approximation and Singer in 1970 gave another characterization of these elements which is more concrete for

application in convenient spaces. Franchetti and Furi in 1972 introduced the concept of coapproximation. Throughout this research, at first we define the concept of ε- coapproximation and prove some theorems about it. Then we define ε-orthogonality and obtain the relation between this concept and ε- coapproximation.

On Representation of Functions from the space ReH_1

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ReH_1 is the space which consists of boundary values of real parts of functions from the Hardy space H_1. Ch. Fefferman [1] proved that any function $f(x)$ from ReH_1 can be represented in the form

$$f(x) = \sum_{k=1}^{\infty} c_k \varphi_k(x), \qquad \sum_{k=1}^{\infty} |c_k| < \infty,$$

where for each function $\varphi_k(x)$ there exists an interval I_k, such that

1. $supp \varphi_k \subseteq I_k$;
2. $\int_{-\infty}^{\infty} \varphi_k(x) dx = 0$;
3. $||\varphi_k||_\infty \leq 1/|I_k|$.

Using the duality theorem of S.Havinson [2], we show that the following additional condition on the functions $\varphi_k(x)$ may be imposed:

$$|\varphi_k(x)| = \frac{1}{|I_k|} \quad a.e. \quad x \in I_k.$$

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Denoising problem for predictable signals

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We consider the problem of estimation the original signal $f(n)$ in presence of an additive noise.

Definition. We say that the bounded signal $f(n) \in LF(\sigma)$, if there is a sequence $h_k, \quad k = 1, 2, \dots$ such that

$$|h(k)| \leq Aexp\{-k^{\sigma}\}, \quad k = 1, 2, \dots \tag{1}$$

and

$$f(n) = \sum_{k=1}^{\infty} f(n-k)h(k), \quad n = 0, \pm 1 \dots$$

Theorem. Let $1 < p < 2$, $\frac{2}{p+3} < \sigma < 1/2$ and

$$x(n) = f(n) + \xi(n), \quad n = 0, \pm 1, \dots$$

where $f(n) \in LF(\sigma)$,

$$\lim_{m\to\infty} \frac{1}{2m+1} \sum_{k=-m}^{m} |f(k)|^2 = 0$$

and $\xi_k, \quad k = 0, \pm 1, \dots$ be a sequence of independent random variables with the same distributions, $\mathbf{E}(\xi_k) = 0, \quad k = 1, 2, \dots$ and $\mathbf{E}(|\xi_k|^p) < \infty$.

Then with probability one we have

$$\lim_{r\to 1-} \int_{E_r} X(rz)e^{-inx}|dz| = f(n), \quad n = 0, \pm 1, \dots$$

where

$$X(rz) = \sum_{n=-\infty}^{\infty} x(n) r^{|n|} z^n, \quad |z| = 1,$$

and

$$E_r = \{z; \quad |z| = 1, \quad (1-r)^{1/p} |X(rz)| > 1\}.$$

Remark. The condition $\sigma > \frac{2}{p+3}$, in the previous theorem, very likely, is related with the concrete method of denoising. Maybe, that the more suitable method will permit to put weaker condition $\sigma > \frac{1}{p+1}$.

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Approximation by entire harmonic functions with optimal growth

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In this talk we discuss the problem of uniform approximation on the real axes $\mathbb{R}$ by entire harmonic functions (i.e. functions harmonic in $\mathbb{R}^2$) and having optimal, possibly slow growth at infinity. This growth is expressed in terms of the growth and the differential properties of functions to be approximated. The analog problems on entire holomorphic approximations on R has been considered earlier by M.V. Keldish and others (see [1]-[3]). The methods used here are modifications or analogs of those developed in [3]. A useful mean in our constructions is the Green identity for functions twice continuously differentiable in a domain with piecewise smooth boundary.

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Decomposition techniques in function spaces with dominating mixed smoothness

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We consider the function spaces with dominating mixed smoothness and develop several decomposition techniques of these spaces - atomic, quarkonial and wavelet decomposition. To show some applications, we give estimates of entropy numbers of their embeddings and study the trace spaces on diagonals.

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