International Conference

HARMONIC ANALYSIS AND APPROXIMATIONS, IV

Dedicated to 80th anniversary of academician

Alexandr Talalian

19 - 26 September, 2008, Tsaghkadzor, Armenia

Sponsors:

- The Research Mathematics Fund
- Promethey Science Foundation
- State Committee of Science, Armenia
- International Mathematical Union
- Hyur Service LLC

CONTENTS

Cyclicity of bicyclic operators and completeness of translates

E. Abakumov (France)

evgueni.abakoumov@univ-mlv.fr

We study cyclicity of operators on a separable Banach space which admit a bicyclic vector. A vector x is called bicyclic for an invertible operator T if the vectors $T^n x,\ n \in \mathbb{Z}$, span the whole space. A simple consequence of our main result is that a bicyclic unitary operator on a Banach space with separable dual is cyclic. We obtain some necessary and some sufficient conditions for cyclicity of weighted shifts. We also discuss completeness results for translates in certain Banach spaces of functions on the real line. The talk is based on joint work with A. Atzmon and S. Grivaux.

Least squares autocorrelation function estimation of noisy unequally spaced data series

M. Abbasi (Iran)

madjid.abbasi@iasbs.ac.ir

The sine and cosine functions of all frequencies are orthogonal to each other with L_2 norm on a given interval $[a, b]$, $a, b \in \mathbb{R}$. This is true for an equally spaced sample (data series), but just for some specific frequencies named as 'integer frequencies'. This is the base for FFT technique in discrete Fourier spectral analysis. The problem is that when the sampling rate is not constant over $[a, b]$, the sine and cosine base functions are not orthogonal for the integer frequencies. Then the discrete Fourier transform of such a data series is not defined.

The continuous autocorrelation function is the inverse Fourier transform of the power spectrum function. Traditionally, for a non-equally spaced time series, the evaluation of its spectrum is done using the Fourier transformation of the resampled data series. However, resampling changes the spectral properties of the original data series specially in the high frequency band.

Least squares spectral analysis (LSSA), introduced by Vaníček (1969), is a technique that takes into account any form of data series, even if it is not equally spaced. It is proved that Fourier spectral analysis is a special case of LSSA. This method does not restricted to integer frequencies. It is also possible to use other types of base functions, like wavelets for spectral analysis or as the base functions of known constituents beside the sine and cosine spectral base functions.

Using LSSA the autocorrelation function can be obtained for an unequally spaced data series through the 'Inverse Least Squares Transform' of its power spectrum. This autocorrelation function is then used to detect the presence of systematic effects over the data series as well as to construct the corresponding variance-covariance matrix of the data series.

This method was applied to the Iranian precise leveling data sets. The results of this study are presented in the talk.

A Chebyshev Spectral Collocation Method for the Coupled Nonlinear Schrödinger Equations

R. Abdur (Malaysia)

rashid_himat@yahoo.com

In the talk, we use the Chebyshev spectral collocation method to obtain numerical solutions for the coupled nonlinear Schrödinger equations. The Schrödinger equations are reduced to a system of ordinary differential equations that are solved by the fourth order Runge-Kutta method. The comparison between the numerical solution and the exact solution for the test cases shows good accuracy of the Chebyshev spectral collocation method.

Solution of Beltram's Equation in Some Special cases

S. Afyan (Armenia)

afyan@ysu.am

A number of algebra and geometry problems lead to the following equation:

$$W_{\overline{z}} - g(z)W_z = 0 \tag{1}$$

For instance, the problem of reducing differential quadratic form to the canonic form, the problem of an elliptic system to the canonic form, the problem of conform mapping of surface on a plane, etc.. These problems were studied by such eminent mathematicians as: L. Lichtenstein, M. Lavrentev, I. Vekua, L. Ahlfors, B. Boyarski, B. Shabat, etc.. It is known, that if $W_0(z)$ is a homeomorphic solution of equation (1), then the general solution is $\phi(W_0(z))$, where ϕ is an arbitrary analytic function. Finding such a $W_0(z)$ leads to an integral equation, which can be solved by the successive approximation method only. A question arises: Is it possible to find, at least in most simple cases, precise homeomorphic solution of equation (1)? In this work, in the special case, when

$$q(z) = \frac{Q(\overline{z})}{P(z)}$$

where $P(z)$ and $Q(z)$ are analytic functions in unit disk and

$$|Q(\overline{z})| < |P(z)|, \quad |z| < 1$$

$$|Q(\overline{z})| \leq |P(|z|)| \quad |z| = 1$$

a formula is obtained for a homeomorphic solution of equation (1).

About the interpolation theorem on $L(p,q)$ spaces

A. Ahmedov (Kyrgyz Republic)

anv_akhmed@mail.ru

In the talk we consider the question on the interpolation of an analitycal family of sublinear operators.

On convergence of generalized Cesáro means of trigonometric Fourier series and their conjugates

T. Akhobadze (Georgia)

t.akhobadze@math.sci.tsu.ge

Let (α_n) be a sequence of real numbers, where $\alpha_n > -1$, $n = 1, 2, \ldots$. Suppose

$$\sigma_n^{\alpha_n}(x,f) = \sum_{\nu=0}^{n} A_{n-\nu}^{\alpha_n - 1} S_\nu(x,f) / A_n^{\alpha_n}, \tag{1}$$

where

$$A_k^{\alpha_n} = (\alpha_n + 1)(\alpha_n + 2)\ldots(\alpha_n + k)/k!$$

and $S_\nu(x,f)$ are partial sums of trigonometric Fourier series.

If a modulus of continuity ω is given then H^ω denotes the class of continuous 2π-periodic functions f for which $\omega(\delta, f) \le \omega(\delta)$, $\delta \in [0, 2\pi)$.

Theorem 1. *Let (α_n) be any sequence on the interval $(-1, d]$, where d is a real number. The summation method defined by (1) is regular if and only if* $\liminf\limits_{n\to\infty}(\alpha_n \ln n) > -\infty$.

In the sequel C and C_ω will denote, respectively, positive absolute constant and a positive constant depending only on ω.

Theorem 2. *If $f \in H^\omega$ and $\alpha_n \in (0,1]$, $n = 3, 4, ...$, then*

$$||\sigma_n^{\alpha_n}(\cdot, f) \ - \ f(\cdot)||_C \le C \max\left\{ \frac{n^{\alpha_n} - 1}{\alpha_n \cdot n^{\alpha_n}} \omega(1/n), \frac{\alpha_n}{n} \int\limits_{\pi/n}^{\pi} \frac{\omega(t)}{t^2} dt \right\}.$$

Theorem 3. *Let (α_n) be any sequence on the interval $(0,1)$ and $f \in H^\omega$. Then*

$$||\sigma_n^{-\alpha_n}(\cdot, f) \ - \ f(\cdot)||_C \le C_\omega \omega(1/n) \frac{n^{\alpha_n} - 1}{\alpha_n \cdot (1 - \alpha_n)}, \ n = 3, 4, ...$$

Analogous estimations are valid for conjugate trigonometric Fourier series. The sharpness of the statements are obtained.

Approximation of function classes in Lorentz spaces with mixed norm

G. Akishev (Kazakhstan)

akishev@ksu.kz

Let $\overline{x} = (x_1, ..., x_m) \in I^m = [0, 2\pi)^m$ and let $\theta_j, q_j \in [1, +\infty)$, $j = 1, ..., m$.

We denote by $L_{\overline{p},\overline{\theta}}(I^m)$ the Lorentz space of Lebesgue - measurable functions $f(\overline{x})$ of period 2π in each variable with mixed norm $\|f\|_{\overline{p},\overline{\theta}} = \|...\|f\|_{p_1,\theta_1} ...\|_{p_m,\theta_m} < +\infty$, where

$$\|g\|_{p,\theta} = \left\{ \int\limits_0^{2\pi} (g^*(t))^\theta t^{\frac{\theta}{p} - 1} dt \right\}^{\frac{1}{\theta}},$$

and g^* is the non-increasing rearrangement of the function $|g|$.

Let $a_{\overline{n}}(f)$ be the Fourier coefficients of $f \in L_1(I^m)$ with respect to the multiple trigonometric system . Then we set

$$\delta_{\overline{s}}(f, \overline{x}) = \sum_{\overline{n} \in \rho(\overline{s})} a_{\overline{n}}(f) \, e^{i\langle \overline{n}, \overline{x} \rangle},$$

where $\langle \bar{y}, \bar{x} \rangle = \sum\limits_{j=1}^{m} y_j x_j$,

$$\rho(\bar{s}) = \left\{ \overline{k} = (k_1, ..., k_m) \in Z^m : \quad 2^{s_j - 1} \le |k_j| < 2^{s_j}, j = 1, ..., m \right\}.$$

For a number sequence we write $\{a_{\overline{n}}\}_{\overline{n} \in Z^m,} \in l_{\overline{p}}$ if

$$\left\| \{a_{\overline{n}}\}_{\overline{n} \in Z^m} \right\|_{l_{\overline{p}}} = \left\{ \sum_{n_m = -\infty}^{\infty} \left[... \left[\sum_{n_1 = -\infty}^{\infty} |a_{\overline{n}}|^{p_1} \right]^{\frac{p_2}{p_1}} ... \right]^{\frac{p_m}{p_{m-1}}} \right\}^{\frac{1}{p_m}} < +\infty,$$

where $\overline{p} = (p_1, ..., p_m)$, $1 \le p_j < +\infty$, $j = 1, 2, ..., m$. By analogy consider the Besov class

$$S^{\overline{r}}_{\overline{p}, \overline{\theta}, \overline{\tau}} B = \left\{ f \in \overset{\circ}{L}_{\overline{p}, \overline{\theta}}(I^m) : \|f\|_{S^{\overline{r}}_{\overline{p}, \overline{\theta}, \overline{\tau}} B} \le 1 \right\},$$

where

$$\|f\|_{S^{\overline{r}}_{\overline{p}, \overline{\theta}, \overline{\tau}} B} = \|f\|_{\overline{p}, \overline{\theta}} + \left\| \left\{ 2^{\langle \overline{s}, \overline{r} \rangle} \|\delta_{\overline{s}}(f)\|_{\overline{p}, \overline{\theta}} \right\} \right\|_{l_{\overline{\tau}}}$$

and $\overline{p} = (p_1, ..., p_m)$, $\overline{\theta} = (\theta_1, ..., \theta_m)$, $\overline{\tau} = (\tau_1, ..., \tau_m)$, $1 \le p_j, \theta_j, \tau_j < +\infty$, $j = 1, ..., m$. For a fixed vector $\bar{\gamma} = (\gamma_1, ..., \gamma_m)$, we set $\gamma_j > 0, j = 1, ..., m$ and $Q_n^{\bar{\gamma}} = \cup_{\langle \bar{s}, \bar{\gamma} \rangle < n} \rho(\bar{s})$, $S_n^{\bar{\gamma}}(f, \bar{x}) = \sum_{\bar{k} \in Q_n^{\bar{\gamma}}} a_{\bar{k}}(f) \cdot e^{i\langle \bar{k}, \bar{x} \rangle}$ is a partial sum of the Fourier series of f.

Approximation of various classes of smooth functions by this method was considered by K.I. Babenko, S.A. Telyakovskii, B.S. Mityagin, Ya. S. Bugrov, N.S. Nikol'skaya, E.M. Galeev, V.N. Temlyakov, Dinh-Dung, N.N. Pustovoitov, E.S. Belinskii, B.S. Kashin and V.N. Temlyakov, A.S. Romanyuk, A.I. Stepanets, Sun Yongsheng and Wang Heping. The main aim of the present paper is to estimate the order of the quantity $S_n^{\overline{\gamma}}\left(S^{\overline{r}}_{\overline{p},\overline{\theta},\overline{\tau}}B\right)_{\overline{q},\overline{\theta}} = \sup\limits_{f\in S^{\overline{r}}_{\overline{p},\overline{\theta},\overline{\tau}}B} \|f - S_n^{\bar{\gamma}}(f)\|_{\overline{q},\overline{\theta}}$.

Theorem. *Let* $\bar{\theta}^{(1)} = (\theta_1^{(1)}, ..., \theta_m^{(1)})$, $\bar{\theta^{(2)}} = (\theta_1^{(2)}, ..., \theta_m^{(2)})$, $\bar{\tau} = (\tau_1, ..., \tau_m)$, $\bar{p} = (p_1, ..., p_m)$, $\bar{q} = (q_1, ..., q_m)$, $\bar{r} = (r_1, ..., r_m)$, $\bar{\gamma} = (\gamma_1, ..., \gamma_m)$, $\gamma_j = \frac{r_j}{r_1}$, *and assume that* $1 \le \theta_j^{(1)}$, $\theta_j^{(2)}$, $\tau_j < +\infty$, $1 \le p_j < q_j < +\infty$, $\max_{j=1,...,m-1}\{\theta_j^{(2)}\} < \min_{j=2,...,m}\{q_j\}$ $\frac{1}{p_j} - \frac{1}{q_j} < r_j$, $j = 1, ..., m$, $0 < r_1 = ... = r_\nu < r_{\nu+1} \le ... \le r_m$, $\frac{1}{p_1} - \frac{1}{q_1} = ... = \frac{1}{p_\nu} - \frac{1}{q_\nu}$, $r_1(\frac{1}{p_j} - \frac{1}{q_j}) < r_j(\frac{1}{p_1} - \frac{1}{q_1})$ $j = \nu + 1, ..., m$. *Then*

$$S_n^{\bar{\gamma}}\left(S^{\bar{r}}_{\bar{p},\bar{\theta}^{(1)},\bar{\tau}}B\right)_{\bar{q},\bar{\theta}^{(2)}} \asymp \begin{cases} 2^{-n\left(r_1+\frac{1}{q_1}-\frac{1}{p_1}\right)} \cdot n^{\sum\limits_{j=2}^{\nu}\left(\frac{1}{\theta_j^{(2)}}-\frac{1}{\tau_j}\right)}, & \theta_j^{(2)} < \tau_j, j=\overline{1,m} \\ 2^{-n\left(r_1+\frac{1}{q_1}-\frac{1}{p_1}\right)}, & \tau_j \le \theta_j^{(2)}, j=\overline{1,m}. \end{cases}$$

Uniform and tangential approximation by meromorphic functions, having optimal growth

S. Aleksanian (Armenia)

asargis@instmath.sci.am

In this talk we discuss the problem of uniform and tangential approximation on stripes S_h and on the sectors Δ_α by meromorphic functions, having optimal growth at infinity. This growth is linked with the growth of the function $f \in A'(S_h)$ (correspondingly $f \in A'(\Delta_\alpha)$) to be approximated, and the growth of its derivative f' on the boundary. In case of approximation on stripes, one can assume in addition, that the poles of approximating meromorphic functions are lying on the imaginary axes.

The problem of approximation on sectors Δ_α has been discussed in [1] and [2], assuming $f \in A(\Delta_{\alpha+\delta})$ with $\delta > 0$.

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Convergence of greedy approximation in $L_p(0,1)$ and $C(0,1)$ with respect to Walsh system

G. Amirkhanyan (Armenia)

a_gagik@mail.ru

We consider the convergence of greedy approximation with respect to Walsh system. For function f we take as an approximant

$$G_m(f) := \sum_{k \in \Lambda} c_k(f) w_k,$$

where $\Lambda \subset \mathbb{N}$ is set of cardinality m containing the indices of the m biggest (in absolute value) Fourier-Walsh coefficients $c_k(f)$ of function f. $G_m(f)$ gives the best m-term approximant in L_2. We construct a continuous function f for which $\{G_m(f)\}$ does not converge in $L_p(0,1)$ for any $p > 2$. For $1 < p < 2$, a function $f \in L_p(0,1)$ is constructed such that $\{G_m(f)\}$ does not converge in measure. So the condition $f \in L_p(0,1)$, $p \neq 2$ does not guarantee the convergence of $\{G_m(f)\}$

in $L_p(0,1)$. For $p > 2$ we give a sufficient condition on $f \in L_p(0,1)$ in terms of decreasing rearrangement of Fourier-Walsh coefficients $c_k(f)$ which guarantee the convergence of $\{G_m(f)\}$ in $L_p(0,1)$.

For $f \in C(0,1)$ we consider convergence of $\{G_m(f)\}$ in $C(0,1)$. It's given sufficient conditions for uniform convergence of $\{G_m(f)\}$ to f. We see that these conditions don't guarantee convergence of Fourier-Walsh series in $C(0,1)$.

For the trigonometric system similar results were obtained by Korner, Konyagin and Temlyakov.

On Dirichlet and Neumann problems for harmonic functions

N. Arakelian (Armenia)

arakelian@instmath.sci.am, narakel@sci.am

The aim of this lecture is to present some aspects of the boundary value problems for harmonic functions in half-spaces. In particular, we discuss the following fact, noticed by M.V. Keldysh. For a pair of functions $f \in C^2$ and $\varphi \in C$ there is a function u harmonic in upper half-plane and continuous on its closure, such that u solves the the Dirichlet problem: $u = f$ on $\mathbb{R}$, and simultaneously u represents itself an "approximate" solution of the Neumann problem $\partial_n u = \varphi$ on $\mathbb{R}$, in sense that the normal derivative $\partial_n u$ on $\mathbb{R}$ is uniformly (and asymptotically) close to φ with any given degree of accuracy.

Some extensions and multi-dimensional versions of the result are obtained. Related questions on existence, representation and richness of solutions for the Dirichlet and Neumann problems and for Poisson equations are discussed.

Tensor harmonics on non-spherical homology classes

C. Arthur (USA)

chris.arthur1@gmail.com

Staring with Lie algebras on the three-sphere we explore more exotic CW-complexes and prepare reasonable interpretations through general relativistic calculations, applying cohomology as a simplified method.

Independent functions in rearrangement invariant spaces

S. Astashkin (Russia)

astashkn@ssu.samara.ru

Let $r_1, r_2, \ldots$ be a sequence of independent random variables with symmetric two-point distribution $\mathbf{P}(r_i = 1) = \mathbf{P}(r_i = -1) = \frac{1}{2}$ (for example, r_i could be the classical Rademacher functions defined on $[0,1]$). By the famous Khintchine inequality, for every $p > 0$ there exists a constant $C_p > 0$ such that for arbitrary real a_k we have

$$\Big\|\sum_{k=1}^{\infty} a_k r_k\Big\|_{L_p[0,1]} \le C_p \Big(\sum_{k=1}^{\infty} a_k^2\Big)^{1/2}.$$

Suppose that X is an rearrangement invariant space on $[0,1]$. We show that the following generalized Khintchine inequality

$$\Big\|\sum_{k=1}^{\infty} f_k\Big\|_X \le C \Big\|\Big(\sum_{k=1}^{\infty} f_k^2\Big)^{1/2}\Big\|_X$$

holds for arbitrary sequence $\{f_k\}_{k=1}^{\infty}$ of independent mean zero random variables from X if and only if X has the Kruglov property. Moreover, a version of well-known Maurey's inequality for vector-valued Rademacher series with independent coefficients will be presented.

Research was partially supported by RFBR (Grant 07-01-96603).

Harmonic analysis in time-frequency plane and Balian-Low theorem

G. Avanesyan (Armenia)

avangt@instmath.sci.am

Gabor frames and time-frequency plane representations are at the heart of modern applied harmonic analysis [1]. A way of circumventing the obstacles in realization of original ideas by von Neumann and Gabor that are posed by Balian-Low theorem on localization was shown recently [2], by using a special entire function with strong exponential localization property. The set of shifts of the latter function serves as a basis for the representation of the discrete *anticommutative* subgroup of the Weyl-Heisenberg group. The set is twice overcomplete, and it consists of a quartet of slightly nonorthogonal sublattices.

The representation for arbitrary functions - elements of separable Hilbert space - using the quartet of sublattices may be viewed as a discretized integral transform, closely related to a Gabor transform or a wavelet transform. The procedure of these uniform discretized expansions has to be contrasted with integral representations. The respective reconstruction algorithm, ready for practical applications, includes the following steps:

1. Calculation of a discrete set of scalar products, and their division into four subsets related to the quartet mentioned;

2. Fourier transformation, separately in each of the subsets of the quartet, to obtain four periodic functions, which are coupled by two linear relations;

3. Projection of the four functions onto two complementary subspaces, which do respect both the refined functional values and the error of approximation;

4. After refining the four Fourier transforms, smoothing procedure for every component may be applied. This possibility is a consequence of the general fact that smooth Fourier transforms do represent localized functions.

Reconstruction includes error correction, due to twice over-completeness.

Applications of the method include the problem of recognition of a standard signal shifted in complex plane, i.e. shifted and 'modulated'; the restrictions of Heisenberg uncertainty relation are completely overcome, in this problem, with the reconstruction algorithm presented.

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Lacunary Series in Mixed Norm Spaces

K. Avetisyan (Armenia)

avetkaren@ysu.am

Let $H(p,q,\alpha)\big(0 < p,q \le \infty, \alpha > 0\big)$ be the space of those functions $f(z)$ holomorphic in the disc $\mathbb{D}$, for which the quasi-norm

$$\|f\|_{p,q,\alpha} = \begin{cases} \left(\displaystyle\int_0^1 (1-r)^{\alpha q-1} M_p^q(f;r)\,dr\right)^{1/q}, & 0 < q < \infty, \\ \displaystyle\sup_{0\le r<1} (1-r)^{\alpha} M_p(f;r), & q = \infty, \end{cases}$$

is finite. Here $M_p(f;r)$ is the pth integral mean of f. For $p = q < \infty$ the spaces $H(p,q,\alpha)$ coincide with the well-known weighted Bergman spaces.

Recall that a sequence $\{m_k\}_{k=0}^{\infty}$ of positive integers is said to be lacunary (or Hadamard) if there exists a constant $\lambda > 1$ such that $\frac{m_{k+1}}{m_k} \geq \lambda$ for all $k = 0, 1, 2, \ldots$. The main theorem characterizes lacunary series in $H(p,q,\alpha)$, which is a generalization of some recent results of Aulaskari, Girela, Peláez, Stević, Zhu, and the author.

Theorem. *Let* $0 < q < \infty, \alpha > 0$, $\{m_k\}_{k=0}^{\infty}$ *be a lacunary sequence, and* $f(z)$ *be a holomorphic function in* $\mathbb{D}$ *given by a convergent lacunary series* $f(z) = \sum_{k=0}^{\infty} a_k z^{m_k}$. *Then the following statements are norm equivalent:*

(a) $f(z) \in H(\infty, q, \alpha)$;

(b) $f(z) \in H(p,q,\alpha)$ *for some* $p \in (0,\infty)$;

(c) $f(z) \in H(p,q,\alpha)$ *for all* $p \in (0,\infty)$;

(d) $\sum_{k=0}^{\infty} \frac{|a_k|^q}{m_k^{\alpha q}} < +\infty$.

Kolmogorov type inequalities for hypersingular integrals with homogeneous characteristic

V. Babenko (Ukraine)

v_babenko@dp.ukrtel.net

Let $C(\mathbb{R}^n)$ be the space of all bounded continuous functions $f : \mathbb{R}^n \to \mathbb{R}$ endowed with the norm $\|f\|_C := \sup\{|f(x)| \ : \ x \in \mathbb{R}^n\}$. For $x = (x_1, \ldots, x_n) \in \mathbb{R}^n$ $|x|$ denotes the usual Euclidean norm in $\mathbb{R}^n$. For a given modulus of continuity $\omega(t)$ we shall consider the space $H^{\omega}(\mathbb{R}^n)$ of functions $f \in C(\mathbb{R}^n)$ such that

$$\|f\|_{H^{\omega}} := \sup_{\substack{x,y \in \mathbb{R}^n \\ x \neq y}} \frac{|f(x) - f(y)|}{\omega(|x-y|)} < \infty.$$

For $\alpha \in (0,1)$ the hypersingular integral is defined in the following way:

$$\left(D^{\alpha}_{\Omega} f\right)(x) = \frac{1}{d_{n,1}(\alpha)} \int\limits_{\mathbb{R}^n} \frac{f(x) - f(x-\xi)}{|\xi|^{n+\alpha}} \Omega\left(\frac{\xi}{|\xi|}\right) d\xi,$$

where

$$d_{n,1}(\alpha) = \frac{\pi^{1+n/2}}{2^{\alpha}\Gamma\left(1+\frac{\alpha}{2}\right)\Gamma\left(\frac{n+\alpha}{2}\right)\sin\frac{\alpha\pi}{2}}.$$

and the function Ω (which is called characteristic) is homogeneous of degree 0 with respect to ξ.

We shall present some new sharp Kolmogorov type inequalities for hypersingular integrals with homogeneous characteristic of multivariate functions and give some applications of such inequalities. An example of obtained inequalities is given in the following

Theorem. *Let $\Omega(x)$ be a non-negative homogeneous of degree 0 with respect to x function, integrable on the unit sphere $S^{n-1} \subset \mathbb{R}^n$. Let also $\alpha \in (0;1)$ and $\omega(t)$ be such that $\int\limits_0^1 \frac{\omega(t)}{t^{\alpha+1}} dt < \infty$. Then for any function $f \in H^{\omega}(\mathbb{R}^n)$ the following sharp inequality holds*

$$\|D^{\alpha}_{\Omega} f\|_C \leq \frac{1}{d_{n,1}(\alpha)} \int\limits_{\mathbb{R}^n} \Omega\left(\frac{\xi}{|\xi|}\right) \frac{\min\{2\|f\|_C,\ \omega(|\xi|)\|f\|_{H^{\omega}}\}}{|\xi|^{n+\alpha}} d\xi'.$$

Gibbs phenomenon for the one-dimensional Dirac system

R. Barkhudaryan (Armenia)

rafayel@instmath.sci.am

We study decompositions by eigenfunctions of the one-dimensional regular Dirac equation

$$L(y) := \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \frac{dy}{dx} - \begin{pmatrix} p(x) & 0 \\ 0 & r(x) \end{pmatrix} y = \lambda y \tag{1}$$

with separated homogeneous boundary conditions

$$y_2(-1)\cos\alpha + y_1(-1)\sin\alpha = 0, \tag{2}$$

$$y_2(1)\cos\beta + y_1(1)\sin\beta = 0, \tag{3}$$

where $y = (y_1, y_2)^T$, $\alpha, \beta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. It is well known (see [1]) that the system $\{v_n\}_{n=-\infty}^{\infty}$ of eigenfunctions of (1) - (3) is complete in the Hilbert space $L_2^2[-1,1] = L_2[-1,1] \times L_2[-1,1]$. For functions from the domain of the corresponding operator we have uniform convergence of decompositions by eigenfunctions. Outside of the domain, however, convergence is slower and non-uniform. Moreover, we showed that the well-known Gibbs phenomenon appears.

For any function $f(x) = (f_1(x), f_2(x))^T \in L_2^2[-1,1]$ denote

$$BC(f, x, \gamma) = f_1(x)\sin\gamma + f_2(x)\cos\gamma,$$

$$S_N(f) = \sum_{n=-N}^{N} c_n v_n(x), \quad c_n = \int\limits_{-1}^{1} v_n^T(x) f(x) dx.$$

The following theorem shows the existence of Gibbs phenomenon.

Theorem. *Let $p, r \in C^1[-1,1]$, $f \in C_2^1[-1,1]$ and suppose that the function f does not satisfy boundary conditions ($BC(f,-1,\alpha) \neq 0$, $BC(f,1,\beta) \neq 0$). Then if, $\alpha, \beta \neq \frac{\pi}{4}$, the following holds:*

$$\limsup_{\substack{N\to\infty \\ x\to 1}} \frac{|BC(D_N(f), x, \beta)|}{|BC(f,1,\beta)|} = \limsup_{\substack{N\to\infty \\ x\to -1}} \frac{|BC(D_N(f), x, \alpha)|}{|BC(f,-1,\alpha)|} = \frac{2}{\pi}\int_0^{\pi} \frac{\sin t}{t} dt.$$

It's important to note that the Gibbs constant above is exactly the same as in the case of classical Fourier series.

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Some identities and inequalities

G. Barsegian (Armenia)

barseg@instmath.sci.am

In this talk we present some new and sharp identities and inequalities:

- in real analysis (for arbitrary smooth function of two real variables);
- in differential geometry (for arbitrary plane curve and arbitrary smooth three dimensional surface);
- in algebraic geometry (for arbitrary real algebraic function).

Also we present an identity and two inequalities in complex analysis (for arbitrary analytic function). In particular we have the following

Theorem. *Assume D is a bounded domain with piecewise smooth boundary, f is a meromorphic function in $\bar{D}$, $k \geq 1$ is an integer. Then*

$$\int\int_D \left|\frac{f'(z)}{f(z)}\right| d\sigma \leq \int\int_D \left|\frac{f^{(k+1)}(z)}{f^{(k)}(z)}\right| d\sigma + \frac{k\pi}{2} l(D),$$

where $l(D)$ is the length of ∂D.

The methods we utilize are in touch with Gamma-lines [1]. The problems considered were partly posed in [2].

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Branching points of the spectrum of algebraic extension of the continuous function's algebra

B. Batikyan (Armenia)

bag@instmath.sci.am

Let X be a locally compact topological space and A be an algebra of continuous complex-valued functions on X. We assume that A contains the identity, and the spectrum of A (the space of characters) coincides with X. Then the spectrum of algebra $A[t]$ of all polynomials over A identifies with $X \times \mathbb{C}$, and every polynomial $r = \sum_{k=0}^{m} a_k t^k$ identifies with the continuous function $\widehat{r}(x, \lambda) = \sum_{k=0}^{m} a_k(x)\lambda^k$ which is the Gel'fand's transform of r.

Let $p = c_n t^n + c_{n-1}t^{n-1} + \ldots + c_0$ be some polynomial over A of the degree $n > 1$, such that the first n coefficients of p does not have common zeros. Denote $B = A[t]/(p)$, where (p) is the principal ideal of $A[t]$, generated by p. Thanks to our assumptions, B appear to be the extension of A, the spectrum Y of algebra B coincides with the set of zeros of the function $\widehat{p}(x, \lambda)$, and the map $\pi : Y \to X$, which is the dual of the embedding of A into B, is finite-sheeted covering map (generally ramified). Remember that the point $y \in Y$ is called the branching point if there is no neighborhood V of this point, such that $\pi|V$ is injective.

The investigations of such extensions and, correspondingly, of covers, has began from the works [1], [2], [3] (see also the survey [4]). Here, and in the following investigations, there have been assumed that A is a Banach algebra, and the polynomial p is an monic polynomial (i.e. $c_n = 1$). In the monic case B is the integral extension of the initial algebra. But if the leading coefficient c_n is not invertible in A, then corank of the A-module B is equal to n, and its rank is equal to infinity. So in the non-monic case B is algebraic but not integral extension of A.

In this thesis we give an algebraic description of the branch points, we define and investigate the A-index of branching of the cover π. Fix any point $y_0 = (x_0, \lambda_0)$ from Y. We will say that the function f is A-holomorphic at x_0, if there exists a compact neighborhood U of the point x_0, such that $f|U$ belongs to the Banach uniform algebra $\overline{A|U}$. Let $\mathbf{A}$ is the germ of all functions that are A-holomorphic at x_0, $\mathbf{p}$ is the image of the polynomial p in $\mathbf{A}[t]$, $\mathbf{B} = \mathbf{A}[t]/(\mathbf{p})$ is the algebraic extension of $\mathbf{A}$, $\mathbf{D}$ is the direct summand of $\mathbf{B}$ corresponding to y_0, and $\widehat{\mathbf{D}}$ is the algebra of germs at y_0 (as a point from Y) of continuous functions of the type $\sum_{k=0}^{m} f_k \lambda^k$, where f_k are A-holomorphic at the point x_0. Denote by $\mathbf{N}$ the pre-image in $\mathbf{A}[t]$ of the null-germ from $\widehat{\mathbf{D}}$ and let us assume that $i_A(y_0) = \min\{\deg \mathbf{q} : \mathbf{q} \in \mathbf{N}, \mathbf{q} \text{ is monic}\}$.

Theorem 1. *If $y_0 = (x_0, \lambda_0)$ belongs to Y, then*

(i) $\sigma(y_0) \le i_A(y_0) \le \nu(y_0)$, where $\sigma(y_0)$ is the topological index of the branching of the cover π at y_0, and $\nu(y_0)$ is the multiplicity of the root λ_0 of the polynomial $c_n(x_0)t^n + c_{n-1}(x_0)t^{n-1} + \ldots + c_0(x_0)$;

(ii) y_0 is a branching point for π if and only if $i_A(y_0) > 1$.

It is natural to call the number $i_A(y_0)$ which depends on A by the term A-index of branching of the point y_0. The topological index of branching $\sigma(y_0)$, in general, is less than $i_A(y_0)$ (even in the case $A = C(X)$). In the meantime, for the monic and irreducible polynomial over the uniform maximal algebra A we have $\sigma(y) = i_A(y) = \nu(y)$ for all "interior" points from Y.

The branching points can be characterized also by point derivations on algebraic extension. Let the multiplicity $\nu(y_0)$ is greater than one. Then for $1 \leq k \leq \nu(y_0) - 1$ there exist nontrivial point derivation $d^{(k)}$ of the order k on algebras B and $\mathbf{B}$, corresponding to the point y_0. The functional $d^{(k)}$ is called local derivation on B, if its kernel contains all the elements whose Gel'fand's transform is equal to zero in the neighborhood of y_0.

Theorem 2. *y_0 is a branching point for cover π if and only if the derivation $d^{(1)}$ on B is local.*

The functional $d^{(k)}$ is called local derivation on $\mathbf{B}$, if its kernel contains the image in $\mathbf{B}$ of the ideal $\mathbf{N}$.

Theorem 3. *$i_A(y_0) = s$ if and only if the derivation $d^{(k)}$ on $\mathbf{B}$ is local for $k < s$, and non-local for $k \geq s$.*

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On Subspaces of the Space of Continuous Functions Consisting of Nonsmooth Functions [1]

E. Berezhnoi (Russia)

ber@uniyar.ac.ru

In the geometric theory of Banach spaces of smooth functions, the following problem is of considerable interest.

Problem A. Given Banach space X of smooth functions, does there exist an infinite-dimensional closed subspace $Y \subset X$ such that each function $y \in Y$ not identically zero is not smoother than the nonsmoothest function from X?

This question was studied by many mathematicians. In the present report, for each Holder space H^ω, we construct an infinite-dimensional closed subspace $G \subset C[0,1]$, isomorphic to l^1 such that, for each function $x \in G$ not identically zero, its restriction to any set of positive measure does not belong to the Holder space H^ω.

Let a set $D \subset I = [0,1]$ of positive measure be given. For each $x : D \to R$, we define the modulus of continuity of x on D as follows. For each $h > 0$, we set $D_h = \{t \in D : t + h \in D\}$. Then by the modulus of continuity of the function x on D we mean $\Omega(x, h; D) = \sup_{0 \le \delta \le h} \sup_{t \in D_\delta} |x(t+h) - x(t)|$. If $D = I$, then, instead of $\Omega(x, h; I)$, we write $\Omega(x, h)$. Suppose that ω is a modulus of continuity. As is customary, by $H^\omega(D)$ we denote the function space

$$\|f|H^\omega(D)\| = \sup_{t \in D} |f(t)| + \sup_{h>0} \frac{\Omega(f, h; D)}{\omega(h)},$$

Theorem 1. *Given Holder space H^ω, there exists a closed infinite-dimensional subspace $G \subset C[0,1]$, isomorphic to l^1 and such that, for each function $x \in G$ not identically zero, its restriction to any set D of positive measure does not belong to $H^\omega(D)$.*

[1]Supported by RFFI, N 08-01-00669

Theorem 2. *Given Holder space H^{ω}, there exists a closed infinite-dimensional subspace $G_0 \subset C[0,1]$, isomorphic to l^1 such that, for each function $x \in G_0$ not identically zero at each point $t_0 \in (0,1)$, the following relations hold:*

$$\overline{\lim_{h \to +0}} \frac{|x(t_0+h) - x(t_0)|}{\omega(h)} = \overline{\lim_{h \to +0}} \frac{|x(t_0-h) - x(t_0)|}{\omega(h)} = \infty.$$

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On Divergence for Rational Approximation

H.-P. Blatt (Germany)

hans.blatt@ku-eichstaett.de

The convergence behavior of best uniform rational approximations $r^*_{n,m}$ with numerator degree n and denominator degree m on $[-1,1]$ is investigated for non-analytic functions if ray sequences in the lower half of the Walsh table are considered, i. e. for sequences $\{(n, m(n))\}_{n=1}^{\infty}$ with

$$\frac{n}{m(n)} \to c \in (1, \infty] \quad \text{as} \quad n \to \infty.$$

For $f(x) = |x|^\alpha$, $\alpha \in \mathbb{R}_+ \setminus 2\mathbb{N}$, it is known that the best rational approximants diverge everywhere outside $[-1, 1]$. We investigate the situation for functions f that are not analytic on $[-1, 1]$ under a condition on the poles of the best approximants. The results can be generalized for the approximation on compact sets K with simply connected complement.

Multiplicative inequalities for the L norm; applications to analysis and number theory

S. Bochkarev (Russia)

bochkarv@mi.ras.ru

This is a survey of results concerning developed by the author new method in the problem of lower estimating of the L norm. Principal tools here are multiplicative inequalities with a multiplicator that can be either decreasing or increasing. These inequalities are elaborated in three different forms.

On uniform spaces, uniformly continuous maps an their categorial characteristics

A. Borubaev and T. Kasymova (Kyrgyz Republic)

tumar2000@mail.ru

By means of absolute closedness we have found a categorical characteristic of the compact complete uniform spaces and complete topological groups in the Raykov's sence, which is the solution of the problem posed by Z. Frolic in 1983 on the topology seminar in Carl's university (Prague).

Find a categorial characteristics of compact and complete spaces.

It is known, that the Hausdorff's topological group G is called *complete in the Raykov's sense*, if G is complete with respect to its two-sided uniformity.

Hausdorff uniform space (X, U) is called *absolutely closed*, if (X, U) is closed in every Hausdorff uniform space, which contains it as a subspace.

1. The topological space is compact if and only if it is absolutely closed ([1]).

2. The uniform space is complete if and only if it is absolutely closed uniform space.

3. The topological group is complete in the Raykov's sense if and only if it is absolutely closed topological group ([2]).

Below we give an answer to the problem posed by Z. Frolic (Theorem 1), also we give a categorial characteristics of complete uniform spaces (Theorem 2) and of complete in the Raykov's sense topological groups (Theorem 3).

The next definition plays crucial role in this note.

Definition 1. *Let K is an arbitrary category, A is some class of morphisms of the category K. The object X of the category K is called A* ***- closed,*** *if every morphism $f : X \to Y$ for any object Y, belongs to the class A.*

The notion of A - closedness was introduced in the connection with above mentioned propositions 1-3.

Theorem 1. *Let K be the category of topological spaces and continuous maps, and A is the class of perfect maps. The topological space X is compact if and only if the object X of the category K is A - closed.*

Theorem 2. *Let K be the category of uniform spaces and uniformly continuous maps, and A is the class of complete uniformly continuous maps. The uniform space (X, U) is complete if and only if the object (X, U) of the category K is A-closed.*

Theorem 3. *Let K be the category of topological groups and such continuous homomorphisms, that are uniformly continuous and complete with respect to two-sided uniformities of topological groups. The topological group G is complete in the Raykov's sense if and only if the object G of the category K is A-closed.*

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Fatou-type theorems for general approximate identities

M. Carlsson (USA)

mcarlsso@math.purdue.edu

For functions $f \in L^1(\mathbb{R}^n)$ we consider extensions to $\mathbb{R}^n \times \mathbb{R}^+$ given by convolving f with an approximate identity. For a large class of approximate identities we obtain a Fatou-type theorem where the convergence regions are sometimes effectively larger than the non-tangential ones. We also give general conditions under which the convergence regions are shown to be optimal. The results extend previous results by Sjögren, Rönning and Brundin concerning the square root of the Poisson kernel.

The modulus of convexity and inner product in linear normed spaces

G. Chelidze (Georgia)

g.chelidze@mail.ru

In the present paper we prove the theorem which gives a positive answer of Nordlander's conjecture that is related to the modulus of convexity of the real normed space X. Let us recall this conjecture and the definition of the modulus of convexity of X: The modulus of convexity of the space X is the function $\delta_X : [0,2] \to [0,1]$ defined by the equality

$$\delta_X(a) = inf\{1 - \frac{1}{2}||x+y|| : x, y \in S, ||x-y|| = a\}.$$

Day [1] proved that if

$$\delta_X(a) \geq 1 - \sqrt{1 - \frac{a^2}{4}}, \quad \forall a \in [0,2]$$

then X is an i.p.s. Nordlander [2] conjectured, that it suffices that this inequality holds for some $a \in (0,2)$. Alonso and Benitez [3] proved that this assertion is true exactly for $a \in (0,2) \setminus D$, where

$$D = \left\{ 2\cos\frac{k\pi}{2n} : k = 1, ..., n-1; n = 2, 3, ... \right\}.$$

More precisely, they showed that if $dimX = 2$, then Nordlander's conjecture is true if $a \in (0,2) \setminus D$ and false if $a \in D$. However, the validity of this conjecture for $dimX \geq 3$ was open [4]. The next Theorem gives an affirmative answer on this question.

Theorem. *Let X be a real normed space with $dimX \geq 3$ and let a be a fixed number from the interval $(0,2)$. The following statements are equivalent:*

(i) X is an inner-product space.

(ii) The set $\{||x+y|| : x, y \in S, ||x-y|| = a\}$ is a singleton.

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On the modulus of continuity of conjugate functions in the space $L(T^n)$

A. Danelia (Georgia)

annadanelia2000@yahoo.com, a.danelia@math.sci.tsu.ge

Let B be an arbitrary non-empty subset of the set $M = \{1, \ldots, n\}$ and let $|B|$ denote the cardinality of B. Let x_B denote a point in $\mathbb{R}^n$ all of whose coordinates with indices in $M \backslash B$ are zero.

If $f \in L(T^n)$, then following Zhizhiashvili , we call the expression

$$\widetilde{f}_B(x) = \Big(-\frac{1}{2\pi} \Big)^{|B|} \int_{T^{|B|}} f(x + s_B) \prod_{i \in B} \operatorname{ctg} \frac{s_i}{2} \, ds_B$$

the conjugate function of n variables with respect to those variables whose indices form the set B .

In the present work, the exact estimates of the partial moduli of continuity of the conjugate functions of many variables are received in the space $L(T^n)$.

The following theorem is valid.

Theorem. a) *Let* $f \in L(T^n)$, $\omega_i(f;\delta) = O(\omega(\delta))$, $\delta \to 0+$, $i = 1,\dots,n$, $\lim\limits_{\delta\to 0+} \frac{\omega(\delta)}{\delta} = +\infty$ *and for each* $B \subseteq M$

$$\int_{\left[0,\frac{\pi}{2}\right]^{|B|}} \min_{i\in B} \omega(s_i) \prod_{i\in B} \frac{ds_i}{s_i} < \infty.$$

Then

$$\omega_k(\widetilde{f}_B;\delta) = O\left\{\left(\int\limits_0^\delta \frac{\omega(s)}{s} ds + \delta \int\limits_\delta^\pi \frac{\omega(s)}{s^2} ds\right) |\ln\delta|^{|B|-1}\right\}, \ \delta \to 0+, \ k \in B,$$

$$\omega_k(\widetilde{f}_B;\delta) = O\left\{\omega(\delta)|\ln\delta|^{|B|}\right\}, \ \delta \to 0+, \ k \in M\setminus B.$$

b) *For each* $B \subseteq M$ *there exist functions* F *and* G *such that* F, G *satisfy conditions of (a) and*

$$\omega_k(\widetilde{F}_B;\delta) \geq C\left\{\left(\int\limits_0^\delta \frac{\omega(s)}{s} ds + \delta \int\limits_\delta^\pi \frac{\omega(s)}{s^2} ds\right) |\ln\delta|^{|B|-1}\right\}, \ 0 \leq \delta \leq \delta_0, \ k \in B,$$

$$\omega_k(\widetilde{G}_B;\delta) \geq C\left\{\omega(\delta)|\ln\delta|^{|B|}\right\}, \ 0 \leq \delta \leq \delta_0, \ k \in M\setminus B.$$

where C *and* δ_0 *are positive constants.*

Measures of smoothness on the unit sphere

Z. Ditzian (Canada)

zditzian@math.ualberta.ca

For function spaces on the unit sphere $S^{d-1} = \{(x_1,\dots,x_d) : x_1^2 + \dots + x_d^2 = 1\}$ different measures of smoothness are discussed. The classical moduli will be compared to recently introduced ones and new results on both will be described. K-functionals, realizations and best approximation by spherical harmonic polynomials could also be construed as measures of smoothness. Relations among the different measures will be given.

Divergence of greedy algorithms for generalized Walsh subsystem in $L^1[0,1]$

S. Episkoposian (Armenia)

sergoep@ysu.am

Let X be a Banach space with a norm $||\cdot|| = ||\cdot||_X$ and a basis $\Phi = \{\phi_k\}_{k=1}^{\infty}$, $||\phi_k||_X = 1$, $k = 1, 2, ...$

For a function $f \in X$ we consider the expansion $f = \sum_{k=1}^{\infty} a_k(f)\phi_k$.

Definition 1. *Given $f \in X$, the m-th greedy approximant of function f with regard to the basis Φ is defined by formula $G_m(f,\phi) = \sum\limits_{k\in\Lambda} a_k(f)\phi_k$, where $\Lambda \subset \{1,2,...\}$ is a set of cardinality m such that $|a_n(f)| \geq |a_k(f)|$, $n \in \Lambda$, $k \notin \Lambda$.*

Let a be a fixed integer, $a \geq 2$ and put $\omega_a = e^{\frac{2\pi i}{a}}$.

Definition 2. *The Rademacher system of order a is defined by*

$$\varphi_0(x) = \omega_a^k \;\; if \;\; x \in \left[\frac{k}{a}, \frac{k+1}{a}\right), \;\; k = 0, 1, ..., a-1,$$

and for $n \geq 0$, $\varphi_n(x+1) = \varphi_n(x) = \varphi_0(a^n x)$.

Definition 3. *The generalized Walsh system Ψ_a of order a is defined by: $\psi_0(x) = 1$, and if $n = \alpha_{n_1} a^{n_1} + ... + \alpha_{n_s} a^{n_s}$ where $n_1 > ... > n_s$, then*

$$\psi_n(x) = \varphi_{n_1}^{\alpha_{n_1}}(x) \cdot ... \cdot \varphi_{n_s}^{\alpha_{n_s}}(x).$$

The following theorem holds (see [1]):

Theorem. *There exists a function $f \in L^1[0,1)$ such that the approximant $G_n(f, \Psi_a)$ with regard to the generalized Walsh system does not converge to $f(x)$ by $L^1[0,1)$ norm, i.e. the generalized Walsh subsystem $\{\psi_{n_k}\}_{k=1}^{\infty}$ is not a quasi-greedy basis in its linear span in L^1.*

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Biorthogonal wavelets on the Cantor dyadic group

Yu. Farkov (Russia)

farkov@list.ru

The Cantor dyadic group can be defined as a locally compact abelian group which is weak direct product of a countable set of the cyclic groups of order 2. Orthogonal wavelets for this group were introduced in 1996 by W.C. Lang; see [1], [2] for some further results and the detailed bibliography on this subject. In the talk, the Cantor dyadic group will be considered in framework of the biorthogonal wavelet theory.

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Electrons in boxes

A. Frumkin and A. Goldberger (Israel)

frumkin@math.tau.ac.il, *assafg@post.tau.ac.il*

Given a sequence of real numbers $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n$ we are thinking about the consecutive gapes $[\lambda_i, \lambda_{i+1}]$ as a sequence of boxes on the line.

Let us assume that electrons are put, one for a box, and the electric field around is decreased as $1/R$ where R is the distance from the charges (sometimes it is called a field of logarithmic potential). We show that the potential of a configuration $\lambda_1 \leq \eta_1 \leq \lambda_2 \leq \cdots \leq \eta_{n-1} \leq \lambda_n$, where the etas are the places of the electrons in the boxes, is related to the probability density of the etas to occur as the eigenvalues of a $n-1$ by $n-1$ principal sub matrix of Hermitian matrix whose eigenvalues are the lambdas. The probability measure we use is the Haar measure on the unitarian group.

So one can approximate the stable configuration by using Monte Carlo process on the unitarian group.

When the Hermitian procedure is changed to a symmetric one, the probability density with respect to the Haar measure on the real orthogonal group involves an interaction of the electrons with the walls of the boxes.

The moment problem for a finite interval

L. Galoyan (Armenia)

levon010887@mail.ru

We consider the following Moment Problem: given a sequence of numbers $S = \{s_k\}$, does there exists a function f for which

$$s_k = \int_0^1 f(x)x^k dx, \quad k = 1, 2, ...?$$

If yes, we call f the solution of problem of moments. This report is devoted to the observation of the following problem: under what conditions on the sequence S the solution of moment problem will belong to the given functional class.

Functional Central Limit Theorem for O-Martingales

K. Gasparian (Armenia)

kargasp@gmail.com

The limit theorems for semimartingales defined on a standard stochastic basises $(\Omega^n, \Im^n, F^n = (\Im^n_t)_{t\geq 0}, P^n)$ with paths from the Skorokhod space D were studied in fundamental work [1]. Here we do not imposed the standard conditions on a basises. All considering martingales will be supposed O(Optional)-measurable from the space L of the processes with paths having both one-sided limits for all $t \in R_+$. On the space L we introduce the Skorokhod type topology (see [2]) and we say that the sequence $X^n \in L$ of processes converges in Law to $X \in L$ $(X^n \xrightarrow{L} X)$ if respective distributions $P_{X^n} = P^n(X^n)^{-1}$ weakly converges to P_X.

Let $X^n \in M^2_{loc}$ be the sequence of square-integrable O-martingales. We consider their quadratic characteristics

$$C^n =< X^n > \quad ((X^n)^2 - < X^n > \in M_{loc}),$$

quadratic covariations

$$[X^n, X^n] =< X^{nc} > + \sum_{s\leq 0} \Delta X^2_s + \sum_{s<0} \Delta^+ X^2_s,$$

where $X^{nc} \in M^2_{loc}$ are the continuous martingale parts of X^n and the compensators ν^{1n} and ν^{2n} of the integer-valued random measures μ^{1n} and μ^{2n}, generating by the jumps of $\Delta X^n_t = X^n_t - X^n_{t-}$ and $\Delta X^n_t = X^n_{t+} - X^n_t$, correspondingly. Besides we supposed that X be some continuous Gaussian O-martingale with non-random quadratic characteristic $C =< X >$ and S be dense subset of R_+.

The following *Lindeberg-Feller* type theorem to be valid (comp. [1]).

Theorem 1. *If the Lindeberg Conditions*

$$|x|^2 1_{|x|>\varepsilon} \times \nu^{in}_t = \int_0^t \int_R |x|^2 1_{|x|>\varepsilon} \nu^{in}(dx, ds) \longrightarrow 0$$

for all $t \geq 0$, $\varepsilon > 0$, $i = 1, 2$ is fulfilled, then the following equivalence holds

$$[X,X]_t \xrightarrow{P} C_t(or < X^n >_t \xrightarrow{P} C_t) \text{ for all } t \in S \Longleftrightarrow X^n \xrightarrow{L} X.$$

For a local O-martingales $X^n \in M_{loc}$ the following variant of Central Limit Theorem also holds (comp. [1]):

Theorem 2. *Under the conditions*

$$\lim_{\varepsilon\uparrow 0} \lim_n supP^n(|x|1_{|x|>\varepsilon} \times \nu_t^{in} > \delta) = 0$$

for all $t \geq 0, \varepsilon > 0, \delta > 0, i = 1, 2$ we have equivalence (for all $t \in S$)

(i) $[X^n, X^n]_t \xrightarrow{P} C_t$

(ii) $\begin{cases} [X^n(\varepsilon), X^n(\varepsilon)]_t \xrightarrow{P} C_t(or \ < X^n(\varepsilon) >_t \xrightarrow{P} C_t) \Longleftrightarrow X^n \xrightarrow{L} X \\ \nu^{in}([0,t] \times \{|x| > \varepsilon\}) \xrightarrow{P} 0 \end{cases}$

where $X^n(\varepsilon) = X^n - X_0^n - \sum_{s\leq .} \Delta X_s 1_{|\Delta X_s|>\varepsilon} - \sum_{s<.} \Delta^+ X_s 1_{\{|\Delta^+ X_s|>\varepsilon\}} \in M_{loc}^2$.

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On regularity of Bivariate Hermite Interpolation

H. Gevorgian (Armenia)

ghovik@r.am

Hermite interpolation by bivariate algebraic polynomials of total degree $\leq n$ is considered. The interpolation parameters are the values of a function and its partial derivatives up to some order $n_\nu - 1$ at the points $z_\nu = (x_\nu, y_\nu)$, $\nu = 1, \ldots, s$, where n_ν is the multiplicity of z_ν. The sequence $N := \{n_1, \ldots, n_s; n\}$ of multiplicities associated with the degree of interpolating polynomials is called an interpolation scheme ($\mathcal{N} \in \mathcal{IS}$), if

$$\sum_{\nu=1}^{s} n_\nu(n_\nu + 1) = (n+1)(n+2)$$

It is proved in [1], that N is regular (correct) in the case

$$\sum_{\{\nu;\ n_\nu > 1\}} n_\nu \leq 3n. \tag{1}$$

In [2] cubic transformations of interpolation schemes was investigated and regularity for a large class of schemes, including the schemes satisfying condition (1) was obtained.

Using new properties of the cubic transformations, we generalize the above mentioned results from [1] and [2].

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On one-step Krylov subspace method

L. Gevorgyan (Armenia)

levgev@hotmail.com

Let H be a Hilbert space and A be a bounded linear operator acting in H. One of the most familiar ways of solving the equation

$$Ax = b$$

is the Richardson's iterative method

$$x_{n+1} = x_n - \alpha_n \left(A x_n - b \right), \;\; n \in \mathbb{Z}^+,$$

where x_0 is an initial guess and α_n is a numerical parameter. This leads to the well-known Krylov subspace method.

Almost all known algorithms are based on two principal projection methods - the minimal residual and the orthogonal residual. We propose a new method, based on the quantity

$$m\left(A\right) = \inf_{t \in \mathbb{C}} \left\| I - tA \right\|.$$

Concentration and stable reconstruction of continuous Gabor Transform

A. Ghaani Farashahi and R. Kamyabi-Gol (Iran)

ghaanifarashahi_arash@yahoo.com

In this article we generalize some uncertainty principles related with the continuous Gabor transform for strong commutative hypergroups. More precisely, in the following we prove that for a locally compact strong commutative hypergroup X with the Haar measure μ and the Plancherel measure λ on its dual $\widehat{X}$, window function ψ and each $f \in L^2(X)$, the portion of $\mathcal{G}_\psi f$ lying outside some small U of finite $\mu \times \lambda$-measure in $X \times \widehat{X}$ cannot be arbitrary small, either. For sufficiently small U, this can be seen immediately by estimating the Hilbert-Schmidt norm of a suitable defined operator. Also we generalize the stable reconstruction of Gabor transform from incomplete noisy data, for strong commutative hypergroups. As an example we show how these techniques apply to the Locally compact groups and Bessel$-$Kingman hypergroups.

Maximal operators of $(C,\alpha)-$ Means of cubic partial sums of d-Dimensional Walsh-Fourier Series

U. Goginava (Georgia)

u.goginava@math.sci.tsu.ge

For the martingale f we consider the maximal operator

$$\sigma_*^\alpha f = \sup_n |\sigma_n^\alpha(f, \overrightarrow{x})|,$$

where σ_n^α is $(C,\alpha)-$ means of cubic partial sums of d-dimensional Walsh-Fourier series

In 1939 for the two-dimensional trigonometric Fourier series Marcinkiewicz has proved that for $f \in L\log L([0,2\pi]^2)$ the means $\sigma_n^1 f$ converge a.e. to f as $n \to \infty$. Zhizhiashvili [5] improved this result and proved that for $f \in L_1([0,2\pi]^2)$ the (C,α)-means $\sigma_n^\alpha f$ converge a.e. to f as $n \to \infty$. Dyachenko [1] proved this result for dimension more than 2.

In [4] Weisz proved that the maximal operator $\sigma_n^1 f$ of double Walsh-Fourier series is bounded from the two-dimensional dyadic martingale Hardy space H_p to the space L_p for $p > 2/3$. The author [2] generalized the theorem of Weisz for the d-dimensional Walsh-Fourier series and proved that the maximal operator σ_*^1 is bounded from the d-dimensional dyadic martingale Hardy space H_p to the space L_p for $p > d/\left(d+1\right)$. We also proved [3] that for the boundedness of the maximal operator σ_*^1 from the Hardy space H_p to the space L_p the assumption $p > d/\left(1+d\right)$ is essential.

Theorem 1. *Let $f \in H_p$, $0 < \alpha < 1$, $p > d/\left(d+\alpha\right)$. Then the maximal operator σ_*^α of the (C,α) means of the d-dimensional Walsh-Fourier series is bounded from martingale Hardy space H_p to the space L_p.*

Theorem 2. *Let $\alpha \in (0,1)$ and $p \le d/\left(d+\alpha\right)$. Then the maximal operator σ_*^α is not bounded from the Hardy space H_p to the space L_p.*

Theorem 3. *Let $d = 2$ and $p = 2/\left(2+\alpha\right)$. Then σ_α^* operator is bounded from martingale Hardy space H_p to the space $weak-H_p$.*

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An example of quasi-greedy basis in $L^1(0,1)$

S. Gogyan (Armenia)

gogyan@gmail.com

Let $\Psi = \{\psi_k\}$ be a basis in a Banach space X and $\inf \|\psi_k\|_X > 0$. For any $f \in X$ we have

$$f = \sum_{k=1}^{\infty} c_k \psi_k,$$

where $c_k \to 0$ for $k \to +\infty$. Define a permutation $\{k_j\}$ of natural numbers to satisfy

$$| c_{k_1} | \geq | c_{k_2} | \geq \cdots.$$

We define the m-th **greedy approximant** of f with regard to the basis Ψ by

$$G_m(f) = \sum_{j=1}^{m} c_{k_j}(f, \Psi) \psi_{k_j}.$$

This nonlinear method of approximation is known as greedy algorithm.

Definition 1. *A basis* $\Psi = \{\psi_k\}_{k=1}^{\infty}$ *is called quasi-greedy if there exists* $C \geq 1$ *such that for any* $f \in X$ *and* $m \geq 1$

$$\|G_m(f)\| \leq C\|f\|.$$

Theorem. *(P. Wojtaszczyk) The basis is quasi-greedy if and only if for every* $f \in X$

$$\lim_{m\to\infty} G_m(f) = f.$$

Trigonometric system is quasi-greedy basis in $L^p(0,1)$, $1 < p < \infty$ (S. Konyagin, V. Temlyakov).

It is known that there exists a quasi-greedy basis in $L^1(0,1)$, but no such basis was constructed. In particular the Haar system isn't quasi-greedy basis in $L^1(0,1)$. We will discuss how to get convergence in three aspects.

1) By modifying the Haar system.

2) By modifying the Greedy Algorithm.

3) By modifying the function.

In the end we will construct quasi-greedy basis in $L^1(0,1)$. We will show one very nice property of this basis as well.

Nonlinear approximation in L^p spaces

M. Grigoryan (Armenia)

gmarting@ysu.am

Let $\{\varphi_n(x)\}$ be the Walsh system.

The following theorems are true:

Theorem 1. *For any* $0 < \epsilon < 1$, $p \geq 1$ *and for each function* $f \in L^p(0,1)$ *one can find a function* $\tilde{f} \in L^p(0,1)$, $mes\{x,\ f \neq \tilde{f}\} < \epsilon$, *such that the* *nonzero terms of the sequence* $\{|c_n(\tilde{f})|\}$, *where* $\{c_n(\tilde{f})\}$ *are Fourier-Walsh coefficients of* $\tilde{f}$, *are monotonically decreasing.*

Theorem 2. *For each* $0 < \epsilon < 1$ *there exists a measurable set* $E \subset [0,1]$ *of measure* $| E |> 1-\epsilon$, *a function* $\mu(x)$, $\mu(x) = 1$ *for* $x \in E$ *and a series of the form*

$$\sum_{i=1}^{\infty} a_i \varphi_i, \quad \text{with} \quad |a_i| \searrow 0,$$

such that for every $p \in [1,\infty)$ *and each function* $f \in L^p_\mu(0,1)$ *one can find a function* $\tilde{f} \in L^1(0,1)$, *which coincides with* f *on* E, *and a series of the form*

$$\sum_{i=1}^{\infty} \delta_i a_i \varphi_i \ , \quad \text{where} \ \ \delta_i = 0 \text{ or } 1 \ ,$$

which converges to $\tilde{f}$ *in* $L^p_\mu(0,1)$, *in* $L^1(0,1)$ *and a.e..*

Theorem 3. *For any* $r \in (0,1)$ *and any function* $f \in L^r(0,1)$ *there exists a series of the following form*

$$\sum_{i=1}^{\infty} a_i \varphi_{k_i} \ , \quad \text{where} \quad |a_i| \searrow 0 \quad \text{and} \quad k_1 < k_2 < \dots,$$

which converges to f *in the* $L^r(E)$ *metric.*

The following questions remain open:
Question 1. *Are Theorems 1-3 true for trigonometric system?*
Question 2. *Is it possible to take* $\mu(x) \equiv 1$ *in Theorem 2?*

Dirac operator with linear potential and its perturbations

T. Harutyunyan (Armenia)

hartigr@yahoo.co.uk

We consider the system of differential equations

$$ly \equiv \{\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \frac{d}{dx} + \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} x\} y = \lambda y, \ y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \lambda \in \mathbb{C} \quad (1)$$

We prove that (1) has $L^2[(-\infty,\infty);\mathbb{C}^2]$-solutions only for $\lambda = \lambda_{\pm n} = \pm\sqrt{2n}$ for $n = 0, 1, 2, \ldots$, and corresponding solutions are

$$u_{\pm n}(x) = \begin{pmatrix} \pm\varphi_{n-1}(x) \\ \varphi_n(x) \end{pmatrix},\ n = 1, 2, \ldots,\ u_0(x) = \begin{pmatrix} 0 \\ \varphi_0(x) \end{pmatrix}, \quad (2)$$

where $\varphi_n(x) = \dfrac{H_n(x)e^{\frac{x^2}{2}}}{\sqrt{2^n n!\sqrt{\pi}}}$ and $H_n(x) = (-1)^n e^{x^2}\dfrac{d^n e^{-x^2}}{dx^n}$ (Chebishev-Hermite polynomials).

The eigenvalues of the boundary value problem (BVP) on $(0,\infty)$

$$ly = \lambda y,\ y_1(0) = 0 \quad (3)$$

are only $\lambda_{\pm 2k} = \pm\sqrt{2(2k)}$, $k = 0, 1, \ldots$, and for BVP

$$ly = \lambda y, y_2(0) = 0 \quad (4)$$

are $\lambda_{\pm(2k+1)} = \pm\sqrt{2(2k+1)}$, $k = 0, 1, \ldots$. We give the description of all BVP (with locally integrable coefficients), isospectral with (3) or (4).

We also show how to change the arbitrary finite number of the spectral data, i.e. to add or to diminish the finite number of eigenvalues, to change the "norming constants" without change the eigenvalues.

Dirichlet Problem in weighted spaces and some uniqueness theorems for harmonic functions

H. Hayrapetyan (Armenia)

hhairapet@seua.am

Let B_1 be the class of harmonic functions u in the upper half-plane $\Pi^+ = \{z;\ \operatorname{Im} z > 0\}$ of the complex plane $\mathbb{C}$ satisfying the following condition:

$$|u(z)| < Me^{|z|^\delta},\ 0 \le \delta < 1,\ \Im z > y_0 > 0,$$

where M is a constant depending on y_0. We consider the following Dirichlet problem:
Problem D. Find a harmonic function $u(z) \in B_1$ such that u satisfies the following boundary condition

$$\lim_{y \to 0} \|u(x,y) - f(x)\|_{L^p(\rho)} = 0,$$

where $f \in L^p(\rho)$, $\rho(x) = \rho_1(x)(1+|x|)^\alpha$, and $\rho_1(x) \neq 0$ is a slowly varying function on plus and minus infinity, α is a real number, $\|\cdot\|_{L^p(\rho)}$ is the norm of $L^p(\rho)$:

$$\|f\|_{L^p(\rho)} = \int_{-\infty}^{\infty} |f(x)|^p \rho(x) dx < \infty.$$

The problem D in the case of $p = 1,\ \alpha \leq 0$ has been studied in more general assumptions on the weight function in [1], [2]. It has been proved, that the problem has a solution for every $f \in L^p(\rho)$ and moreover, the general solution has been obtained.

The same problem for the unit disk in the spaces $L^p,\ p > 1$ and L^1 in the case of $\alpha \leq 0$ has been studied in [3], [4].

In this talk we assume that α is an arbitrary real number. In the case of $\alpha \geq 1$ we obtain sufficient conditions on $f(x)$, for the problem D to have a solution. We use the obtained results for investigation of the problems of uniqueness of harmonic functions in the half-plane and in the half-space.

Also we have obtained a theorem concerning uniqueness of harmonic functions in the disk.

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Riemann-Hilbert's type problem for Bitsadze equation in the weighted classes $L^1(\rho)$

M. Hayrapetyan and H. Hayrapetyan (Armenia)

mushegh.h@gmail.com, hhairapet@seua.am

Let $D^+ = \{z; |z| < 1\}$, $T = \{t; |t| = 1\}$. We consider Riemann-Hilbert's type boundary-value problem for Bitsadze equation

$$\frac{\partial^2 u}{\partial \bar{z}^2} = 0, z \in D^+ \tag{1}$$

in the following form:

Problem B**.** Find a function $u(z)$, satisfying (1) and the following boundary conditions

$$\lim_{r\to 1-0} \left\| Re\, a_0(t)u(rt) - f_0(t) \right\|_{L^1(\rho)} = 0$$

$$\lim_{r\to 1-0} \left\| Re\, a_1(t)\frac{\partial u(rt)}{\partial r} - f_1(t) \right\|_{L^1(\rho)} = 0$$

where $f_0'(t), f_1(t) \in L^1(\rho)$, $\rho(t) = |1-t|^\alpha$, $\alpha > 0$, $a_0(t), a_1(t) \in C^{1+\delta}(T)$, $\delta \in [0;1]$, $a_0(t), a_1(t) \neq 0$.

We prove the following statements:

a) If $\min\{n-2\kappa_0; n-2\kappa_1\} > -2$ or $\min\{n-2\kappa_0; n-2\kappa_1\} = -2$ and $\kappa_0 \neq \kappa_1$, then the homogeneous problem B admits $4+2n-2(\kappa_0+\kappa_1)$ linearly independent solutions.

b) If $\min\{n-2\kappa_0; n-2\kappa_1\} < -2$, then the homogeneous problem B admits $4+\widetilde{\kappa_0}+\widetilde{\kappa_1}$ linearly independent solutions, where $\widetilde{\kappa_j} = \max\{n-2\kappa_j-1; -2\}$.

c) If $\min\{n-2\kappa_0; n-2\kappa_1\} = -2$ and $\beta_0^2 = \beta_1^2$, then the homogeneous problem B has one nontrivial solution.

d) If $\min\{n-2\kappa_0; n-2\kappa_1\} = -2$ and $\beta_0^2 \neq \beta_1^2$, then the homogeneous problem B has no nontrivial solutions.

Here $n = [\alpha]+1$, in the case when α is not an integer and $n=\alpha$, if α is integer number, $\kappa_j = ind a_j(t)$.

$$\beta_j = \exp\left(\frac{1}{4\pi i}\int\limits_T \frac{d_j(\tau)}{\tau}d\tau\right),\ d_j(\tau) = \ln\left(t^{2\kappa_j}\overline{a_j(t)}a_j^{-1}(t)\right),\ t\in T,\ j=0,1.$$

We investigate also the non-homogeneous problem. In the case of $\alpha=0$ this problem has been investigated in [1].

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On one class of trigonometric series

R. Hovsepian (Armenia)

We consider series

$$a_0 + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$$

satisfying the following condition:

$$\sum_{k=1}^{n} k \cdot \rho_k = o(n), \quad \rho_k = \sqrt{a_k^2 + b_k^2}. \tag{1}$$

We prove the following

Theorem 1. *For every measurable finite function $f(x)$ there exists a trigonometric series satisfying (1) and converging almost everywhere (a.e.) to $f(x)$.*

From here we get

Theorem 2. *For every measurable finite function $f(x)$ there exists a uniformly smooth primitive function $F(x)$, i.e. there exists a function F such that $F'(x) = f(x)$ a.e. and the modulus of continuity of the function F is of order $o(\delta \ln \delta)$.*

Note that in the N. N. Luzin's famous theorem only the continuity of F is asserted.

Earlier we have proved (negative answer to one question of P. L. Ul'yanov) that in the class of trigonometric series satisfying (1) there exist series with positive partial sums converging everywhere to $f \in \bigcap_{p<\infty} L^p$ that are not Fourier series (the reason is that f can be infinite).

Below we state several versions of uniqueness theorems for the class of trigonometric series satisfying (1).

Theorem 3. *If the partial sums $S_n(x)$ of the trigonometric series satisfying (1) are such that*

1. $\liminf S_n(x)$ *is finite everywhere except a countable set;*
2. $\liminf S_n(x) \geq \varphi(x)$, *where* $\varphi \in L^1$,

then it is a Fourier series.

Theorem 4. *If the trigonometric series satisfying (1) is such that*

1. $S_{n_k}(x) \to f(x)$ *everywhere except a countable set, and* f *is finite;*
2. $f(x) \geq \varphi(x)$ *a.e., where* φ *is an integrable finite derivative,*

then it is a Fourier series.

The following theorem can be considered as an analogue of the Cantor's theorem:

Theorem 5. *If the trigonometric series satisfying (1) a.e. converges to zero, and at the remaining points (except a countable set) $S_{n_k}(x)$ converges (possibly to ∞), and the lower density of the sequence $n_k(x)$ is positive, then the coefficients of the series are all equal to zero.*

Note also that there exist trigonometric null-series satisfying (1) with positive partial sums. From here we deduce the existence of uniformly smooth increasing singular functions.

On compact embeddings of Sobolev's type spaces on metric spaces endowed with measure

I. Ivanishko (Belarus)

iya.ivanishko@tut.by

Let (X, d, μ) be a bounded Hausdorff space endowed with quasimetric d (which generates the topology of X) and positive Borel measure μ. We assume that d and μ are related to each other with the condition of doubling of order $\gamma > 0$: there exists a constant $c > 0$, such that

$$\mu B(x,s) \le c \left(\frac{s}{r}\right)^{\gamma} \mu B(x,r), \quad x \in X, \quad 0 < r < s.$$

Here $B(x,r) = \{y \in X : d(x,y) < r\}$ is the ball with center $x \in X$ and radius $r > 0$.

Also let Ω be the class of all positive increasing functions $\eta : (0,1] \to (0,1]$, such that $\eta(r)r^{-a}$ almost decreases for some $a > 0$ (we say η **a**lmost decreases, if $\eta(r_2) \le c\,\eta(r_1)$ for some $c \ge 1$ and all $0 < r_1 \le r_2$).

For $f \in L^1_{\mathrm{loc}}(X)$ and $\eta \in \Omega$ define the maximal operator as

$$\mathcal{N}_\eta f(x) = \sup_B \frac{1}{\eta(r_B)} \fint_B |f - f(x)|\, d\mu, \quad \fint_B f\, d\mu \equiv \frac{1}{\mu B} \int_B f\, d\mu$$

(we take sup over all balls B, which contain the point $x \in X$, and r_B is the radius of B) and classes

$$C^p_\eta(X) = \{f \in L^p(X) : \|f\|_{S^p_\eta} = \|f\|_p + \|\mathcal{N}_\eta f\|_p < \infty\}.$$

In [1] (in the case of $X = [0,1]^n$, and hence $\gamma = n$) and in [2] (in the general case) the following theorem of embedding of Sobolev's type was proved: *if* $1 < p < q$, $\eta, \sigma \in \Omega$ *and*

$$\eta(t) = \sigma(t) t^{\gamma(1/p - 1/q)}, \tag{1}$$

then

$$C^p_\eta(X) \subset C^q_\sigma(X).$$

Now we'll give conditions for compactness of these embeddings.

Theorem. *Let* $1 < p < q$ *and the function* $\eta, \sigma \in \Omega$ *are connected with the relation (1). Then for every function* $\sigma_0 \in \Omega$ *satisfying* $\lim\limits_{t \to 0} \dfrac{\sigma(t)}{\sigma_0(t)} = 0$ *the follwing embedding*

$$C^p_\eta(X) \subset C^q_{\sigma_0}(X)$$

is compact.

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Turan and Delsarte extremal problems for periodic positive definite functions[2]

V. Ivanov (Russia)

ivaleryi@mail.ru

Continuous positive definite functions appear naturally in function theory, approximation theory, probability theory, discrete geometry, analytic number theory, time series analysis, optics, crystallography, signal processing. Optimization problems in this fields translate into extremal problems for such functions. We discuss some extremal problems for continuous positive definite functions on one dimensional torus $T=[0,1)$, known as Turan and Delsarte problems.

For $0<h\le 1/2$ we define two classes of even continuous positive definite functions:

$$K_T(h)=\Big\{f(x)=\sum_{k\in\mathbb{Z}}\widehat{f}_k e^{ikx}\in C(T):f(0)=1,\ \widehat{f}_k\ge 0,\ \mathrm{supp} f\in[-h,h]\Big\},$$

$$K_D(h)=\Big\{f\in C(T):f(0)=1,\ \widehat{f}_k\ge 0,\ f(x)\le 0,\ h\le |x|\le 1/2\Big\}.$$

In Turan problem it is necessary to calculate the value

$$A_T(h)=\sup\{\widehat{f}_0:f\in K_T(h)\}. \tag{1}$$

Problem (1) was set up by Turan in 1970. It has applications in the analytic number theory, signal processing. This problem was considered by S.B. Stechkin, A.A. Andreev, A.Yu. Popov, D.V. Gorbachev and A.S. Manoshina. S.B. Stechkin found that $A_T(1/q)=1/q$, $q\in\mathbb{N}$.

In Delsarte problem it is necessary to calculate the value

$$A_D(h)=\sup\Big\{\widehat{f}_0:f\in K_D(h)\Big\}. \tag{2}$$

[2]Supported by RFBR (grants no. 05-01-39005 and no. 06-01-00372)

Problem (2) is a particular case of the common Delsarte problem for polynomial expansion of Jacobi which allows to estimate the upper bound of maximal power of spherical codes, contact number of sphere S^{n-1}, density of packing of $\mathbb{R}^n$-space. The common Delsarte problem is exactly solved only for particular cases. Let's mention the works of G.A. Kabatyanskii, V.I. Levenshtein, V.M. Sidelnikov, V.V. Arestov and A.G. Babenko, V.A Yudin, H.J.A. Sloane and A.M. Odlyzko and other authors. Problem (2) corresponds to the case $n = 2$.

We have solved problems (1), (2) for all h, showing particulary that $A_T(h) = A_D(h)$ [1,2]. Our solution of these problems for rational h is based on solution of discrete variant of a well known Fejer problem about the greatest value at zero of nonnegative trigonometric polynomial with fixed average value. Reduction of problem (1) to discrete variant of Fejer problem was realized by D.V. Gorbachev and A.S. Manoshina.

Together with integral Turan problem (1) we are interested in pointwise Turan problem, where it is necessary to find

$$A_T(x,h) = \sup\{f(x) : f \in K_T(h)\}, \quad 0 \le |x| \le h. \tag{3}$$

For $h = 1/2$ the function (3) was found by V.V. Arestov, E.E. Berdysheva and H. Berens. For remaining h functions (3) are unknown. Let's note that for rational x, h problem (3) is reduced to discrete variant of Fejer problem about the greatest coefficients of nonnegative trigonometric polynomial with fixed average value.

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Orthonormal spline systems with arbitrary knots as bases in $H_1[0,1]$

A. Kamont, G. Gevorgian (Poland, Armenia)

a.kamont@impan.gda.pl, ggg@ysu.am

One of the results in our study of general Franklin systems – i.e. orthonormal systems of piecewise linear functions corresponding to a given sequence of knots – is the characterization of sequences of knots for which the corresponding general Franklin system is a basis or an unconditional basis in $H_1[0,1]$. In this talk, we present a version of these results for higher order orthonormal spline systems. We give a simple geometric characterization of sequences of knots for which the corresponding orthonormal spline system of order r is a basis in the atomic Hardy space $H_1[0,1]$. We give also some results and examples in this direction for $H_p[0,1]$, $1/2 < p < 1$.

Some Equivalent Multiresolution Conditions on Locally Compact Abelian Groups

R.A. Kamyabi-Gol and R. Raisi Tousi (Iran)

kamyabi@ferdowsi.um.ac.ir, re_ra97@stu-mail.um.ac.ir

Conditions under which a function generates a multiresolution analysis are investigated. The definition of the spectral function of a shift invariant space is generalized from $\mathbb{R}^n$ to a locally compact abelian group and the union density and intersection triviality properties of a multiresolution analysis are characterized in terms of the spectral functions. Finally an alternative definition of a multiresolution analysis is obtained.

On the theory of uniformly continuous mappings

B. Kanetov and T. Kasymova (Kyrgyz Republic)

tumar2000@mail.ru

It is known that for big cardinals the uniform τ-finally paracompact spaces, which were introduced by A. A. Borubaev [1], coincide with the uniform paracompacts in the M. Rice's sense [2].

Since any space can be considered as a map into a point, in this work the uniform τ-finally paracompact spaces are extended to maps of uniform spaces. In particular, we prove that uniform τ-finally paracompacticity is preserved in the side of the preimage under the uniform τ-finally paracompact mappings.

Definition 1. *The mapping $f : (X, U) \to (Y, V)$ of the uniform space (X, U) on the uniform space (Y, V) is called uniformly τ-finally paracompact, if for every open covering $\alpha \in U$ of the uniform space (X, U) there exist open covering $\beta \in V$ of the space (Y, V), $\gamma \in U$ and subcovering $\alpha_0 \subset \alpha$ with the cardinal $\leq \tau$ such that $f^{-1}\beta \wedge \gamma \succ \alpha_0^{\angle}$.*

If $f : (X, U) \to (Y, V)$ is an uniformly τ-finally paracompact map and $Y = \{y\}$, then (X, U) is uniformly τ-finally paracompact space.

The composition of two uniformly τ-finally paracompact maps is again uniformly τ-finally paracompact map.

Theorem. *If the mapping f and the space (Y, V) are uniformly τ-finally paracompact, then the space (X, U) is also uniformly τ-finally paracompact, and conversely, if the space (X, U) is uniformly τ-finally paracompact, then the mapping f is uniformly τ-finally paracompact.*

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On the spherical grassmannian

K. Kangni (Cote d'Ivoire)

kangnikinvi@yahoo.fr

Let G be a locally compact group and let K be a compact subgroup of G. We study some spherical function according to an unitary dual of K which is an extension of a classical K-spherical function. This class of functions is very important in noncommutative harmonic analysis.

On Riemann sums and maximal functions

G. Karagulyan (Armenia)

g.karagulyan@yahoo.com

We investigate problems on almost everywhere convergence of subsequences of Riemann sums

$$R_n f(x) = \frac{1}{n}\sum_{k=0}^{n-1} f\left(x + \frac{k}{n}\right), \quad x \in \mathbb{T}.$$

We establish a relevant connection between Riemann and ordinary maximal functions, which allows to use techniques and results of the theory of differentiations of integrals in $\mathbb{R}^n$ in mentioned problems. In particular, we prove that for a definite sequence of infinite dimension n_k Riemann sums $R_{n_k} f(x)$ converge almost everywhere for any $f \in L^p$ with $p > 1$.

A remark about asymptotic property of commutators

M. Karakhanyan (Armenia)

karakhanyan@yahoo.com

It has been proved in [1] that if a, b, x are elements of complex Banach algebra with unite element, such that $[a, b] = ab - ba = 0$ and $\|\exp(ita)\| = o\left(|t|^{1/2}\right)$, $\|\exp(itb)\| = o\left(|t|^{1/2}\right)$, for real t, as $t \to \pm\infty$ and $[a + ib, x] = 0$, then $[a - ib, x] = 0$.

Later in [2] was shown that in the above mentioned theorem the condition $o\left(|t|^{1/2}\right)$ cannot be replaced by $O\left(|t|^{1/2}\right)$, but the central role here played weakening of the condition $[a, b] = 0$.

In [3] the class $Gr(A)$ was introduced, weakening the condition $[a, b] = 0$. In this work we consider asymptotic variants of this results.

Let A be a Banach algebra with unite element $\mathbf{1}$ over the field of complex numbers $\mathbb{C}$ (we assume that $\|\mathbf{1}\| = 1$ and $\|xy\| \leqslant \|x\| \cdot \|y\|$ for all $x, y \in A$).

$\mathbb{C}$ – linear functional $\varphi : A \to \mathbb{C}$ is called "state", if $\|\varphi\| = \varphi(\mathbf{1}) = 1$. The set of all states $St(A)$ is $\sigma(A^*, A)$ – compact, convex subset of the dual space A^*. Remember that (see [3, 4]) the element $\mathbf{h} \in A$ is called "hermitian", if $\varphi(\mathbf{h}) \subset \mathbb{R}$ for all $\varphi \in St(A)$, and the later is equivalent to the condition $\|\exp(it\mathbf{h})\| = 1$ for all real t. the set of all hermitian elements $H(A)$ of the algebra A is a closed $\mathbb{R}$-linear subspace of the algebra A. Note that the element $a \in A$ is called hermite-decomposable, if it admits a representation of the form $a = \mathbf{h} + i\mathbf{k}$, where $\mathbf{h}, \mathbf{k} \in H(A)$. Such representation, if exists, is unique. The class of all hermite-decomposable elements of algebra A is denoted by $H_{\mathbb{C}}(A)$, and this is closed, $\mathbb{C}$-linear subspace of A, which appears in the same time to be Lie algebra with respect to commutator.

Let's choose on algebra A a local convex topology τ with the following properties: the mapping $(A, \|\cdot\|) \to (A, \tau)$ is continuous, the multiplication is separately τ-continuous. Note that standard topologies in algebras of operators possess this properties. Remember (see [3]), that the element $a \in A$ belongs to the class $Gr(A)$, if there exists an element $b \in A$ such that

$$\max\left\{\left\|\exp(-\lambda b)\cdot\exp\left(\bar{\lambda}a\right)\right\|;\left\|\exp(-\bar{\lambda}a)\cdot\exp\left(\lambda b\right)\right\|\right\} = o\left(|\lambda|^{1/2}\right),$$

as $|\lambda| \to \infty$, $\lambda \in \mathbb{C}$.

Theorem 1. *Let A be a complex Banach algebra with unite element, with given above mentioned local-convex topology τ. Then for every neighborhood $U \subset A$ of zero in topology τ, there exists a neighborhood $V \subset A$ of zero in the same topology τ, such that if $x \in A$, $\|x\| \leqslant 1$, $a \in Gr(A)$, $[a, x] \in V$, then $[b, x] \in U$.*

As a consequence we obtain

Theorem 2. *Let A be a complex Banach algebra with unite element and the element $a \in Gr(A)$. Then for every $\varepsilon > 0$ there exists $\delta > 0$, such that if $x \in A$, $\|x\| \leqslant 1$ and $\|[a, x]\| < \delta$, then $\|[b, x]\| < \varepsilon$.*

Corollary 1. *Let A be a complex Banach algebra with unite element and $a \in Gr(A) \bigcap H_{\mathbb{C}}(A)$. Then for every $\varepsilon > 0$ there exists $\delta > 0$ such that if $x \in A$, $\|x\| \leqslant 1$ and $\|[a, x]\| < \delta$, then $\|[a^{+}, x]\| < \varepsilon$.*

Using Theorem 1, we can prove the following result

Theorem 3. *Let A be a complex Banach algebra with unite element, with given above mentioned local-convex topology τ. Then for every neighborhood $U \subset A$ of zero in the topology τ, there exists a neighborhood $V \subset A$ of zero in the same topology τ, such that if $x \in A$, $\|x\| \leqslant 1$, $a \in Gr(A)$ and $ax - xb \in V$, then $bx - xa \in U$.*

In the case when the topology τ coincides with the topology of the norm on A the following is true:

Theorem 4. *Let A be a complex Banach algebra with unite element and $a \in Gr(A)$. Then for every $\varepsilon > 0$ there exists $\delta > 0$, such that if $x \in A$, $\|x\| \leqslant 1$ and $\|ax - xb\| < \delta$, then $\|bx - xa\| < \varepsilon$.*

Corollary 2. *Let A be a complex Banach algebra with unite element, $a \in Gr(A) \bigcap H_{\mathbb{C}}(A)$. Then for every $\varepsilon > 0$ there exists $\delta > 0$ such that if $x \in A$, $\|x\| \leqslant 1$, and $\|ax - xa^{+}\| < \delta$, then $\|a^{+}x - xa\| < \varepsilon$.*

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Weighted Anisotropic Integral Representations of Holomorphic Functions in the Unit Ball in $\mathbb{C}^n$

A. Karapetyan (Armenia)

armankar2005@rambler.ru

Denote by B_n the unit ball in the complex n-dimensional space $\mathbb{C}^n$: $B_n = \{w \in \mathbb{C}^n : |w| < 1\}$ and let dm be the Lebesgue measure in $\mathbb{C}^n \equiv \mathbb{R}^{2n}$.

Theorem. *Assume that* $1 \leq p < +\infty, \alpha = (\alpha_1, \cdots, \alpha_n) \in \mathbb{R}^n$ *and for* $1 \leq k \leq n$

$$\alpha_n + \alpha_{n-1} + \cdots + \alpha_k > -(n+1-k).$$

Further, assume that $\beta = (\beta_1, \cdots, \beta_n) \in \mathbb{C}^n$ *and for* $1 \leq k \leq n$

$$\sum_{i=k}^{n} Re\beta_i \geq \sum_{i=k}^{n} \alpha_i, \quad if \quad p = 1,$$

$$\sum_{i=k}^{n} Re\beta_i > \frac{\sum\limits_{i=k}^{n}\alpha_i + n + 1 - k}{p} - (n+1-k), \quad if \quad 1 < p < +\infty.$$

Then each function f holomorphic in B_n and satisfying the condition

$$\int_{B_n} |f(w)|^p \prod_{k=1}^{n} (1 - |w_1|^2 - |w_2|^2 - \cdots - |w_k|^2)^{\alpha_k} dm(w) < +\infty,$$

admits the following integral representations:

$$f(z) = \int_{B_n} f(w) S_\beta(z; w) \prod_{k=1}^{n} \left(1 - \sum_{i=1}^{k} |w_i|^2\right)^{\beta_k} dm(w), \quad z \in B_n,$$

$$\overline{f(0)} = \int_{B_n} \overline{f(w)} S_\beta(z; w) \prod_{k=1}^{n} \left(1 - \sum_{i=1}^{k} |w_i|^2\right)^{\beta_k} dm(w), \quad z \in B_n,$$

where the kernel $S_\beta(z; w)$ has the following properties ($z \in B_n, w \in \overline{B_n}$):

(a) $S_\beta(z; w)$ is holomorphic in $z \in B_n$;

(b) $S_\beta(z; w)$ is holomorphic in $w \in B_n$ and continuous in $w \in \overline{B_n}$;

(c) the estimate

$$|S_\beta(z; w)| \leq const(n; \beta)(1 - |z|)^{-(n+1+Re\beta_n+l)}$$

is valid, where $l = \sum\limits_{Re\beta_i > 0} Re\beta_i$.

Remark. If $n = 2$ and $Re\beta_2 > -1$, $Re\beta_2 + Re\beta_1 > -2$, then

$$S_\beta(z; w) = \frac{1}{\pi^2}\left[\frac{\beta_1(\beta_2 + 1)}{(1 - z_1\overline{w_1})^{\beta_1+1}(1 - z_1\overline{w_1} - z_2\overline{w_2})^{\beta_2+2}} + \right.$$

$$\left. + \frac{(\beta_2 + 1)(\beta_2 + 2)}{(1 - z_1\overline{w_1})^{\beta_1}(1 - z_1\overline{w_1} - z_2\overline{w_2})^{\beta_2+3}}\right].$$

Neotherianity of the regular operator with constant coefficients in the domain

G. Karapetyan and A. Darbinyan (Armenia)

armankri@yahoo.com

Definition 1. *For the polyhedron $\Re$ and bounded domain $\Omega \subset \mathbb{R}^n$ denote by $H^{\Re}(\Omega)$ the space of all measurable functions $\{u\}$ with finite norm*

$$||u||_{\Re}(\Omega) \equiv \left(\sum_{\alpha\in\Re} \int_{\Omega} |D^{\alpha}u(x)|^2 dx \right)^{\frac{1}{2}}, \tag{1}$$

and by $\dot{H}^{\Re}(\Omega)$ denote the completion of the set $C_0^{\infty}(\Omega)$ with norm (1).

For an open bounded cube Δ ($\overline{\Omega} \subset \Delta$) and function $\Phi \in \dot{H}^{\Re}(\Delta)$ we denote $H^{\Re}(\Omega, \Phi) = \{u \in H^{\Re}(\Omega); (u - \Phi) \in \dot{H}^{\Re}(\Omega)\}$.

We assume that k is natural number, $\Phi \in \dot{H}^{(k+1)\Re}(\Delta)$ is a fixed function, the domain $\Omega \subset \mathbb{R}^n$ satisfies to the rectangle's condition (cf. [1]), $k\Re = \{k\alpha = (k\alpha_1, k\alpha_2, ..., k\alpha_n), \alpha \in \Re\}$, and $\Re$ is completely regular polyhedron (cf. [2]).

Definition 2. *We say that the linear differential operator $P(D) \equiv \sum_{\alpha\in\Re} p_{\alpha}D^{\alpha}$ is regular, if for some constant $\chi > 0$*

$$\left|P^0(\xi)\right| \geq \chi \sum_{\alpha\in\partial'\Re} |\xi^{\alpha}| \quad \forall \xi \in \mathbb{R}^n,$$

where $P^0(\xi) \equiv \sum_{\alpha\in\Re} p_{\alpha}\xi^{\alpha}$.

The following theorems are true.

Theorem 1. *The index of the regular operator with constant coefficients of the form $P(D) \equiv \sum_{\alpha\in\Re} p_{\alpha}D^{\alpha}$, as an operator acting from $H^{(k+1)\Re}(\Omega, \Phi)$ to $H^{k\Re}(\Omega, \Phi)$, is finite and equal to zero.*

Theorem 2. *Let* $P(D) \equiv \sum\limits_{\alpha\in\Re} p_\alpha D^\alpha$ *is a linear differential operator with constant coefficients, acting from* $H^{(k+1)\Re}(\Omega,\Phi)$ *to* $H^{k\Re}(\Omega,\Phi)$. *Then the following conditions are equivalent:*

1. $P(D)$ *is regular;*

2. $P(D)$ *is neotherian from* $H^{(k+1)\Re}(\Omega,\Phi)$ *to* $H^{k\Re}(\Omega,\Phi)$*;*

3. *there exists* $C > 0$ *such that* $\forall u \in H^{(k+1)\Re}(\Omega,\Phi)$

$$\|u\|_{(k+1)\Re}(\Omega) \leq C\left(\|P(D)u\|_{k\Re}(\Omega) + \|u\|_{L_2}(\Omega) + \|\Phi\|_{(k+1)\Re}(\Omega)\right)$$

4. *the operator* $P(D)$ *is bounded from* $H^{(k+1)\Re}(\Omega,\Phi)$ *to* $H^{k\Re}(\Omega,\Phi)$ *and possesses regularizer.*

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Remark on the Compressed Sensing

B. Kashin (Russia)

kashin@mi.ras.ru

In this talk we clarify the connections between the new direction in the signal analysis called "Compressed Sensing" and problems of estimation of n-widths, which were studied in 70-80's of the past century.

Weighted estimates of the boundary behavior of the differentiable functions and elliptic boundary problems

I. Katkovskaya and V. Krotov (Belarus)

vkrotov@cosmostv.by

Let $n \geq 2$, $\Omega \subset \mathbb{R}^n$ be a bounded Lipschitz domain, $B_r(x) \subset \partial\Omega$ be the surface ball with center at the point $x \in \partial\Omega$ and radius $r > 0$. Also let $m \in \mathbb{N}$, $0 < p < \frac{n-1}{m}$, and suppose we are given a function ε satisfying $\varepsilon(t) \uparrow$, $\varepsilon(t)t^{-1+\frac{mp}{n-1}} \downarrow$ and

$$D_\varepsilon(x) = \{y \in \Omega : |x - y| < \varepsilon(\operatorname{dist}(y, \partial\Omega)\},$$

$$N_\varepsilon u(x) = \sup\{|u(y)| : y \in D_\varepsilon(x)\}$$

(in the case of $\varepsilon(r) = r$ we simply write Nu).

Theorem. *Let ν be an outer measure on $\partial\Omega$ satisfying*

$$\nu(B_r(x)) \leq c[\varepsilon^{-1}(r)]^{n-mp-1}$$

Then for every $u \in C^m(\Omega)$, $N(\nabla^m u) \in L^p(\partial\Omega)$, the following inequality holds:

$$\left(\int_0^\infty \lambda^{p-1}\nu\{N_\varepsilon u > \lambda\}\, d\lambda\right)^{1/p} \leq c\left\{\|\nabla^m u\|_{L^p(\partial\Omega)} + \sum_{k=1}^{m} |\nabla^k u(x_0)|\right\}$$

($x_0 \in \Omega$ is a fixed point). Moreover, for ν-almost all $x \in \partial\Omega$ The function u has $D_\varepsilon(x)$-limit.

In particular, we can take in this theorem $\nu = H^s_\infty$ - the Hausdorff capacity, generated by the function $s(r) = [\varepsilon^{-1}(r)]^{n-mp-1}$.

Analogous propositions are true also for the case $p > \frac{n-1}{m}$, and in this case we can obtain the same type estimates also for partial derivatives of u. Furthermore, all of this results are of local nature (behavior).

This propositions complete some investigations of [1–3] and are based on methods of [4]. The natural field of applications for above mentioned results is the tangential boundary behavior of the solutions of the elliptic boundary problems (cf. [1]).

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A Generalization of Frames in Banach Spaces

S. Kaushik (India)

shikk2003@yahoo.co.in

Fusion Banach frames satisfying property S have been introduced. A sufficient condition under which a weakly compactly generated Banach space has a fusion Banach frame satisfying property S has been obtained. Also, a necessary and sufficient condition for a fusion Banach frame to satisfy property S has been given. Further, fusion Banach frames satisfying property S have been characterized in terms of closedness of certain subspaces, of the dual space, in the $weak^*$- topology. Finally, a sufficient condition under which a fusion Banach frame satisfies property S has been given.

An example of solution of an inverse problem, using Needlet tight frame : Denoising of Radon Transform

G. Kerkyacharian (France)

kerk@math.jussieu.fr

The introduction of Wavelets orthonormal basis, more then twenty years ago, was a great improvement for people dealing with signal analysis, statistic, image processing,... . Lot of problems were solved with this tool, for instance the construction of adaptive estimators in non parametric statistic, specially : density estimation, regression, inverse problems... (e.g. [1], [2]). Actually, the main point was the localization properties of these basis.

It appears now that, for statistical purpose, we have to built basis which are linked with the geometry of the problem, and suitably localized. Of course it is not always possible to have orthonormal basis. These last years, Narcowich, Petrushev, Ward, Xu, have build tight localized frames on the sphere, the interval, the ball. ([3],[4], [5]) These frames, named "Needlets" by these authors, are exactly modelized on the geometry of some operators and their singular value decomposition, which are linked with some statistical inverse problems : Wicksell problem ([6]), dealing with Jacobi polynomials, Radon transform on the ball, the study of the Cosmological Microwave Background ([7])...

In this talk, I will try to show an application of these tools to Tomography, and to show how needlets build on the singular value decomposition of the Radon transform, could provide an adaptive estimator for the denoising problem.

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On absolute and unconditional convergence of series by general Franklin system

K. Keryan and G. Gevorgian (Armenia)

karenkeryan@yahoo.com, ggg@ysu.am

It is well known that the numerical series $\sum_{n=0}^{\infty} a_n$ absolutely converges if and only if it converges unconditionally. For functional series the notion of absolute a.e. convergence is not equivalent to the notion of unconditional a.e. convergence. The main point is that the exceptional set depends on the rearrangement. For example, if $\{R_n(x)\}_{n=1}^{\infty}$ is the Rademacher's system, $\sum_{n=1}^{\infty} a_n^2 < \infty$ and $\sum_{n=1}^{\infty} |a_n| = \infty$, then the series $\sum_{n=1}^{\infty} a_n R_n(x)$ converges unconditionally a.e. (see [1], page. 36), but diverges absolutely a.e.. However, for the Haar system the following is true (see [2])

Theorem. *(E. Nikishin, P. Ul'yanov). The series*

$$\sum_{n=0}^{\infty} a_n \chi_n(x),$$

where $\{\chi_n(x)\}_{n=1}^{\infty}$ *is the Haar system, converges unconditionally a.e. on the set* $E \subset [0;1]$ *if and only if for almost all* $x \in E$ *the sum* $\sum\limits_{n=0}^{\infty} |a_n \chi_n(x)|$ *is finite.*

G. Gevorgian [3] extended this theorem to the series by classical Franklin system.

In [4] this theorem was proved for series by general Franklin system under some conditions on the partition of $[0;1]$.

Here we prove the theorem for general Franklin series without any restriction. Namely,

Theorem. *Let* $\{f_n(x)\}_{n=0}^{\infty}$ *be the Franklin system generated by admissible sequence* $\mathcal{T}$. *Then the series* $\sum_{n=0}^{\infty} a_n f_n(x)$ *converges unconditionally a.e. on some set E if and only if, if it converges absolutely a.e. on E.*

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Lipschitz multivalued mappings in optimization linear problems

R. Khachatryan and R. Avetisyan (Armenia)

protectdrive@gmail.com

Different aspects related to Lipschitz properties of multivalued mappings were considered in [1, 2]. In [1] it is shown that a convex multivalued mapping with compact set of values is a Lipschitz mapping, and the result was extended to the so-called polyhedral multivalued mappings. Under some natural assumption, it is shown in [2] that the multivalued mapping of the form $a(x) = \{y \in F : f(x, y) \leqslant 0\}$ is Lipschitz. However, such multivalued mappings cover only a rather narrow class of parameterized extremal problems.

The present paper points at a sufficient condition under which the intersection of a Lipschitz mapping and a convex mapping is a Lipschitz mapping. Also some parameterized linear programming problems are considered and the marginal mappings are found to satisfy the Lipschitz condition

Theorem 1. *Let $a(x) = \{y \in R^m : \langle f_i(x), y\rangle \leqslant b_i,\, i = 1, \ldots, p\}$, where the functions $f_i(x)$ $(i = 1, \ldots, p)$ satisfy the Lipschitz condition at a point $x_0 \in R^n$. If $a(x) \neq \emptyset$ and $\mathrm{Im}\, \mathrm{a} = \bigcup\limits_{\mathrm{x} \in \mathrm{U}(\mathrm{x}_0)} \mathrm{a}(\mathrm{x})$ is a bounded set, then the* *mapping a is Lipschitz from above at a point x_0, i.e. there exists a neighborhood $U(x_0)$ of x_0 and constant $L > 0$ such that $a(x) \subseteq a(x_0) + L\|x - x_0\|B_1(0)$, $\forall x \in U(x_0)$.*

Theorem 2. *Let X and Y be Banach spaces and let $a_1 : X \to 2^Y$ be a multivalued mapping with convex, compact values, satisfying Lipschitz condition at a neighborhood of $x_0 \in X$. If $a_2 : X \to 2^Y$ is a closed, convex mapping such that $x_0 \in \mathrm{intdoma_2}$ and $0 \subseteq \mathrm{int}\{\mathrm{a_1(x_0) - a_1(x_0)}\}$, then the mapping $a(x) \equiv a_1(x) \cap a_2(x)$ satisfies the Lipschitz condition at a neighborhood of x_0.*

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Convergence of the Fourier series for functions of countably many variables

N. Kholshchevnikova (Russia)

Kholshchevnikova@gmail.com

Let $\mathbb{T}^\infty$ and $[0,1]^\infty$ are the Cartesian products of countably many one-dimensional tori $\mathbb{T} = \mathbb{R}/\mathbb{Z}$ or segments $[0,1]$, respectively. The Lebesgue measure is defined on $\mathbb{T}^\infty$ and $[0,1]^\infty$ as on the product of the spaces with one-dimensional Lebesgue measure. Thus, it is possible to consider L^p and different functional spaces on $\mathbb{T}^\infty$ and $[0,1]^\infty$.

The infinite dimensional trigonometric system:

$$\theta_{n_1,\ldots,n_p}(x) = \prod_{r=1}^{p} e^{2\pi i n_r x_r},\ p \in \mathbb{N},\ n_r \in \mathbb{Z},\ x = (x_1, \ldots, x_p, \ldots) \in \mathbb{T}^\infty$$

was examined by B. Jessen and is called the Jessen system.

Similarly, infinite dimensional Walsh, Haar and other systems on $[0,1]^\infty$ are defined. The rectangular, cubic and another kinds of the convergence are defined for the series on infinite dimensional systems. We'll consider some questions on the convergence of the Fourier series for the functions of countably many variables from special functional classes.

On the Fourier-Haar coefficients of functions from L_p

A. Kobelyan (Armenia)

a_kobelyan@ysu.am

Let $\{\chi_k\}_{k=1}^\infty$ is a Haar system.

In 1991 M.G. Grigorian proved (see [1],[2]) the following fact: for any sequence $\{a_k\}_{k=1}^\infty \notin l_2$, $a_n \searrow 0$, and for every $\varepsilon > 0$ there exists a measurable set $E \subset [1,0]$ with measure $|E| > 1-\varepsilon$, such that for every $f \in L^1[0,1]$ there exist a function $\tilde{f} \in L^1[0,1]$ and a sequence $n_k \uparrow$, so that $\tilde{f}(x) = f(x)$ on E and Fourier-Haar coefficients of the modified function $\tilde{f}$ are

$$c_n(\tilde{f}) = \int_0^1 \tilde{f}(x)\chi_n(x)dx = \begin{cases} a_{n_k}, & \text{if } n = n_k; \\ 0, & n \notin \{n_k\}_{k=1}^\infty. \end{cases}$$

The following theorem take place:

Theorem. *Let $\sum_{k=1}^\infty a_k\chi_k$ is a series , for which*

$$\overline{\lim_{m\to\infty}} \sum_{n=1}^m a_n\chi_n(x) \overset{a.e.}{=} +\infty, \quad \lim_{n\to\infty} a_n\chi_n(x) \overset{a.e.}{=} 0.$$

Then for any measurable function f and $p > 1$, $\varepsilon > 0$ there exist a function $g(x) \in L^p[0,1]$ and a sequence $\{\delta_k\}_{k=1}^\infty$, $\delta_k \in \{1;0\}$, such that $mes\{x, g(x) \neq f(x)\} < \varepsilon$ and $c_k(g) = \delta_k a_k$, $k = 1,2,....$

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Fourier operators in weighted function spaces with non-standard growth and applications to BVP

V. Kokilashvili (Georgia)

kokil@rmi.acnet.ge

The goal of our talk is to present the solution of some two-weighted problems for Fourier operators and approximation theory in various Banach function spaces with non-standard growth conditions. Applications to the boundary value problems of analytic and harmonic functions in "bad" domains in frame of weighted Lebesgue spaces variable exponent are aimed.

On approximation of functions in L_p, $0 < p < 1$ by trigonometric polynomials with gaps

Yu. Kolomoitsev (Ukraine)

kolomus1@mail.ru

Let A be a proper subset of $\mathbb{Z}$. Then the system $\{e^{ikx}\}_{k\in A}$ is not complete in the space $L_p(0,2\pi)$ for $p \geq 1$. A somewhat different situation arises in L_p with $p < 1$. A.A. Talalyan was the first who proved that there exists an infinite set $B \subset \mathbb{Z}$ such that the system $\{e^{ikx}\}_{k\in\mathbb{Z}\setminus B}$ is complete in $L_p(0,2\pi)$, $0<p<1$. In [1] it is proved the following result: if $B=\{n_k\}_{k\in\mathbb{Z}}$ is a convex sequence for $k\geq 1$ and $n_{-k}=-n_k$, then the system $\{e^{ikx}\}_{k\in\mathbb{Z}\setminus B}$ is complete in $L_p(0,2\pi)$, $0<p<1$, if and only if $\lim_{k\to\infty}(n_{k+1}-n_k)=\infty$.

For some sets $A\subset\mathbb{Z}$ that possess certain arithmetic properties, the estimates for the best approximation

$$E_n(f,A)_p := \inf_{T\in span\{e^{ikx}\}_{k\in A\cap(-n,n)}} \|f-T\|_{L_p(0,2\pi)}$$

are obtained.

Let us consider the class of functions

$$H^{\alpha}_{1,p} := \{ f\in L : \sup_{n\geq 1} n^{\alpha} E_{n-2}(f,\mathbb{Z})_p \leq 1\}.$$

Theorem *Let* $0<p<1$, $Q=\mathbb{Z}\setminus\{\pm q^k\}_{k\in\mathbb{N}}$ $(q\geq 2)$ *and* $n\in\mathbb{N}$. *The following statements hold:*

(i) *if* $0<\alpha<\frac{1}{p}-1$, *then*

$$\sup_{f\in H^{\alpha}_{1,p}} E_n(f,Q)_p \asymp \frac{1}{n^{\alpha}};$$

(ii) *if* $\frac{1}{p}-1\leq\alpha\leq\frac{2}{p}-2$, *then*

$$\frac{C_1}{n^{\frac{1}{p}-1}} \leq \sup_{f\in H^{\alpha}_{1,p}} E_n(f,Q)_p \leq \frac{C_2 \ln^{\frac{1}{p}}(n+1)}{n^{\frac{1}{p}-1}};$$

(iii) *if* $\alpha>\frac{2}{p}-2$, *then*

$$\sup_{f\in H^{\alpha}_{1,p}} E_n(f,Q)_p \asymp \frac{1}{n^{\frac{1}{p}-1}}$$

where $\asymp$ *is two-sided inequalities with positive constants that depend only on* p, q *and* α; C_1 *and* C_2 *are the positive constants that depend only on* p *and* q.

Similar results are obtained for the sets $S=\mathbb{Z}\setminus\{\pm k^s\}_{k\in\mathbb{N}}$ $(s\geq 2)$ and $M=\mathbb{Z}\setminus(-m,m)$.

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Sharp relations between different norms in Lorentz spaces

V. Kolyada (Sweden)

viktor.kolyada@kau.se

We study the Lorentz spaces $L^{p,s}(R,\mu)$ in the range $1 < p < s \leq \infty$, for which the standard functional

$$||f||_{p,s} = \left(\int_0^\infty (t^{1/p} f^*(t))^s \frac{dt}{t} \right)^{1/s}$$

is only a quasi-norm. We find the optimal constant in the triangle inequality for this quasi-norm, which leads us to consider the following decomposition norm:

$$||f||_{(p,s)} = \inf \left\{ \sum_k ||f_k||_{p,s} \right\},$$

where the infimum is taken over all finite representations $f = \sum_k f_k$. We prove that the decomposition norm and the dual norm

$$||f||'_{p,s} = \sup \left\{ \int_R f g \, d\mu : ||g||_{p',s'} = 1 \right\}$$

agree for all values $p, s > 1$. We find also optimal constants in inequalities between the norms $||f||_{p,s}$, $||f||'_{p,s}$, and

$$||f||^*_{p,s} = \left(\int_0^\infty (t^{1/p} f^{**}(t))^s \frac{dt}{t} \right)^{1/s},$$

where

$$f^{**}(t) = \frac{1}{t} \int_0^t f^*(u) du.$$

The talk is essentially based on our joint work with S. Barza and J. Soria (to appear).

On convergence of greedy approximations for the trigonometric system

S. Konyagin (Russia)

konyagin@ok.ru

We study the following nonlinear method of summation of trigonometric Fourier series. Consider a periodic function $f \in L_p(\mathbb{T})$, $1 \le p \le \infty$, ($L_\infty(\mathbb{T}) = C(\mathbb{T})$), defined on the torus $\mathbb{T}$. Let a number $m \in \mathbb{N}$ be given and Λ_m be a set of $k \in \mathbb{Z}$ with the properties:

$$\min_{k\in\Lambda_m} |\hat{f}(k)| \ge \max_{k\notin\Lambda_m} |\hat{f}(k)|, \quad |\Lambda_m| = m,$$

where

$$\hat{f}(k) := (2\pi)^{-1} \int_{\mathbb{T}} f(x) e^{-ikx} dx$$

is a Fourier coefficient of f. We define

$$G_m(f) := S_{\Lambda_m}(f) := \sum_{k\in\Lambda_m} \hat{f}(k) e^{i(k,x)}$$

and call it a m-th greedy approximant of f with regard to the trigonometric system $\mathcal{T} := \{e^{i(k,x)}\}_{k\in\mathbb{Z}}$. Clearly, a m-th greedy approximant may not be unique. We discuss the convergence of greedy approximants for trigonometric Fourier expansion in $L_p(\mathbb{T})$.

Greediness of the wavelet system in variable exponent Lebesgue spaces

T. Kopaliani (Georgia)

t.kopaliani@math.sci.tsu.ge

Wavelet systems are well known examples of greedy bases for many function and distribution spaces. Temlyakov showed that the Haar basis (and any wavelet systems L^p-equivalent to it) is greedy in the Lebesgue spaces $L^p(\mathbb{R}^n)$ for $1 < p < \infty$. When wavelets have sufficient smoothness and decay, they are also greedy bases for the Sobolev and Triebel-Lizorkin spaces. The Haar system is the "best" basis in a rearrangement invariant space. It is true that if a rearrangement invariant space has an unconditional basis then the Haar system is such a basis. P.Wojtaszczyk showed that if X is a rearrangement invariant space on $[0,1]^n$ and Haar system normalized in X is a greedy basis in X then $X = L^p[0,1]^n$ for some $1 < p < \infty$ (with equivalent norm). Note that there are examples of rearrangement invariant space with symmetric, and also with greedy basis.

We point out that when Hardy-Littlewood maximal operator is bounded on the space $L^{p(t)}(\mathbb{R})$, $(1 < a \leq p(t) \leq b < \infty;\ t \in \mathbb{R})$, well known characterization of $L^p(\mathbb{R})$ spaces $(1 < p < \infty)$ in terms of orthogonal wavelet bases extends to the space $L^{p(t)}(\mathbb{R})$. As an application of this result, we show that such bases normalized in $L^{p(t)}(\mathbb{R})$ are not a greedy bases of $L^{p(t)}(\mathbb{R})$ if $p(t) \neq \text{const}$.

Square Roots of Convolutions

T. Korner (UK)

T.W.Korner@dpmms.cam.ac.uk

Wiener and Wintner gave an example of a singular measure μ whose convolution square $\mu * \mu$ lay in L^1. Later Saeki improved this to make $\mu *$

μ continuous. I shall give results along these lines linking the Hausdorff dimension of the support of μ with the Lipschitz (or Hölder) smoothness of $\mu * \mu$ in a way which is close to best possible.

On Constants in Evaluations of Spline Approximation for Binary Grid

O. Kosogorov and Yu. Demjanovich (Russia)

okosogorov@mail.ru, Yuri.Demjanovich@JD16531.spb.edu

Evaluations of spline approximation are discussed in a number of works (for example, see [1-5]). In the case of equidistant grid the exact evaluations are well-known, and also there are such evaluations for quasi-equidistant grids. Because of the digit grids of computers are binary grids, the values of a function $u(t)$ defined in grid knots generate a table which is often transmitted to addressee from the sender; therefore it may be arise a requirement of approximate restoration of a values of the function between of the grid knots (it is possible if the addressee applies a powerful computer with fine digit grid). For example, the problem is actual for transmission of data from artificial earth satellite to earth station equipped with powerful computer complex.

The problem of evaluation for approximate restoration of a values of the function between of the binary grid knots is discussed in [5] (see §6, ch.VI).

In the offered paper the problem is examined for minimal splines from class C^2. It is very important that spaces of such splines gives the telescopical system for embedded grids; the mentioned spaces are suitable for spline-wavelet decompositions.

Let $X(k)$ be grid of positive number with k-digit mantissa that is set of number in the form of $\overline{0.1, \varepsilon_2, \ldots, \varepsilon_k} \cdot 2^n$, where $\varepsilon_i \in \{0,1\}, i = 2, 3, \ldots, n$, and n is an integer; here horizontal bar indicates representation of binary number.

In offered paper the value $|u(t) - \widetilde{u}(t)|$, $t > 0$, is evaluated; here $\widetilde{u}(t) \stackrel{\text{def}}{=} \sum_j \langle f_j, u \rangle \omega_j(t)$, and $\{f_j\}$ is the system of functionals which is biorthogonal to the system of coordinate splines $\omega_j \in C^2$ conformed with the grid $X(k)$.

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Classification of functions by local smoothness on metric spaces endowed with measure

V. Krotov (Belarus)

vkrotov@cosmostv.by

Let (X, d, μ) be a metric space endowed with metric d and regular Borel measure μ, which are related to each other with the following doubling-condition:

$$\mu(B(x, 2r)) \leq c\mu(B(x, r)), \quad B(x, r) = \{y \in X : d(x, y) < r\}.$$

Also let Ω be the class of increasing positive functions $\eta : (0, \mathrm{diam}X) \to (0, \mathrm{diam}X)$, such that the function $\eta(t)t^{-a}$ almost decreases for some $a > 0$. For $\eta \in \Omega$ we consider maximal functions

$$\mathcal{N}_\eta f(x) = \sup \frac{1}{\eta(r_B)} \cdot \frac{1}{\mu(B} \int_B |f(x) - f(y)| \, d\mu(y)$$

where sup is to be taken over all balls B, that contain the point $x \in X$, and r_B is the radius of B. These maximal functions were considered first by A. Calderon [1] (in the case of $\eta(t) = t^\alpha$) and V. I. Kolyada [2] in the general case. In light of the obvious inequality

$$|f(x) - f(y)| \leq [\mathcal{N}_\eta f(x) + \mathcal{N}_\eta f(y)]\eta(d(x,y)), \quad x, y \in X$$

$\mathcal{N}_\eta f(x)$ measures the local smoothness.

First of all, note that $\mathcal{N}_\eta f$ is measurable for any locally-integrable function f on X. The next result shows that the maximal operators $\mathcal{N}_\eta$ classify all of the functions from $L^p(X)$, $p > 1$, by their local smoothness.

Theorem 1. *For any $f \in L^p(X)$, $p > 1$ there exists $\eta \in \Omega$, such that*

$$\|\mathcal{N}_\eta f\|_{L^p(X)} \leq \|f\|_{L^p(X)}.$$

This implies that $L^p(X) = \bigcup_{\eta \in \Omega} C^p_\eta(X)$, where

$$C^p_\eta(X) = \{f \in L^p(X) : \mathcal{N}_\eta f \in L^p(X)\}.$$

For $X = [0,1]^n$ this statement can be obtained from the results of K. I . Oskolkov [3] in the case of $n = 1$ and V. I. Kolyada [2] for any $n \in \mathbb{N}$, concerning estimates of functions from $\mathcal{N}_\eta f$ by the L^p-modulus of continuity of f.

In the case of $p = 1$ the statement of the Theorem 1 is not true, but we can prove the following:

Theorem 2. *For any $f \in L^1(X)$ there exists $\eta \in \Omega$, such that*

$$\mu\{\mathcal{N}_\eta f > \lambda\} \leq \lambda^{-1}\|f\|_{L^1(X)}, \quad \lambda \geq 1.$$

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The K-functional and near minimizers for Sobolev and Morrey-Campanato spaces with applications to Harmonic Analysis and Inverse Problems

N. Kruglyak (Sweden)

natan@ltu.se

I plan to discuss recent achievements in a rather old (more than 40 years) problem: to find a formula for the K-functional for a couple of Sobolev spaces. New covering theorems (which are used in the proofs) and already obtained applications of these results will be also discussed.

Strong Spherical Means of Multiple Fourier Series

O. Kuznetsova (Ukraine)

kuznets@iamm.ac.donetsk.ua

For any $p \geq 1$ we have found in [1] an exact order of growth for norms $H_{n,p}$ of p-strong means

$$H_{n,p}(f,x) = \left(\frac{1}{n+1} \sum_{j=0}^{n} |S_j(f,x)|^p \right)^{\frac{1}{p}} \tag{1}$$

of spherical partial Fourier sums

$$S_R(f,x) = \sum_{|k|\le R} \widehat{f}(k)e^{ikx} \quad (R>0)$$

in the space of measurable a.e. bounded functions f defined on the m-dimensional tours $\mathbb{T}^m = [-\pi,\pi)^m$ for $m \ge 3$.

$$H_{n,p} \asymp n^{\frac{m-1}{2}-\min(\frac{1}{2},\frac{1}{p})} \quad (n\to\infty).$$

For norms $h_{R,p}$ of integral analogs of (1)

$$h_{R,p}(f,x) = \left(\frac{1}{R}\int_0^R |S_r(f,x)|^p dr\right)^{\frac{1}{p}}$$

we find lower and upper bounds.

$$R^{\frac{m-1}{2}-\min(1,\frac{2}{p})} \ll h_{R,p} \ll R^{\frac{m-1}{2}-\min(\frac{1}{2},\frac{1}{p})} \quad (R\to\infty, p\ge 1).$$

It has been obtained estimates which can not be improved, for integral norms of linear means of spherical Dirichlet kernels in terms of the coefficients of these means (Sidon's inequality type). For multiple trigonometric series with coefficients that have radial symmetry, we have obtained conditions for the considered series to be Fourier series, and also conditions that are necessary and sufficient for convergence of these series on spheres in $L(\mathbb{T}^m)$. In particular, it is proved criterion for the series

$$\sum_{k\in\mathbb{Z}^m} \frac{b(|k|)}{|k|^\alpha} e^{ikx}$$

to converge on spheres in $L(\mathbb{T}^m)$, where $\alpha > 0$, $b(t)$ is a function that is slowly changing in Zigmund's sense.

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New Bounds for Discrepancy Function and Small Ball Inequality in All Dimensions

M. Lacey (USA)

lacey@math.gatech.edu

We will address a questions of relevance to Irregularities of Distribution, Approximation Theory, and Probability Theory. The most elementary formulation of the set of questions comes from Irregularities of Distribution. Given a collection P of N points in the unit cube in dimension d, define the Discrepancy Function

$$D_N(x) = |[0,x) \cap P| - N|[0,x)|\,, \qquad x \in [0,1]^d\,.$$

By $[0,x)$ we mean the d-dimensional rectangle anchored at the origin and at point x. The Discrepancy Function at each point x is the difference between the number of points of the collection P that are in the rectangle $[0,x)$, and number of points expected to be in the rectangle. A classical result of Klaus Roth shows that D_N can never be very small, regardless how cleverly the collection P is chosen. Namely, $\|D_N\|_2 \gtrsim (\log N)^{(d-1)/2}$. This is a sharp estimate. The L^∞ norm of the Discrepancy Function is conjectured to be larger. In two dimensions, one has the definitive bound of Wolfgang Schmidt, $\|D_N\|_\infty \gtrsim \log N$. We will discus an extension of this result to arbitrary dimensions: For dimensions $d \geq 3$, there is an $\epsilon(d) > 0$ so that we have the universal estimate

$$\|D_N\|_\infty \gtrsim (\log N)^{(d-1)/2+\epsilon(d)}\,.$$

That is, the gain over the Roth bound is $(\log N)^{\epsilon(d)}$. This result extends a result of Jozef Beck, in dimension $d=3$, which gave a doubly-logarithmic improvement of the Roth bound. The connection to Approximation Theory and Probability Theory is made through the so-called Small Ball Inequality, a non-trivial lower bound on the L^∞ norm of sums of Haar functions in higher dimensions, supported on rectangles of a fixed volume. In this context, we extend results of Talagrand to dimensions $d \geq 3$. Joint work with Dmitriy Bilyk and Armen Vagharshakyan.

Densities connected with overconvergence

W. Luh (Germany)

gawrone@uni.trier.de

It is well known that universal Taylor series have a strong connection with overconvergence. While by Ostrowski's classical results there exist interdependencies with the occurrence of gaps in the sequence of coefficients and the phenomenon of overconvergence.

Supplement to R.Nevanlinna's principle of harmonic measure

A.L. Lukashov[3] (Russia)

LukashovAL@info.sgu.ru

Let D and G be domains in $\overline{\mathbb{C}}$ that are bounded by a finite number of Jordan arcs, ω be the harmonic measure of a part of the boundary for a domain with respect to a point in that domain.

Theorem. *If $w(z)$ is a meromorphic function in D and continuous in $\overline{D}$ such that its derivative does not vanish in D, $w(D) \supseteq G$ and $w(\partial D) = \partial G$ then for any finite system of arcs $\alpha_z \subset \partial D$, and for any $w_0 \in G$*

$$\omega(w_0, \alpha_w, G) \leq \sum_k \omega(z_k(w_0), \alpha_z, D),$$

[3]Work is partially supported by RFBR (grant N07-01-00167) and by presidential grant of leading scientific schools (NSh-2970.2008.01).

where the sign of summation is over all solutions z_k of the equation $w(z) = w_0$ and $\alpha_w = w(\alpha_z)$.

The same inequality is proved also under different conditions with the help of recent V.N. Dubinin and S.I. Kalmykov's majoration principle (see [1]).

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Strong extrapolation spaces

K.V. Lykov and S.V. Astashkin (Russia)

alkv@list.ru, astashkn@ssu.samara.ru

Let E be a symmetric space of measurable functions on $[0,1]$. Let us call E **strong extrapolation space** and denote $E \in \mathcal{SE}$ if there exists $C > 0$ such that for any $x = x(t) \in E$

$$C^{-1}\|y\|_E \leqslant \|x\|_E \leqslant C\|y\|_E,$$

where

$$y(t) := \|x\|_{\log_2 2/t} = \left(\int_0^1 |x(s)|^{\log_2 2/t}\, ds\right)^{\log_2^{-1} 2/t}.$$

Theorem 1. *A symmetric space $E \in \mathcal{SE}$ if and only if the operator $Sx(t) = x(t^2)$ is bounded in E. Moreover, if $E \in \mathcal{SE}$, then for the K-functional of the couple (E, L_∞)*

$$\mathcal{K}(t, f(u); E, L_\infty) \asymp \mathcal{K}(t, \|f\|_{\log_2 2/u}; E, L_\infty),$$

with constants independent of t and f.

Example. For the Zygmund space $\mathrm{Exp}L^{\beta}$ ($\beta > 0$) with the norm

$$\|x\|_{\mathrm{Exp}L^{\beta}} = \sup_{0<t\leq 1} \log_2^{-1/\beta}\left(\frac{2}{t}\right) x^*(t),$$

where $x^*(t)$ is nonincreasing rearrangement of $|x(t)|$, the following holds

$$\mathrm{Exp}L^{\beta} \in \mathcal{SE} \ \text{ and } \ \mathcal{K}(t, f; L_\infty, \mathrm{Exp}L^{\beta}) \asymp t \sup_{p \geqslant t} \frac{\|f\|_{p^\beta}}{p} \ \ (t \geqslant 1).$$

If E is any symmetric space then $E(\log^{-1})$ stands for the space with the norm equivalent to the quasi-norm $\|x^*(t)\log_2^{-1} 2/t\|_E$. Applying Theorem 1, we get

Theorem 2. *Let T be a linear operator bounded in L_p for any $p \geqslant p_0 \geqslant 1$. Suppose that*

$$\|T\|_{L_p \to L_p} \leqslant Cp \ \ (p \geqslant p_0). \tag{$*$}$$

Then T is bounded from E into $E(\log^{-1})$ for any $E \in \mathcal{SE}$. Moreover, there exists a linear operator T_0 satisfying $(*)$ with the following property: for any $E \in \mathcal{SE}$ and any symmetric space $F \supset T_0(E)$ holds

$$E(\log^{-1}) \subset F.$$

Research was partially supported by RFBR (Grant 07-01-96603).

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Geometric properties of the ridge function manifold

V. Maiorov (Israel)

maiorov@tx.technion.ac.il

We study geometrical properties of the ridge function manifold $\mathcal{R}_n$ consisting of all possible linear combinations of n functions of the form $g(a \cdot x)$, where $a \cdot x$ is the inner product in $\mathbb{R}^d$.

We obtain an estimate for the ε-entropy numbers in terms of smaller ε-covering numbers of the compact class $G_{n,s}$ formed by the intersection of the class $\mathcal{R}_n$ with the unit ball $B\mathcal{P}_s^d$ in the space of polynomials on $\mathbb{R}^d$ of degree s. In particular we show that for $n \leq s^{d-1}$ the ε-entropy number $H_\varepsilon(G_{n,s}, L_q)$ of the class $G_{n,s}$ in the space L_q is of order $ns \log 1/\varepsilon$ (modulo a logarithmic factor). Note that the ε-entropy number $H_\varepsilon(B\mathcal{P}_s^d, L_q)$ of the unit ball is of order $s^d \log 1/\varepsilon$.

Moreover, we obtain an estimate for the pseudo-dimension of the ridge function class $G_{n,s}$.

On Exponential Spline Spaces and Wavelet Decomposition

A. Makarov (Russia)

Antony.Makarov@gmail.com

The source of B_φ-splines is approximation relations regarded as a system of linear algebraic equations. Normalized exponential splines of the first order are constructed in the paper. The obtained splines are continuous and have minimal compact support. A realization of system of biorthogonal functionals for B_φ-splines is proposed. The solutions of some interpolation problems generated by mentioned biorthogonal system are offered.

An embedding of the general spline spaces is established for arbitrary refinements of irregular grid, wavelet decomposition in the case of sequence of refining irregular grids is discussed, compactly supported wavelet basis is constructed, the formulas of decomposition and reconstruction are done. The obtained formulas are easily parallelized.

On approximation in the mean by polynomials with gaps on non-Caratheodory domains

V. Martirosian and S. Mkrtchyan (Armenia)

mart@instmath.sci.am

This report intends to present new results on possibility of approximation by polynomials with gaps. Approximations are accomplished in norm of Lebesgue space L_p on the non-Caratheodory domains of the complex plane. Lacunary versions of some results of A.L.Shahinyan, M.M.Djrbashyan, J.E.Brennan and others authors are obtained. Analogous approximations by real parts of polynomials with gaps are considered too.

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On geometrically decreasing Kolmogorov $n-$widths in Hilbert spaces

M. Martirosyan and S. Samarchyan (Armenia)

approximation@gmail.com

Let H be a Hilbert space, S be a bounded and infinite subset of H and $d_n(S,H)$ be the Kolmogorov $n-$width of S in H. For $n \in \mathbb{N}$ consider

$$D_n(S,H) = \sup_{\{e_k\}_{k=1}^{n+1} \subset S} \min_{1 \leqslant k \leqslant n+1} \frac{1}{\|\varphi_k\|},$$

where $\{e_k\}_{k=1}^{n+1}$ and $\{\varphi_k\}_{k=1}^{n+1}$ are biorthogonal systems with coinciding linear spans. We will call such systems generated biorthogonal systems or shorter GBS. Further consider

$$C_n(S,H) = \inf_{\{e_k\}_{k=1}^{n} \subset S} \sup_{e \in S} \frac{1}{\|\varphi\|},$$

where $e, e_1, \ldots, e_n$ and $\varphi, \varphi_1, \ldots, \varphi_n$ are GBS for some $\varphi_1, \ldots, \varphi_n$.

We prove the lemma which establishes relation between Kolmogorov $n-$widths and GBS.

Lemma 1. *Let $\{e_k\}_{k=1}^{n+1}$ and $\{\varphi_k\}_{k=1}^{n+1}$ are generated biorthogonal systems in H. Then*

$$d_n(E_{n+1},H) = \frac{1}{\max\limits_{1 \leqslant k \leqslant n+1} \|\varphi_k\|},$$

where $E_{n+1} = \left\{ \sum\limits_{k=1}^{n+1} a_k e_k : \ |a_k| \leqslant 1,\ k = 1,2,\ldots,n+1 \right\}.$

Theorem 1.

$$\frac{C_n(S,H)}{n+1} \leqslant \frac{D_n(S,H)}{n+1} \leqslant d_n(S,H) \leqslant C_n(S,H) \leqslant D_n(S,H).$$

Corollary 3. *Consider three limits* $\lim\limits_{n\to\infty} \sqrt[n]{D_n(S,H)}$, $\lim\limits_{n\to\infty} \sqrt[n]{C_n(S,H)}$ *and* $\lim\limits_{n\to\infty} \sqrt[n]{d_n(S,H)}$. *If at least one of the limits exists then all three limits exist and are equal.*

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Topological Lumpiness and Topological Extreme Amenability

H. P. Masiha (Iran)

masiha@kntu.ac.ir

In this talk we give some characterizations of topological extreme amenability. Also, we answer a question raised by Ling. In particular, we prove that if T is a Borel subset of a locally compact semigroup S such that $M(S)^*$ has a multiplicative topological left invariant mean then T is a topological left lumpy if and only if there is a multiplicative topological left invariant mean M on $M(S)^*$ such that $M(\chi_T) = 1$, where χ_T is the characteristic functional of T. Consequently, if T is a topological left lumpy locally compact Borel subsemigroup of a locally compact semigroup S, then T is extremely topological left amenable if and only if S is.

Prediction Error for Continuous-time Stationary Processes with Singular Spectral Densities

L. Mikaelyan and M. Ginovyan[4] (Armenia, USA)

mikaelyanl@ysu.am, ginovyan@math.bu.edu

Let $X(\phi)$ be a stationary Gaussian generalized process, i.e., $X(\cdot)$ is a linear operator mapping the space of infinitely differentiable functions with finite support $D = D(\mathbb{R})$ into a Gaussian subspace $H = H(X)$ of a space $L^2(dP)$, constructed from a probability measure P. We assume that the process $X(\phi)$ is regular, possesses a spectral density $f(\lambda)$, and $f(\lambda) - 1 \in L^1(\mathbb{R})$.

For $t, s \in \mathbb{R}$, $-\infty \leq s \leq t \leq \infty$, denote by $H_s^t = H_s^t(X)$ the subspace of H spanned by the random variables $X(\varphi)$ with $\text{supp}\{\varphi\} \subset [s, t)$, and let P_s^t be the orthogonal projector in H onto H_s^t. Consider the functionals $\tau(f; T, s) = \text{tr}[P_{-T}^0 P_0^s P_{-T}^0]$ and $\tau(f; s) = \text{tr}[P_{-\infty}^0 P_0^s P_{-\infty}^0]$, where $\text{tr}[A]$ stands for the trace of an operator A. Then (see [1] - [3]) the functional $\delta(f; T, s) = \tau(f; s) - \tau(f; T, s)$ provides a natural measure of accuracy of prediction of the random variable $\xi \in H_0^s$ by the observed values $\eta \in H_{-T}^0$ (the past of length T), compared with their prediction by the observed values $\eta \in H_{-\infty}^0$ (the whole past). The quantity

$$\delta(f; T) = \lim_{s \to 0} \frac{1}{s}\ \delta(f; T, s).$$

is called the relative prediction error of a random variable $\xi \in H_0^s$ by the past of length T compared with the whole past. Clearly $\delta(f; T) \geq 0$ and $\delta(f; T) \to 0$ as $T \to \infty$. We are concerned with the rate of decrease of prediction error $\delta(f; T)$ to zero as $T \to \infty$, depending on the properties of spectral density $f(\lambda)$.

Our approach to the problem is based on the Krein's theory of continual analogs of orthogonal polynomials and the continual analogs of Szegö theorem on Toeplitz determinants (see [2], [3]). To this end first we

[4]M. Ginovyan's research was partially supported by National Science Foundation Grant #DMS-0706786.

obtain formulae for the resolvents and Fredholm determinants of the corresponding Wiener-Hopf truncated operators.

We obtain explicit expressions and/or asymptotic formulae for prediction error $\delta(f;T)$ in the cases where the spectral density $f(\lambda)$ of the underlying process possesses either zeros (the model is an anti-persistent process), or poles (the model is a long memory processes). The results show that the asymptotic relation $\delta(f;T) \sim 1/T$ as $T \to \infty$ is valid, whenever the spectral density $f(\lambda)$ of the underlying process possesses at least one singularity (zero or pole) of power type. Two typical results in this direction are:

Theorem 1. *Let $f(\lambda)$ be the spectral density of a continuous-time stationary process given by $f(\lambda) = \frac{(\lambda-\omega)^2}{(\lambda-\omega)^2+1}$, where ω is a real number. Then for any $T > 0$, $\delta(f;T) = \frac{1}{T+2}$.*

We say that a function $g(\lambda)$ is **regular** if it is a spectral density of a short memory process, that is, $0 < C_1 \le g(\lambda) \le C_2 < \infty$, where C_1 and C_2 are absolute constants, and possesses usual smoothness properties.

Theorem 2. *Let $f(\lambda)$ be the spectral density of a continuous-time stationary process given by*

$$f(\lambda) = g(\lambda) \prod_{k=1}^{n} \left[\frac{(\lambda-\omega_k)^2}{(\lambda-\omega_k)^2+1} \right]^{\alpha_k}, \quad \alpha_k \in (-1/2, 1/2) \setminus \{0\},$$

where $g(\lambda)$ is a regular spectral density and ω_k, $k = \overline{1,n}$, are distinct real numbers. Then $\delta(f;T) \sim \frac{1}{T} \sum_{k=1}^{n} \alpha_k^2$ as $T \to \infty$.

Remark. Observe that the model in Theorem 2 displays a long memory process for $-1/2 < \alpha_k < 0$, and, as a special case, includes the continuous-time fractional $ARIMA(p,d,q)$ model.

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Singular integrals and the Faber-Schauder system

S. Mironova (Russia)

srmironova@yandex.ru

We investigate properties of singular integrals in the plane using the Faber-Schauder system of functions.

Universal power series and analytic continuation

J. Müller (Germany)

jmueller@uni-trier.de

Let $f(z) = \sum\limits_{\nu=0}^{\infty} a_\nu z^\nu$ be a power series with radius of convergence 1. It is well-known that there is a residual set M of functions in $H(\mathbb{D})$, where $\mathbb{D}$ is the unit disk, such that for all $f \in M$ the sequence of partial sums $(S_n f)$ of the corresponding power series shows universality properties outside $\mathbb{D}$. We consider the question which universality properties may still appear if f is analytically continuable.

Strong Solvability of Approximation Problems in Orlicz Space

D. Müller-Wichards and P. Kosmol (Germany)

muewi@etech.haw-hamburg.de

We give a brief introduction to Orlicz-spaces, pointing out that various properties of the defining 1-D Young-function Φ carry over to norms and modulars characteristic for the space. In particular this holds for differentiability and strict convexity but also for numerical and stability properties. Reflexivity can be characterized in terms of the growth properties of the Young function Φ and its convex conjugate Ψ. One of our main results is that for σ-finite non-atomic measures differentiability of Φ, Gâteaux-differentiability and Fréchet-differentiability of the Orlicz-space, strict convexity of Ψ, strict convexity and local uniform convexity of the dual space are all equivalent. For Orlicz-sequence spaces a similar (and slightly weaker) statement holds. Local uniform convexity in turn is equivalent to strong solvability of approximation problems, i.e. every minimizing sequence converges to the minimal solution. As applications we consider Tychonov-type regularizations and weak Chebyshev greedy algorithms.

On the almost everywhere summability of double series with respect to block-orthonormal systems

G. Nadibaidze (Georgia)

g.nadibaidze@math.sci.tsu.ge

Block-orthonormal systems were introduced by Gaposhkin [1]. He proved, that the Menshov-Rademacher‘s theorem and the strong law of large numbers are valid for such systems in certain conditions. In [3, 4] there were obtained some results on convergence and summability of series with respect to block-orthonormal systems. In particular, Menshov-Rademacher's and Gaposhkin's theorems were generalized and the exact Weyl multipliers for the convergence and summability a.e. of series with respect to block-orthogonal systems were established [3].

The necessary and sufficient conditions on the length of blocks were obtained by Nadibaidze [3], for the Kacmarz's theorem on the Cesaro $(C, 1)$ summability a.e. of orthogonal series to be valid for the block-orthonormal series. Using proved theorems it is possible to determine exact Weyl multipliers for the Cesaro $(C, 1)$ summability a.e. of the block-orthonormal series in the case, when the Kacmarz's theorem is not true.

Definition 1. *Let* $\{M_p\}$ *and* $\{N_q\}$ *be the increasing sequences of natural numbers and ,* $\triangle p, q = (M_p, M_{p+1}] \times (N_q, N_{q+1}]$, $(p, q \geq 1)$.. *Let* $\{\varphi_{m,n}\}$ *be a system of functions from* $L^2((0,1)^2)$. *The system* $\{\varphi_{m,n}\}$ *will be called a* $\triangle p, q$*-orthonormal system if:*

1) $\|\varphi_{m,n}\|_2 = 1$ $n = 1, 2, ..., m = 1, 2, ...;$

2) $(\varphi_{i,j}, \varphi_{k,l}) = 0$, *for* $(i, j), (k, l) \in \triangle_{p,q}$, $(i, j) \neq (k, l)$, $(p, q \geq 1)$.

Moricz [2] established the analogue of Kacmarz's theorems for the $(C, 1, 1)$, $(C, 1, 0)$ and $(C, 0, 1)$ summability a.e. of double orthogonal series. Now We obtained the necessary and sufficient conditions on the M_p and N_q sequences when the Moricz's theorems [2] are valid for the $(C, 1, 1)$, $(C, 1, 0)$ and $(C, 0, 1)$ summability a.e. of double block-orthonormal series.

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Representation of measurable functions by subsystems of Walsh system

M. Nalbandyan (Armenia)

mikayeln@freenet.am

The problem of representing a function f by a series in classical and general orthonormal systems has a long history.

A question posed by Lusin in 1915 asks whether it is possible to find for every measurable function $[0, 2\pi]$ a trigonometric series, with coefficient sequence converging to zero, that converges to the function almost everywhere. For non real-valued functions, this question was given an affirmative answer by Men'shov in 1941.

There are many works devoted to representations of functions by series in classical and general orthonormal systems and the existence of different types of universal series in the sense of convergence almost everywhere and by measure.

The following theorems are true for the Walsh system:

Theorem 1. *For any $l \in \{2^s\}_{s=1}^{\infty}$ there exists a subsystem $\{\omega_{n_k}\}_{k=1}^{\infty}$ of the Walsh system, where $n_k = k^l + o(k^{l-1}), k = 1, 2, ...$, such that for every mea-*

surable function $f(x)$ there exists a series by the subsystem $\{\omega_{n_k}\}_{k=1}^{\infty}$, that converges almost everywhere to $f(x)$.

Theorem 2. *For any growing to infinity sequence $\{\lambda(k)\}_{k=1}^{\infty}$ there exists a subsystem $\{\omega_{n_k}\}_{k=1}^{\infty}$ of the Walsh system with $n_k = k^2 + o(\lambda(k)), k = 1, 2, ...,$ such that for every measurable function $f(x)$ there exists a series by the subsystem $\{\omega_{n_k}\}_{k=1}^{\infty}$, that converges almost everywhere to $f(x)$.*

For the trigonometric system Theorem 2 was proved in [1].

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On Series With Monotone Coefficients by Walsh System

K. Navasardyan (Armenia)

knavasard@ysu.am

We consider series by Walsh system

$$\sum_{n=0}^{\infty} a_n W_n(x),$$

satisfying the conditions

$$a_0 \geq a_1 \geq \ldots \geq a_n \geq \ldots, \qquad \lim_{n\to\infty} a_n = 0. \tag{1}$$

and

$$\sum_{n=0}^{\infty} a_n^2 = \infty. \tag{2}$$

We prove the following theorems.

Theorem 1. *Let* (1) *and* (2) *hold . Then for any* $\varepsilon > 0$ *there exists a measurable set* $E \subset [0,1]$, $\mu E > 1 - \varepsilon$, *such that for any function* $f \in L^1(0,1)$ *there is some* $g \in L^1(0,1)$ *and numbers* $\delta_n = 0$ *or* ± 1 *such that* $g(x) = f(x)$, $x \in E$, *and the series* $\sum_{n=1}^{\infty} \delta_n a_n W_n(x)$ *converges to* g *in the metric of* $L^1(0,1)$.

Theorem 2. *Let* (1) *and* (2) *hold . Then there exists numbers* $\delta_n = \pm 1$ *such that for any* $p \in (0,1)$ *the series* $\sum_{n=1}^{\infty} \delta_n a_n W_n(x)$ *is universal with respect to subseries in* $L^p(0,1)$, *in the sense of convergence in the space* $L^p(0,1)$.

On universal relatives of the Riemann zeta-function

M. Niess (Germany)

markus.niess@ku-eichstaett.de

The Riemann zeta-function ζ has the following well-known properties:

(1) It is holomorphic in the complex plane except for a simple pole at $z = 1$ with residue 1.

(2) The symmetry relation $\zeta(z) = \overline{\zeta(\bar{z})}$ holds for $z \neq 1$.

(3) The functional equation

$$\zeta(z)\Gamma(z/2)\pi^{-z/2} = \zeta(1-z)\Gamma((1-z)/2)\pi^{-(1-z)/2}$$

holds.

Moreover, ζ has a universality property due to Voronin (1975). We show that arbitrarily close approximations of the Riemann zeta-function which satisfy (1)-(3) may have a different universality property. Consequently, these approximations do not satisfy the Riemann hypothesis. Moreover, we investigate the set of all "Birkhoff-universal" functions satisfying (1)-(3).

Inequalities for real-root polynomials and bounds for the extreme zeros of classical orthogonal polynomials

G. Nikolov (Bulgaria)

geno@fmi.uni-sofia.bg

A recent conjecture by W. Forster and I. Krasikov about a family of inequalities for real-root polynomials has been confirmed in a recent work (joint with with R. Uluchev). It turns out that, restricted to the polynomial case, these inequalities are a refinement of the classical Jensen inequalities for functions from the Laguerre-Polya class. Moreover, they provide a method for derivation of bounds for the extreme zeros of the classical orthogonal polynomials.

Condition Numbers of Large Matrices under Spectral Restrictions

N. Nikolski (France, Russia)

Nikolai.Nikolski@math.u-bordeaux1.fr

We study possible discrepancies between the condition number

$$CN(T) = \|T\| \cdot \|T^{-1}\|$$

and the spectral condition number $SCN(T) = \|T\| r(T^{-1})$ for large (nonselfadjoint) matrices or operators T, $r(\cdot)$ being the spectral radius (so that $r(T^{-1}) = 1/min_j|\lambda_j(T)|$ for the matrix case, $\lambda_j(T)$ are eigenvalues). Given a family $\mathcal{F}$ of matrices/operators we are looking for a function φ such that $CN(T) \leq \varphi(SCN(T))$ for every $T \in \mathcal{F}$. The growth rate of $\varphi(\delta)$ as $\delta \to 0+$ describes the complexity of the inversion problem in $\mathcal{F}$ with respect to the selfadjoint case.

We examine from this point of view several useful classes $\mathcal{F}$, as

1) all $n \times n$ matrices acting on an euclidean space (Kronecker),

2) all $n \times n$ matrices acting on an arbitrary n-dimensional Banach space (Van der Waerden's problem),

3) operators / matrices having given eigenvalues and a prescribed functional calculus/growth rate of the resolvent,

4) algebras of matrices/operators with a given spectrum.

A link with the Kadison-Singer problem is briefly discussed.

Stationary and nonstationary biorthogonal compactly supported wavelets

I. Novikov (Russia)

igorno@icmail.ru

Basis and approximation properties of stationary and nonstationary biorthogonal compactly supported wavelets in different function spaces are considered. In particular stationary biorthogonal compactly supported wavelets preserving localization with the growth of smoothness is examined. Two examples of infinitely differentiable nonstationary biorthogonal compactly supported wavelets are investigated also.

Partially supported by RFBR Grant 08-01-00226-a.

On the Convergence of Multiple Fourier-Haar Series

G. Oniani (Georgia)

oniani@atsu.edu.ge

Let $n \geq 2$ and $I^n = [0,1]^n$. For $f \in L(I^n), x \in I^n, m \in \mathbb{N}^n$, and $r > 0$ denote by $S_m(f)(x)$,$S_r(f)(x)$,$G_m(f)(x)$ and $G_r(f)(x)$ rectangular partial sum,spherical partial sum, rectangular general term and spherical general term of the Fourier series of f at the point x with respect to multiple Haar system $(\chi_{m_1}...\chi_{m_n})$.

From the well-known strong maximal theorem of B. Jessen, J. Marcinkiewicz and A. Zygmund and corresponding results of S.Saks [1] and T.Zerekidze [2] it follows that for every $n \geq 2$, $L(\ln^+ L)^{n-1}(I^n)$ is the widest integral class in which the almost everywhere convergence of the rectangular partial sums of multiple Fourier-Haar series is guaranteed. For the spherical partial sums the same is known in two-dimensional case. It follows from the results of G.Kemkhadze [3] and G.Tkebuchava [4].

Getsadze [5] proved that in integral classes wider than $L\ln^+ L(I^2)$ divergence phenomena on the sets of positive measure remain valid even for rectangular and spherical general terms of a double Fourier-Haar series.

We prove the following theorems that generalizes the above mentioned results of G.Kemkhadze, G.Tkebuchava and R.Getsadze

Theorem 1. *Let $n \geq 2$ and $f \in L(I^n)$. If $x \in I^n$ is diadic-irrational and $\lim_{m\to\infty} S_m(f)(x) = f(x)$ then $\lim_{r\to\infty} S_r(f)(x) = f(x)$.*

Theorem 2. *Let $n \geq 2$ and $f \in L\backslash L(\ln^+ L)^{n-1}(I^n)$. Then there exists an equimeasurable with f non-negative function g on I^n such that*

$$\limsup_{m\to\infty} G_m(g)(x) = \infty \text{ and } \limsup_{r\to\infty} G_r(g)(x) = \infty \text{ a.e on } I^n.$$

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On differential and fractal properties of the "extended" Riemann's function

K. Oskolkov (USA)

oskolkov@math.sc.edu

We discuss properties of the function

$$\mathcal{R} := \sum_{n\in\mathbb{Z}\setminus\{0\}} \frac{e^{\pi i(tn^2+2xn)}}{\pi i n^2}, \quad (t,x)\in\mathbb{R}^2,$$

and its partial derivative

$$\mathcal{H} := \frac{\partial_x \mathcal{R}}{4\pi i} = \text{p. v.} \sum_{n\in\mathbb{Z}\setminus\{0\}} \frac{e^{\pi i(tn^2+2xn)}}{2\pi i n}.$$

$\mathcal{R}$ is an extension of Riemann's function $R := \dfrac{2}{\pi}\sum_{n=1}^{\infty}\dfrac{\sin \pi n^2 t}{n^2}$, $t\in\mathbb{R}$, since the real part of $\mathcal{R}$ for $x=0$ coincides with R. The latter was suggested about 170 years ago by B. Riemann as a plausible example of a continuous and nowhere differentiable function.

Both functions $\mathcal{R}$ and $\mathcal{H}$ possess a wide spectrum *of fractal, and multifractal* properties. For example, for each fixed $\alpha \in \left(0, \frac{1}{2}\right)$ the set of values of the parameter t, for which $\mathcal{H}$ as a function of the variable x satisfies the Lipschitz – Hölder condition α *sharply*, has a non-trivial fractional dimension in the sense of Hausdorff.

On the Best Approximation of Periodic Functions by Splines of Defect 2

N. Parfinovych and V. Babenko (Ukraine)

nparfinovich@mail.ru, babenko.vladislav@gmail.com

Let L_p $(1 \le p \le \infty)$ be the space of 2π-periodic functions $f : \mathbb{R} \to \mathbb{R}$ with the corresponding norm $\|f\|_p$. Given $r \in \mathbb{N}$ and rearrangement invariant set $F \subset L_1$ denote by $W^r F$ the class of functions $f \in L_1$ such that $f^{(r-1)}$ is locally absolutely continuous and $f^{(r)} \in F$. By $W^r H^\omega$ let us denote the class of r times continuously differentiable 2π-periodic functions such that $f^{(r)}$ has prescribed concave majorant $\omega(t)$ of modulus of continuity. Let $S^k_{2n,m}$ $(k = 1,2;\ n,m \in \mathbb{N})$ be the space of 2π-periodic polynomial splines of order m and defect k, with knots $\frac{kj\pi}{n}$, $j \in \mathbb{Z}$.

By $E(M,H)_p$ and $d_n(M,L_p)$ denote the best L_p-approximation of the class $M \subset L_p$ by the set $H \subset L_p$ and Kolmogorov n-width of this class in the space L_p respectively.

In is well known that the spaces $S^1_{2n,m}$ (for all $m \ge r$) are the extremal spaces for the widths $d_{2n}(W^r F, L_1)$ and $d_{2n}(W^r H^\omega, L_1)$. For the class $W^r F$ this fact was proved by A.A. Ligun (if F is unit ball in the space L_p) and by V.F. Babenko (if F is an arbitrary rearrangement invariant set). N.P. Korneychuk obtained the corresponding result for the class $W^r H^\omega$.

We found the exact values of the quantities $E(W^r F,\ S^2_{2n,m})_1$ and $E(W^r H^\omega, S^2_{2n,m})_1$. In addition we proved that the spaces $S^2_{2n,m}$ are also extremal spaces for the widths $d_{2n}(W^r F, L_1)$ (for all $m \ge r$) and for the widths $d_{2n}(W^r H^\omega, L_1)$ for all $m \ge r+1$).

Theorem. *Let $r,n = 1,2,...$,let F be an arbitrary rearrangement invariant set of 2π-periodic functions, and let $\omega(t)$ be a concave modulus of continuity.*

Then

$$E(W^rF, S^2_{2n,m})_1 = E(W^rF, S^1_{2n,m})_1 = d_{2n}(W^rF, L_1), \quad m \geq r;$$

$$E(W^rH^\omega, S^2_{2n,m})_1 = E(W^rH^\omega, S^1_{2n,m})_1 = d_{2n}(W^rH^\omega, L_1), \quad m \geq r+1.$$

We also have studied some other approximation and extremal properties of the splines of defect 2.

Sampling of bandlimited functions on Hyperbolic spaces and Homogeneous trees

I. Pesenson (USA)

pesenson@temple.edu

A notion of bandlimited functions is introduced on hyperbolic spaces and homogeneous combinatorial graphs. It is shown that such functions are uniquely determined by their values on some countable subsets of points (subsets of vertices). Such uniqueness sets are described in terms of Plancherel-Polya and Poincare-Wirtinger-type inequalities. A reconstruction algorithm of bandlimited functions from uniqueness sets which uses the idea of frames in Hilbert spaces is developed. The second reconstruction algorithm uses variational splines on manifolds and graphs.

On Extreme Points of Some Sets of Holomorphic Functions

A. Petrosyan (Armenia)

albpet@xter.net

In the Banach space $H^\infty(U)$ of bounded, holomorphic functions in the unit disc U of the complex plane, we consider the subset $H^\infty(U;K)$

of functions that take values in a compact K. The work is aimed at description of the extreme points of the set $H^\infty(U;K)$ for a sufficiently wide class of convex compacts K, namely, for compacts satisfying the condition

(P) *there exists a natural number n such that at any point $\zeta \in \partial K$ there exist a straight line l_ζ supporting K and a parabola of the order $2n$, with vertex at ζ, tangential to l_ζ, such that in some neighborhood D_ζ of the point ζ the branches of the parabola envelope the set $K \cap D_\zeta$.*

Note that the (P) means that the tangency order of the boundary ∂K with the support line does not exceed $2n-1$ at any point of ∂K. We prove

Theorem. *Let K be a convex compact. If a function $f(z)$ is an extreme point of the set $H^\infty(U,K)$, then necessarily K satisfies*

$$\int_0^{2\pi} \log \rho \left(f\left(e^{i\theta}\right), \partial K \right) d\theta = -\infty, \tag{1}$$

where $\rho\left(f\left(e^{i\theta}\right), \partial K\right)$ denotes the distance between $f\left(e^{i\theta}\right)$ and ∂K, and conversely if K satisfies the condition (P) and (1)*, then $f(z)$ is an extreme point of the set $H^\infty(U,K)$.*

A counterpart of this theorem is valid for the Banach space $A(U)$ of functions analytic in the disc and continuous up to its boundary.

The Hadamard product of holomorphic functions on open sets

T. Pohlen (Germany)

pohlen@uni-trier.de

Let $f(z) = \sum\limits_{\nu=0}^{\infty} a_\nu z^\nu$ and $g(z) = \sum\limits_{\nu=0}^{\infty} b_\nu z^\nu$ be power series with positive radii of convergence r_a and r_b, respectively. The Hadamard product is the power series $\sum\limits_{\nu=0}^{\infty} a_\nu b_\nu z^\nu$. Its radius of convergence r satisfies $r \geq r_a \cdot r_b$. For z with small modulus the Hadamard product has the integral representation

$$\sum_{\nu=0}^{\infty} a_\nu b_\nu z^\nu = \frac{1}{2\pi i} \int\limits_{|\zeta|=r} \frac{f(\zeta)}{\zeta} g\left(\frac{z}{\zeta}\right) d\zeta, \tag{1}$$

known as the Parseval integral. The right hand side of (1) can also be regarded for functions $f \in H(\Omega_1)$ and $g \in H(\Omega_2)$, where $\Omega_1, \Omega_2 \subset \mathbb{C}$ are open sets with $0 \in \Omega_1 \cap \Omega_2$. The question is in what way the condition $0 \in \Omega_1 \cap \Omega_2$ can be relaxed. We give a definition how the Hadamard product of f and g can be extended to open sets not necessarily containing the origin.

On the local equatorial characterization of zonoids

D. Ryabogin (USA)

ryabogin@math.kent.edu

Zonotopes and zonoids are convex bodies that are finite and infinite sums of segments. Zonotopes can be easily characterized as polytopes with two-dimensional symmetric faces. Moreover, every zonotope has "zones" - the set of two-dimensional faces containing edges parallel to any given edge. On the other hand, as it was shown by W. Weil in 1977, there is no simple "inner" characterization of zonoids. In particular, W. Weil asked whether one can obtain the so-called "local equatorial" characterization, that would correspond to the "zonal description" of zonotopes, mentioned above. It was shown by G. Panina and independently by P.Goodey and W. Weil that such characterization is, indeed, possible in EVEN dimensions. Recently we (together with F. Nazarov and A. Zvavitch) proved that the local equatorial characterization of zonoids is possible ONLY in EVEN dimensions.

Duality relations in the theory of frames and Bessel sequences

A. Safapour (Iran)

safapour@mail.vru.ac.ir

Frames was first introduced by Duffin and Schaeffer in the context of nonharmonic analysis in 1952. From the 80th of the previous century frames began to be widely studied by mathematicians, physicists, engineers and some other specialists. Recently duality relations have been of interests in frame theory and related topics. They play a fundamental role in analyzing reproducing systems, especially Gabor systems. There are several duality relations in these fields. Dual frames, R-dual sequences, Ron-Shen duality principle and Wexler-Raz orthogonality relation are some of the most important of them. In this talk we have a new look at these theories. A characterization of duals of frames will be given. Duality relation for Gabor frames in $L^2(R)$ as well as for $L^2(G)$ when G is an LCA group will be discussed. Some new results and examples are included.

Approximations of Traces of Products of Truncated Toeplitz Operators

A. Sahakian and M. S. Ginovyan[5] (Armenia, USA)

sart@ysu.am, ginovyan@math.bu.edu

Truncated Toeplitz operators arise commonly in the statistical analysis of continuous-time stationary processes: asymptotic distributions and large deviations of Toeplitz type quadratic functionals, estimation

[5]M. Ginovyan's research was partially supported by National Science Foundation Grant #DMS-0706786.

of the spectral functionals, hypotheses testing about the spectrum, etc. (see, e.g., [2], and references therein).

Here we are concerned with the problem of approximation of the traces of products of truncated Toeplitz operators and bounding the corresponding errors.

This problem is of particular importance in the cases where the underlying model displays a continuous-time stationary process with possibly unbounded or vanishing spectral density. Such cases arise in the study of long-memory (the spectral density is unbounded) and anti-persistent (the spectral density has zeros) continuous-time stationary processes.

Let $f_i(\lambda)$, $i = 1, 2$, be integrable real symmetric functions defined on $\mathbb{R}$. Denote by $B_T(f_i)$ the truncated Toeplitz operators generated by $f_i(\lambda)$, and for a positive integer ν define $S_{T,\nu} = \frac{1}{T}\mathrm{tr}[B_T(f_1)B_T(f_2)]^\nu$, where $\mathrm{tr}[A]$ stands for the trace of A. It is well-known that a good approximation of $S_{T,\nu}$ provides the functional (see, e.g., [2])

$$M_\nu = (2\pi)^{2\nu-1}\int_{-\infty}^{\infty}[f_1(\lambda)f_2(\lambda)]^\nu\,d\lambda. \tag{1}$$

The problem of approximation $S_{T,\nu}$ by M_ν and estimation of the error rate for $\Delta_{T,\nu} := |S_{T,\nu} - M_\nu|$ goes back to the classical monograph by Grenander and Szegö (1958), and has been studied by Ibragimov (1963), Ginovian (1988, 1994), and Ginovyan and Sahakian (2007) (see [2], and references therein), where sufficient conditions were found for asymptotic relation

$$\Delta_{T,2} = |S_{T,2} - M_2| = o(1) \quad \text{as} \quad T \to \infty.$$

In this paper we provide conditions in terms of functions $f_i(\lambda)$ ensuring the asymptotic relation $\Delta_{T,2} = O(T^{-\gamma})$ ($\gamma > 0$) as $T \to \infty$. The results improve on the $o(1)$ rates obtained in Ginovyan and Sahakian [2].

The theorem that follows is a typical result in this direction. For $0 < \alpha \le 1$ and $p \ge 1$, we denote by $\mathrm{Lip}(\mathbb{R}; p, \alpha)$ the L^p-Lipschitz class of functions defined on $\mathbb{R}$.

Theorem. *Let* $f_i(\lambda) \in \mathrm{Lip}(\mathbb{R}; p_i, \alpha)$, $0 < \alpha \leq 1$ *and* $p_i \geq 1$, $i = 1, 2$ *with* $1/p_1 + 1/p_2 \leq 1/2$. *Then for any* $\varepsilon > 0$, $\Delta_{T,2} = O\left(T^{-\alpha+\varepsilon}\right)$ *as* $T \to \infty$.

For Toeplitz matrices generated by spectral densities of FARIMA$(0, d_i, 0)$ ($d_i \in (0, 1/2)$) processes (which are long-memory stationary time series), Lieberman and Philips (see [3]) observed that the first-order approximation for $S_{T,1}$ breaks down (because M_1 has singularity at the boundary of parameter space), and the second-order term in the asymptotic expansion removes this singularity, and provides an improved approximation.

We establish a similar fact for stationary fractional Riesz-Bessel motion (fRBm). A fRBm, introduced in Anh et al. [1], is defined to be a continuous-time Gaussian process $X(t)$ with spectral density

$$f(\lambda; \alpha, \beta) = C|\lambda|^{-2\alpha}(1 + \lambda^2)^{-\beta}, \quad \lambda \in \mathbb{R}, \tag{2}$$

where C is a positive constant, $0 < \alpha < 1$ and $\beta > 1/2$.

Observe that $X(t)$ is stationary if $0 < \alpha < 1/2$ and is nonstationary with stationary increments if $1/2 < \alpha < 1$. The exponent α determines the long-range dependence (LRD) of fRBm, while the exponent β indicates the second-order intermittency of the process.

Assuming that the truncated Toeplitz operators $B_T(f_i)$ are generated by the spectral densities $f_i(\lambda) = f(\lambda; \alpha_i, \beta_i)$ ($i = 1, 2$) of the form (2) with $\alpha_i \in (0, 1/2)$ (i.e., the underlying fRBm processes are stationary), we derive an explicit second-order asymptotic expansion for the functional $S_{T,1}$, and show that the order of magnitude of the second term in this expansion depends on the long-memory parameters α_i of the processes. Also, we show that the pole in the first-order approximation is removed by the second-order term, which provides a substantially improved approximation to the original functional.

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Hermit trigonometric interpolation and quadrature formulae

K. Sahakyan and S. Hayrapetyan (Armenia)

karen_sahakyan@mail.ru, sofia31337@mail.ru

An Hermit trigonometric interpolation formula is developed. This formula is convenient to use for numerical integration. A convergence of obtained quadrature formula is proved as well.

On fast pointwise approximation of Hardy space

S. Samarchyan and V. Musoyan (Armenia)

approximation@gmail.com

Let K be a compact subset of the interval $[0,1)$. We investigate the existence of a system $e_k \in C(K)$, $k = 1,2,\ldots$ such that each function $f \in H^\infty(D)$ is pointwise approximable by $\{e_k(x)\}_{k=1}^{\infty}$ faster than geometrical progression.

In the case, when the coefficients of linear combinations continuously depend on the considered function, one can use Tikhomirov's method and the method of q-bounded systems defined in any locally convex topological vector space ([1]) to obtain the following

Theorem. *There isn't any system $e_k \in C(K)$, $k = 1, 2, \dots$ that for each function $f \in H^\infty(D)$ there exists a triangular matrix of coefficients $\left(a_k^{(n)}\right)_{k \leq n}$, which continuously depends on f and*

$$\sqrt[n]{\left| f\left(x\right) - \sum_{k=1}^{n} a_k^{(n)} e_k\left(x\right) \right|} \to 0, \ x \in K.$$

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L^p estimates of oscillatory integrals

G. Sampson (USA)

sampsgm@auburn.edu

For $a_j, b_j \geq 1, j = 1, 2$, we prove in 2− dimensions that the operator

$$Kf(x) = \int_{\mathbb{R}^2_+} k(x,y) f(y) dy, \quad x \in \mathbb{R}^2_+ = [0, \infty) \times [0, \infty)$$

maps $L^p(\mathbb{R}^2_+)$ into itself for $p \in [\frac{a_1+b_1}{a_1+(\frac{r}{2})b_1}, \frac{a_1+b_1}{a_1(1-\frac{r}{2})}]$ where $\frac{a_1}{b_1} = \frac{a_2}{b_2} \geq 1, b_1 b_2 > 1$, and $k(x,y) = \varphi(x,y) e^{ig(x,y)}$. And $\varphi(x,y)$ satisfies for $|x - y| > 0$ and $0 \leq r < 2$,

$$|\partial_x^\alpha \partial_y^\beta \varphi(x,y)| \leq C_{\alpha\beta} |x-y|^{-r-|\alpha|-|\beta|}, \forall \alpha, \beta \in \mathbb{N}^2, \mathbb{N} = \{0, 1, 2, \cdots\},$$

e.g. $\varphi(x,y) = |x-y|^{-r-i\tau}, \tau$ real. With $x^a \cdot y^b = x_1^{a_1} y_1^{b_1} + x_2^{a_2} y_2^{b_2}$, and $g(x,y) = x^a \cdot y^b + \mu_{\bar{1}}(x)\mu_{\bar{1}}(y)\Phi(x^a, y^b), \Phi(x,y) \in C^\infty(\mathbb{R}^4)$ and except for some minor technical conditions

$$|\partial_x^\alpha \partial_y^\beta \Phi(x,y)| \leq C_{\alpha\beta} \text{ for } x, y \geq \bar{1} \text{ and } \sum_{j=1}^{2} (\alpha_j + \beta_j) \geq 1.$$

Also $\mu_{\bar{1}}(x) = \mu_1(x_1)\mu_1(x_2), \mu_1(t) = 1$ for $t \geq 2, \mu_1(t) = 0$ for $0 \leq t \leq 1, 0 \leq \mu_1(t) \leq 1$ and $\mu_1(t) \in C^\infty(\mathbb{R}_+)$. Examples of Φ's defined for $x, y \geq \bar{1}$ is $log(\sum_{j=1}^2 x_j + y_j)$ or $(\sum_{j=1}^2 x_j + y_j)^l, 0 \leq l < 1$, if $y \geq \bar{4}$ and if $\bar{1} \leq y \leq \bar{3}$, set $\Phi(x,y) = 1$.

Greedy approximation with respect to Faber-Schauder system

A. Sargsyan (Armenia)

asargsyan@ysu.am

It is known that there is no quasi-greedy basis in the space $C[0,1]$, i.e. the Faber-Schauder system $\Phi = \{\varphi_n(x)\}_{n=0}^\infty$ (which is a basis in the space $C[0,1]$) is not quasi-greedy in $C[0,1]$.

The following strengthenings of this result take place:

Theorem 1. *There exists a function $f(x) \in C[0,1]$ with coefficients of the expansion by Faber-Schauder system satisfying the condition*

$$|A_n(f)| = O\left(\log_2^{-1} n\right), \quad \text{when} \quad n \to \infty,$$

and with greedy algorithm divergent in the norm of space $C[0,1]$.

It is not hard to see, that if coefficients of an expansion of a function $f(x) \in C[0,1]$ by Faber-Schauder system satisfy the condition $|A_n(f)| = O\left(\log_2^{-1-\varepsilon} n\right)$ when $n \to \infty$ $(\varepsilon > 0)$, then the series $\sum_{n=0}^\infty A_n(f)\varphi_n(x)$ absolutely (then also unconditionally) converges in the norm of space $C[0,1]$.

Theorem 2. *There exists a function $f(x) \in C[0,1]$ with greedy algorithm with respect to Faber-Schauder system divergent by measure on $[0,1]$.*

So, greedy algorithm with respect to Faber-Schauder system uniformly converges on $[0,1]$ not for all continuous in $[0,1]$ functions. One might ask whether it is always possible to alter the values of an arbitrarily given continuous in $[0,1]$ function on a small set, so that the greedy algorithm with respect to Faber-Schauder system, when applied to the function thus altered, will yield a sequence of approximants that does uniformly converge on $[0,1]$.

The answer of this question is positive. The following theorems take place:

Theorem 3. *For each $0 < \epsilon < 1$ and for every function $f(x) \in C[0,1]$ one can find a function $g(x) \in C[0,1]$, $mes\{g(x) \neq f(x) , x \in [0,1]\} < \epsilon$, with expansion by Faber-Schauder system unconditionally convergent in $C[0,1]$.*

Theorem 4. *For each $\epsilon \in (0,1)$ there exists a measurable set $E \subset [0,1]$ with measure $|E| > 1-\epsilon$, so that to every $f(x) \in C[0,1]$ there corresponds a function $g(x) \in C[0,1]$, that coincides whit $f(x)$ on E, such that every two coefficients of the expansion of that function with respect to Faber-Schauder system are not equal to each other and are not vanishing, the greedy algorithm of that function by Faber-Schauder system uniformly converges to it, and the following inequality takes place:*

$$\|G_m(g,\Phi)\|_C \ \leq \ 5 \cdot ||g||_C \leq 10 \cdot ||f||_C \ , \ \forall m \in \mathbb{N}.$$

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Independent functions and Banach-Saks property

E. Semenov (Russia)

semenov@func.vsu.ru

We study the Banach-Saks property and p-Banach-Saks property in the class of rearrangement invariant spaces. The Banach-Saks index is an important characteristic of Banach spaces. We prove that it is closely related with properties of independent functions. The talk is based on joint article with S.V.Astashkin and F.A.Sukochev. Research is supported by the RFBR grant.

Construction of wavelet packets associated with p-multiresolution analyses

F. A. Shah (India)

fashah_jmi@yahoo.co.in

In the present paper, we construct p-wavelet packets associated with multiresolution p-analysis defined by Farkov for $L^2(\mathbb{R}+)$. The collection of all dilations and translations of the wavelet packets defines the general wavelet packets and is an overcomplete system.

Sharp embedding theorems for analytic spaces in the unit ball defined with the help of Luzin area operator and Bergman metric ball

R. Shamoyan (Russia)

rsham@mail.ru

We will add to the list of sharp embedding theorems in the unit ball in $\mathbb{C}^n$ some new results concerning spaces with unusual quasinorms defined with the help of Bergman metric ball and spaces defined with the help of Luzin area operator. Such type spaces appeared recently in the unit disk in [1],[2] and in the ball in [3]. We,in particular, use "cutting measure" argument to get new sharp embedding theorems from known results and properties of Bergman metric ball. A generalization of a result from [1] to the case of unit ball will be also presented. We apply inequalities for tent spaces in the unit ball from [4].

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Convergence of Wavelet Series

N. Sheikh (India)

neyaznit@yahoo.co.in

The Cesáro means and integral modulus of continuity have been studied in various ways. Here in this paper, we define Cesáro means and integral modulus of continuity for wavelet series. We made a complete investigation concerning the interaction between the rate of convergence of Cesáro means of wavelet series and integral modulus of continuity. We give the best possible sufficient condition with respect to modulus of continuity that implies the convergence at a given rate.

On the weighted interval quadrature formulae on some classes of periodic functions

D. Skorokhodov and V. Babenko (Ukraine)

dmitriy.skorokhodov@gmail.com, babenko.vladislav@gmail.com

Let C and L be the spaces of 2π-periodic functions which are continuous and Lebesgue integrable respectively. By W_L denote the unit ball in the space L.

Let $K * f$ stands for the convolution of functions $f, K \in L$. For an arbitrary $F \subset C$ denote by $K * F$ the set of all functions ϕ which can be presented in the form

$$\phi = a + K * f,$$

where $a \in \mathbb{R}$, $a \perp K$, and $f \in F$.

The function $K \in L$ is called the CVD-kernel if for an arbitrary function $f \in C$

$$\nu(K * f) \leq \nu(f),$$

where $\nu(g)$ denotes the number of sign changes of function $g \in C$ on a period.

Let $n \in \mathbb{N}$ and $h \in (0, \pi/n)$ be fixed. Let also $p \in L$ be a non-negative function such that $\operatorname{supp} f \cap (\pi, \pi) \subset [-h, h]$.

Denote by $Q_n(p)$ the set of all possible weighted interval quadrature formulae of the form

$$\kappa(f) = \sum_{i=1}^{n} a_i \int\limits_{x_i-h}^{x_i+h} f(t)p(t-x_i)\,dt,$$

where $x_1 < x_2 - 2h < \ldots < x_n - 2(n-1)h < 2\pi + x_1 - 2nh$ and $\{a_i\}_{i=1}^{n} \subset \mathbb{R}$.

For a given set F and for a formula $\kappa \in Q_n(p)$ let

$$R(F,\kappa) = \sup_{f\in F} \left| \int\limits_0^{2\pi} f(t)\,dt - \kappa(f) \right|$$

and

$$\mathcal{E}(F,\kappa) = \inf_{\kappa\in Q_n(p)} R(f,\kappa).$$

By $\kappa_n(p)$ we shall denote the formula from $Q_n(p)$ having equidistant node intervals and equal coefficients.

Theorem. *Let $n \in \mathbb{N}$, $h \in \left(0, \frac{\pi}{n}\right)$ and K be a CVD-kernel. Then*

$$\mathcal{E}(K * W_L, Q_n(p)) = R(K * W_L, \kappa_n(p)).$$

The similar results were also proved for the convolutions of W_L with kernels from $\mathcal{A}_n$ and $CVD, \triangle$. Moreover, in the case $p(t) \equiv \chi_{[-h,h]}(t)$ on (π, π), it was proved that $\mathcal{E}(K * F, Q_n(p)) = R(K * F, \kappa_n(p))$ for an arbitrary rearrangement-invariant set F and CVD-kernel K.

Universal series in weighted L^1_μ spaces

M. Soghomonyan (Armenia)

soghomonyanmkrtich@gmail.com

A series $\sum_{n=1}^{\infty} \varphi_n$ is said to be universal in X Banach space with respect to subseries, if $\forall x \in X$ there exists a growing sequence of natural numbers n_k, such that

$$\sum_{k=1}^{\infty} \varphi_{n_k} \overset{X}{=} x$$

Universal series can be defined also with respect to rearrangement: for each $x \in X$ there exists a $\sigma : N \to N$ bijection, such that

$$\sum_{n=1}^{\infty} \varphi_{\sigma(n)} \overset{X}{=} x$$

Definition. A system $\{\varphi_n\}_{n=1}^{\infty}$ in a Hilbert space H is called a frame if there exist two constants $0 < A \leq B < \infty$, such that for all $h \in H$

$$A||h||^2 \leq \sum_{n=1}^{\infty} (\langle h, \varphi_n \rangle)^2 \leq B||h||^2$$

Theorem. *Let $\{\varphi_n(x)\}_{n=1}^{\infty}$ be a frame in $L^2_{[0,1]}$. Then for any $\epsilon > 0$ there exist a function $\mu(x)$ $(0 < \mu(x) \leq 1)$ and a measurable set $E_\epsilon \subset [0,1]$, such that $|E_\epsilon| > 1 - \epsilon$, $\mu(x) = 1$ on E_ϵ, and there exists a series $\sum_{n=1}^{\infty} a_n \varphi_n(x)$ which is universal in $L^p_{\mu(x)}[0,1]$ $(1 \leq p \leq 2)$ with respect to subseries.*

Note that in [1] M.G.Grigorian established the existence of universal series $\sum a_n \varphi_n(x)$ in weighted L^p_μ spaces with respect to subseries by general orthonormal system $\{\varphi_n(x)\}$

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Rational Best Approximation on $(-\infty, 0\,]$

H. Stahl (Germany)

stahl@tfh-berlin.de

We consider rational best approximants

$$r_n^* = r_n^*(f, (-\infty, 0]; \cdot) \in R_{n,n+k}$$

on $(-\infty, 0]$, $k \in \mathbb{Z}$ fixed, to functions of the form

$$f = u_0 + u_1 \exp \tag{1}$$

with u_0, u_1 given rational functions and exp denoting the exponential function.

Starting point of the talk is the famous solution of the $'1/9'$ problem by Gonchar & Rakhmanov from 1986, which deals with the uniform rational approximation of the exponential function on $(-\infty, 0]$. This result has been extended to functions of type (1). Related questions will be addressed in the talk, as for instance, overconvergence throughout the complex plane $\mathbb{C}$, the asymptotic distribution of the poles of the approximants, the convergence of close-to-best approximants, and methods for the numerical calculation of these approximants.

The renewed and extended interest in the by now classical $'1/9'$ problem is motivated by applications in numerical analysis, where good rational approximants are needed for functions of type (1), as for instance, for so-called $'\varphi$ functions' that appear in exponential integrators.

Universality of series by general Franklin system

Arpi Stepanyan (Armenia)

arpistepanyan@gmail.com

For the general Franklin system $\{f_n\}_{n=0}^{\infty}$ corresponding to quasi-dyadic weak regular sequences of partition of $[0,1]$ we prove the existence of universal series with respect to subseries for almost everywhere (a.e.) absolute convergence in the class of a.e. finite measurable functions, namely:

Theorem. *There exists a series $\sum_{n=0}^{\infty} a_n f_n(x)$ such that for any a.e. finite measurable function $f(x)$ there exists an increasing subsequence of natural numbers $n_1 < n_2 < ... < n_k < ...$ such that the series $\sum_{k=0}^{\infty} a_{n_k} f_{n_k}(x)$ almost everywhere absolutely converges to $f(x)$ on $[0,1]$, i.e.*

$$\sum_{k=0}^{\infty} a_{n_k} f_{n_k}(x) = f(x) \quad a.e. \quad on \quad [0,1]$$

and

$$\sum_{k=0}^{\infty} |a_{n_k} f_{n_k}(x)| < +\infty \quad a.e. \quad on \quad [0,1].$$

An analogous theorem holds also for double series by general Franklin system.

On the Summability of Fourier Series with Variable Order

Sh. Tetunashvili (Georgia)

stetun@rmi.acnet.ge, stetun@hotmail.com

Let σ_n^{α} be a sequence of the Cesaro summability means of order α for trigonometric Fourier series of function $f(x)$.

Let α_n be a sequence of numbers from the interval $[0, 1]$.

Consider the means $\sigma_n^{\alpha_n}(x, f)$ with variable order.

The following theorems are valid

Theorem 1. *For any sequence of numbers $\alpha_n \to 0+$, as $n \to \infty$, there exists a continuous function $f(x)$, such that the sequence of means $\sigma_n^{\alpha_n}(x; f)$ diverges at the point x_0.*

On the other hand we have

Theorem 2. *For any continuous function $f(x)$, there exists a sequence of numbers $\alpha_n \downarrow 0$ as $n \to \infty$, such that the equality*

$$\lim_{n\to\infty} \sigma_n^{\alpha_n}(x; f) = f(x)$$

is fulfilled at every point x.

Similar theorems for norm summability are true for integrable functions.

Theorem 2 gives the positive answer to the problem posed by V. Temlyakov.

The polynomial inverse image method

V. Totik (Hungary and USA)

totik@shell.cas.usf.edu

We discuss the polynomial inverse image method that can be used to carry over results from some simple sets like intervals or circles to more general sets like arbitrary compact sets on the real line or on the plane. We shall mention several recent applications including sharp polynomial inequalities, polynomial approximation on general compacts, asymptotics for Christoffel functions on curves, fine zero spacing of orthogonal polynomials and an extension of Lubinsky's universality theorem from an interval to general compact sets on $\mathbb{R}$.

Approximation of functions by polynomials with restrictions

R. Trigub (Ukraine)

postmaster@ok.donbass.com

We study pointwise approximations of functions with a given modulus of continuity of the r-th derivative on a interval (segment) of real axis as a rule with various additional restrictions.

A. The first of these restrictions is the simultaneous approximation of functions and their derivatives by polynomials with Hermitian interpolation of any multiplicity at given points. These theorems can be applied, for example, to obtain precise sufficient conditions for the expansion of functions in Fourier-Jacobi series.

B. The second restriction is that the functions are approximated by polynomials on only one side (from above or below). J. Karamata was the initiator. Direct theorems for one-sided integral approximation of smooth functions were obtained long ago (A.G. Postnikov, G. Freud). These theorems were used in the proofs of Tauberian theorems with remainder term.

Comonotone approximations when the function and the approximating polynomial are supposed monotone, convex et cetera. G.G. Lorentz was the initiator.

C. Restrictions on the coefficients of the approximating polynomials. If the coefficients of polynomials are assumed to be integer, then we have to impose some arithmetical restrictions on the function. The first exact results on the order of approximation on $[0, 1]$(analogues of theorems of Jackson and Bernstein) were obtained by Gel'fond. Pointwise approximations on any line interval of length at most four were studied by author.

Uniform approximations of functions of functions by polynomials with positive coefficients have been studied in connection with a problem on the spectra of positive operators.

An attempt is taken here to combine the mentioned constraints, when it is possible of course.

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Bellman function method in analysis

V. Vasyunin (Russia)

vasyunin@pdmi.ras.ru

The Bellman function method is a rather elementary but very powerful tool for obtaining various inequalities in modern analysis. Using a function with some concavity property you can prove a desired estimate. If you succeded in finding the minimal (maximal) possible such function, you get the inequality with sharp constant. This the best possible function is called the Bellman function of the corresponding problem. In the talk I'll try to illustrate this method on the example of the classical John–Nirenberg inequality for the BMO-functions on the real line.

Universal properties of approximation operators

A. Vogt (Germany)

And-Vogt@gmx.net

We discuss universal properties of some operators $L_n : C[0,1] \to C[0,1]$.

The operators considered are closely related to a theorem of Korovkin, which states the following:

Theorem. *(Korovkin) If a sequence of operators L_n satisfies the conditions*

(1) L_n is linear,

(2) L_n is positive

(3) $L_n f_i \to f_i$ uniformly for $i = 0, 1, 2$, where $f_i(x) = x^i$,

then $L_n f \to f$ uniformly for every continuous function f.

We investigate, which behavior of $L_n f$ is possible, if we modify the assumptions of Korovkin's theorem slightly. There exists for example continuous function f, such that $(L_n f)_{n\in\mathbb{N}}$ is dense in $(C[0,1], \|\cdot\|_\infty)$, even if L_n satisfies the conditions $(1), (2)$ and $L_n f_i \to f_i$ for $i = 1, 2, 3 \ldots$. Similar phenomena occur in polynomial interpolation. If P_n is the polynomial of degree $\leq n$, which interpolates a function f at the nodes $0 \leq x_0^n < x_1^n < \ldots < x_n^n \leq 1$ $(n \in \mathbb{N})$, it is well known, that even if f is a continuous function, $(P_n)_{n\in\mathbb{N}}$ does not necessarily converge to f. Moreover, there exists an infinitely differentiable function f and a system of nodes, such that for any measurable function g, there exists a subsequence of $(P_n)_{n\in\mathbb{N}}$, which converges almost everywhere to g.

Compressed sensing

P. Wojtaszczyk (Poland)

pwojt@mimuw.edu.pl

Compressed sensing is a novel scheme in signal processing developed in the last five years by E.Candes, D.Donoho, T.Tao et al. The aim is to recover k most essential pieces of information about a signal by applying d linear, nonadaptive measurements. The simplest discrete scheme is: The signal is a vector $x \in \mathbb{R}^N$ with N very big and we assume that it has at most k non-zero coordinates. We want to find d linear forms $\phi_1, \ldots \phi_d$ on $\mathbb{R}^N$ and a decoder $\Delta : \mathbb{R}^d \to \mathbb{R}^N$ such that

$$\Delta(\phi_1(x), \ldots, \phi_d(x)) = x$$

An important additional requirement is that the whole procedure must be numerically feasible. I will describe the nature of measurement sets $(\phi_j)_{j=1}^d$ considered in the literature and proposed decoders.

The main interest is in the stability of the decoders i.e. what happens when x is not k-sparse and/or we apply the decoder to a measurement given with some error.

On wavelets with compact support constructed by means of splines

Z. Wronicz (Poland)

wronicz@agh.edu.pl

In 1982 J.O. Strömberg constructed a wavelet by means of piecewise linear functions i.e. a function $\psi \in L^2(\mathbb{R})$, $\|\psi\| = \left(\int_{\mathbb{R}} |\psi(t)|^2 \, dt \right)^{\frac{1}{2}}$ such that the system $\left\{\psi_{j,k}\right\}_{j,k\in\mathbb{Z}}$, $\psi_{j,k}(x) = 2^{\frac{j}{2}}\psi\left(2^j x - k\right)$ is an orthonormal basis in the space $L^2(\mathbb{R})$. Unfortunately, the support of this wavelet is not bounded. In the talk we shall present a construction of a wavelet with support equal to [0,3] by means of piecewise linear functions and we shall give a few open problems connected with orthonormal wavelets constructed by means of splines.