International Conference

HARMONIC ANALYSIS AND APPROXIMATIONS, V

Dedicated to 75th anniversary of academician

Norair Arakelian

10 - 17 September, 2011, Tsaghkadzor, Armenia

Yerevan, 2011

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C O N T E N T S

On order of approximation of the generalized Nikol'skii – Besov class in a Lorentz space

G. Akishev (Karaganda State University, Kazakhstan)
akishev@ksu.kz

Let $\overline{x} = (x_1, ..., x_m) \in I^m = [0, 2\pi)^m$ and let $\theta_j, q_j \in [1, +\infty)$, $j = 1, ..., m$.

We shall denote by $L_{\overline{p},\overline{\theta}}(I^m)$ the Lorentz spaces with mixed norm of Lebesgue - measurable functions $f(\overline{x})$ of period 2π in each variable such that $\|f\|_{\overline{p},\overline{\theta}} = \|...\|f\|_{p_1,\theta_1}...\|_{p_m,\theta_m} < +\infty$, where

$$\|g\|_{p,\theta} = \left\{ \int_0^{2\pi} (g^*(t))^\theta t^{\frac{\theta}{p}-1} dt \right\}^{\frac{1}{\theta}},$$

and g^* is the non-increasing rearrangement of the function $|g|$.

Let $a_{\overline{n}}(f)$ be the Fourier coefficients of $f \in L_1(I^m)$ with respect to the multiple trigonometric system . Then we set $\delta_{\overline{s}}(f, \overline{x}) = \sum\limits_{\overline{n} \in \rho(\overline{s})} a_{\overline{n}}(f) e^{i\langle \overline{n}, \overline{x} \rangle}$,

where $\langle \bar{y}, \bar{x} \rangle = \sum\limits_{j=1}^{m} y_j x_j$,

$$\rho(\bar{s}) = \left\{ \overline{k} = (k_1, ..., k_m) \in Z^m : \quad 2^{s_j - 1} \le |k_j| < 2^{s_j}, j = 1, ..., m \right\}.$$

For a numerical sequence we write $\{a_{\overline{n}}\}_{\overline{n} \in Z^m,} \in l_{\overline{p}}$ if

$$\left\| \{a_{\overline{n}}\}_{\overline{n} \in Z^m} \right\|_{l_{\overline{p}}} = \left\{ \sum_{n_m = -\infty}^{\infty} \left[... \left[\sum_{n_1 = -\infty}^{\infty} |a_{\overline{n}}|^{p_1} \right]^{\frac{p_2}{p_1}} ... \right]^{\frac{p_m}{p_{m-1}}} \right\}^{\frac{1}{p_m}} < +\infty,$$

where $\overline{p} = (p_1, ..., p_m)$, $1 \le p_j < +\infty$, $j = 1, 2, ..., m$. Let the function $\Omega(\bar{t}) = \Omega(t_1, ..., t_m)$ be a function of mixed modulus of continuity type of an order $l \in \mathbb{N}$. We consider the following sets

$$\Gamma(\Omega, N) = \left\{ \bar{s} = (s_1, ..., s_m) \in \mathbb{Z}_+^m : \Omega(2^{-s_1}, ..., 2^{-s_m}) \ge \frac{1}{N} \right\},$$

$$Q(N) = \bigcup_{\bar{s}\in\Gamma(\Omega,N)} \rho(\bar{s}), \quad \text{and denote } \; S_{Q(N)}(f,\bar{x}) = \sum_{\bar{k}\in Q(N)} a_{\bar{k}}(f)\cdot e^{i\langle \bar{k},\bar{x}\rangle}.$$

Consider the generalized Nikol'skii – Besov class

$$S^{\Omega}_{\overline{p},\overline{\tau}}B = \left\{ f \in \overset{\circ}{L}_{\overline{p}}(I^m) : \left\| \left\{ \Omega^{-1}(2^{-\bar{s}}) \, \|\delta_{\overline{s}}(f)\|_{\overline{p}} \right\}_{\bar{n}\in\mathbb{Z}^m_+} \right\|_{l_{\overline{\tau}}} \le 1 \right\},$$

where $\overline{p} = (p_1, ..., p_m)$, $\overline{\tau} = (\tau_1, ..., \tau_m)$, $1 \le p_j, \tau_j < +\infty$, $j = 1, ..., m$ and $\Omega(2^{-\bar{s}}) = \Omega(2^{-s_1}, ..., 2^{-s_m})$. Approximation of the generalized Nikol'skii – Besov classes in Lebesgue spaces with isotropic norm $L_p(I^m)$ was considered by Sun Yongsheng and Wang Heping, N.N. Pustovoitov, D.B. Bazakhanov, S.A. Stasyuk. The main aim of the present talk is to present an estimate of the order of approximation of Nikol'skii–Besov classes in the metric of the Lorentz space.

Theorem. *Let a function $\Omega(\bar{t})$ be a function of mixed modulus of continuity type of an order l satisfying to some additional conditions. If $1 < q_j < p_j < \infty$, and $p_j \ge 2$, $1 \le \theta_j, \tau_j < +\infty$, $j = 1, ..., m$, then*

$$\sup_{f\in S^{\Omega}_{\overline{p},\overline{\tau}}B} \|f - S_{Q(N)}(f)\|_{\overline{q},\overline{\theta}} \asymp \frac{1}{N} (\log_2 N)^{\sum\limits_{j=2}^{m}\left(\frac{1}{2}-\frac{1}{\tau_j}\right)}$$

for $2 < \tau_j < +\infty$, $j = 1, ..., m$. If $\tau_j \le 2$, $j = 1, ..., m$, then

$$\sup_{f\in S^{\Omega}_{\overline{p},\overline{\tau}}B} \|f - S_{Q(N)}(f)\|_{\overline{q},\overline{\theta}} \asymp \frac{1}{N}.$$

Weighted greedy algorithm in $L_p(0,1)$ and $C(0,1)$ with respect to Franklin and trigonometric systems

H. Aleksanyan (Yerevan State University, Armenia)
haik_alexanyan@yahoo.com

Let X be a Banach space and $\Psi = \{\psi_n\}_{n=1}^{\infty}$ be a basis in X with $\inf\limits_n ||\psi_n|| > 0$. For any $f \in X$ one has the expansion

$$f = \sum_{n=1}^{\infty} c_n(f)\psi_n,$$

where $\{c_n(f)\}$ are uniquely determined by f and $\lim\limits_{n\to\infty} c_n(f) = 0$. We take a decreasing sequence of positive numbers $\Gamma = \{\gamma_n\}_{n=1}^{\infty}$ and for $f \in \mathrm{X}$ consider the decreasing rearrangement of absolute values of non-vanishing coefficients of f with the weights γ_n:

$$|\gamma_{\sigma(1)}c_{\sigma(1)}(f)| \geq |\gamma_{\sigma(2)}c_{\sigma(2)}(f)| \geq \ldots \geq |\gamma_{\sigma(n)}c_{\sigma(n)}(f)| \geq \ldots,$$

and define the N-th weighted greedy approximant of f as follows:

$$G_N(f,\Psi,\Gamma,\sigma) := G_N(f,\Psi,\Gamma) := \sum_{n=1}^{N} c_{\sigma(n)}(f)\psi_{\sigma(n)},\ N = 1,2,\ldots$$

Note that in the case $\gamma_n \equiv 1$ we get an ordinary greedy algorithm. We study the convergence of weighted greedy approximants in general setting and in particular the interplay between the normalizing sequence Γ and various types of convergence of weighted greedy algorithm for Franklin and trigonometric systems. We obtain necessary and sufficient conditions on weight sequence for Franklin functions to guarantee uniform convergence of the greedy algorithm for functions from $\mathrm{C}[0,1]$ and almost everywhere convergence for functions from $L^1[0,1]$. Similar conditions are found in the case of trigonometric system and convergence almost everywhere or by the norm of $L^p(\mathbb{T})$, where $p > 2$. We also prove the non-existence of weight sequence for the trigonometric system which gives uniform convergence for all continuous functions or almost everywhere convergence for functions from $L^1(\mathbb{T})$.

Similar questions about convergence of weighted greedy approximation were studied by T. Tao, T. Korner, V. Temlyakov, S. Konyagin and S. Gogyan.

Optimal uniform and tangential approximation on the sector by the entire functions

S. Aleksanyan (Institute of Mathematics of NAS, Armenia)
asargis@instmath.sci.am

In this talk we discuss the problem of optimal uniform and tangential approximation on the sector $\Delta_\alpha = \{z \in \mathbb{C} : |\arg z| \leq \alpha/2\}$ for $\alpha \in (0, 2\pi)$ by entire functions. The problem of uniform and tangential approximation on the sector by entire functions was investigated by H. Kober [1], M. Keldysh [2], S. Mergelyan [3], N. Arakelyan [4]-[5] and other authors. In that works the approximable functions were holomorphic on the interior of Δ_α and there were some conditions on Δ_α.

Here we suppose that the approximable function $f \in A'(\Delta_\alpha)$, i. e. $f \in C'(\Delta_\alpha)$ and is holomorphic on the interior of Δ_α. We estimate the growth of the approximating functions on the complex plane depending on the growth of f on Δ_α and the differential properties of f on the boundary of Δ_α. We give a positive answer to the hypothesis proposed by H. Kober in [1]: *Let $f \in A_b(\Delta_\alpha)$ - is holomorphic on the interior of Δ_α, continuous on Δ_α and bounded on Δ_α. If the function $z \to f(z^{1/\rho})$ with $\rho = \pi/(2\pi - \alpha)$ is uniformly continuous on the boundary of the sector $\Delta_{\alpha\rho}$, then the function f admits uniform approximation on Δ_α by the entire functions of the order $< \rho$ or of the order ρ and finite type.*

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Dirichlet problem for non-self-adjoint degenerate differential equations of fourth order on infinite intervals

H. Ansari (Azad Islamic University of Ahar, Iran)
amirco206@yahoo.com

We consider the following differential equation

$$Lu \equiv (t^{\alpha}u'')'' + au''' + pt^{-2}u = f, \tag{1}$$

where $t \in (1, +\infty)$, $f \in L_{2,2}(1, +\infty)$, i.e. $\int_1^{+\infty} t^2 |f(t)|^2\, dt < \infty$ and $\alpha \geq 2$, $a, p = const$.

Denote by $\dot{W}^2_{\alpha}$ the completion of $\dot{C}^2[1, +\infty) = \{u \in C^2[1, +\infty), u(0) = u'(0) = u(+\infty) = u'(+\infty) = 0\}$ in the norm $||u||^2_{\dot{W}^2_{\alpha}} = \int_1^{+\infty} t^{\alpha} |u''(t)|^2\, dt$.

Note that for $\alpha \geq 2$ there is a continuous embedding $\dot{W}^2_\alpha \subset L_{2,-2}(1,+\infty)$, which is compact for $\alpha > 2$.

Definition. *We say that $u \in \dot{W}^2_\alpha$ is a generalized solution of the equation (1), if for every $v \in \dot{W}^2_\alpha$ the equality*

$$(t^\alpha u'', v'') - a(u'', v') + p(u, v) = (f, v)$$

is valid.

First we consider the particular case of the equation (1) for $p = 0$

$$Mu \equiv (t^\alpha u'')'' + au''' = f. \tag{2}$$

It is easy to see that $D(M) = D(L)$. It follows from $u \in W^2_\alpha$ (since $\alpha \geq 2$) that $u(1) = u'(1) = u'(+\infty) = 0$. From the equality $(u''(t), u'(t)) = \frac{1}{2}\int_1^{+\infty}((u'(t))^2)'\,dt$, which is zero, we conclude that the generalized solution for the equation (2) is unique. The existence of the generalized solution for the equation (2) can be proved using Riesz's lemma on the representation of the continuous linear functionals in Hilbert space. Note that the inverse operator $M^{-1} : L_2(0,b) \to L_2(0,b)$ is compact for $\alpha > 2$. Now we can consider the equation (1) since the number $-p > 0$ can be regarded as spectral parameter for the operator M.

It is important to note that in contrast to the case of a finite interval the statement of the Dirichlet problem for the infinite interval does not depend on the sign of the coefficient a.

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On an example of GC_2 set without maximal hyperplanes

A. Apozyan (Institute of Mathematics of NAS, Armenia)
aapozyan@yahoo.com

The set of interpolation knots $X \subset \mathbb{R}^d$ is said to satisfy the geometric characterization of Chung and Yao for Π_n^d (GC_n for short), if the fundamental polynomial of each knot can be represented as a product of linear factors. In this talk we reject the generalized Gasca-Maeztu conjecture proposed by C. de Boor, on existence of a maximal hyperplane of a GC_n set in $\mathbb{R}^d$, $d \geq 3$, called GM_d-conjecture. According to GM_d-conjecture, for every GC_n set $X \subset \mathbb{R}^d$ there exists a maximal hyperplane, i.e., X is a special case of Berzolari-Radon construction. We construct an example of GC_2 set in $\mathbb{R}^6$ without maximal hyperplanes, thus rejecting the above mentioned conjecture.

Analytic and real analytic continuation

(some new aspects of the Weierstrass theory)

N. Arakelian (Institute of Mathematics of NAS, Armenia)
arakelian@instmath.sci.am

Some problems of the Weierstrass theory of analytic functions are considered, such as the problem on efficient analytic continuation of an analytic element, or the problem on distribution of its possible singularities on the boundary of the circle of convergence, etc. In addition, some analogs of these problems are considered in the case of real analytic continuation of power series of several real variables, including the problems on efficient harmonic continuation of the Laplace series and the localization of their singularities.

An Inversion of Generalized Cosine Transform in Hilbert's Fourth Problem

R. Aramyan (Institute of Mathematics of NAS, Armenia)
rafikaramyan@yahoo.com

The solution of Hilbert's fourth problem interpreted as *to construct all Finsler metrics in* $\mathbf{R}^n$ *whose geodesics are straight lines* leads to integral equation of the *generalized cosine transform* type. It is the same to ask about construction of all projective Finsler metrics in $\mathbf{R}^n$. In $\mathbf{R}^2$ the A.Pogorelov-R.Ambartzumian-R.Aleksander result solves the problem. Pogorelov has shown in [2] that any smooth projective Finsler metrics in $\mathbf{R}^3$ is generated by signed measure in the space of planes.

We denote by $\mathbf{E}$ the space of planes in $\mathbf{R}^3$, and let $\mathbf{S}^2$ be the unit sphere in $\mathbf{R}^3$. We consider locally finite signed measures μ in the space $\mathbf{E}$ possessing densities with respect to the invariant measure, i. e. $\mu(de) = h(e)de$. By $[x]$ we denote the bundle of planes containing the point $x \in \mathbf{R}^3$. By h_x we denote the restriction of h onto $[x]$ as a symmetric function on $\mathbf{S}^2$. In [2], Pogorelov proved the following result.

Theorem. *If H is a smooth projective Finsler metric in $\mathbf{R}^3$ ($H : \mathbf{R}^3 \times \mathbf{S}^2 \to [0,\infty)$), then there exists a uniquely determined locally finite signed measure μ in the space $\mathbf{E}$, with continuous density function h, such that*

$$H(x,\Omega) = \int_{\mathbf{S}^2} |(\Omega,\xi)|\, h_x(\xi)\, d\xi \quad \text{for} \quad (x,\Omega) \in \mathbf{R}^3 \times \mathbf{S}^2. \tag{1}$$

Here h_x is the restriction of h onto $[x]$, $d\xi$ - the Lebesgue measure on $\mathbf{S}^2$.

Thus for smooth projective Finsler metrics, Pogorelov's result establish the existence of the measure μ, in general not positive. We are interested in the inversion of the transform $h \to H$ defined by (1) (i.e. we want to recover h from a given H).

Note that in the case when μ is a translation invariant (t.i.) measure on $\mathbf{E}$ ($d\mu = h(\xi)d\xi \cdot dp$), (1) represents the *cosine transform* playing an important role in convexity. An inversion formula of the cosine transform going back to W. Blaschke is well known.

In [1] the author of the present abstract considers the solution of the integral equation (1) by integral geometry methods and propose an inversion formula for reconstruction of Crofton measures from projective smooth Finsler metrics in $\mathbf{R}^3$.

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Some extensions of the Hardy–Littlewood inequalities for harmonic functions

K. Avetisyan and Y. Tonoyan
(Yerevan State Univeristy, Armenia)
avetkaren@ysu.am, elenatonoyan@gmail.com

Let $B_n (n \geq 2)$ be the open unit ball in $\mathbb{R}^n$, and $h(p,q,\alpha)$ $\big(0 < p,q \leq \infty$, $\alpha > 0\big)$ be the space of those functions $u(x)$ harmonic in B_n, for which the quasi-norm

$$\|u\|_{p,q,\alpha} = \begin{cases} \left(\int_0^1 (1-r)^{\alpha q-1} M_p^q(u;r)dr\right)^{1/q}, & 0 < q < \infty, \\ \sup\limits_{0<r<1} (1-r)^{\alpha} M_p(u;r), & q = \infty, \end{cases}$$

is finite. Here $M_p(u;r)$ is the pth integral mean of u over the sphere $|x| = r$. For holomorphic functions in the unit disc $\mathbb{D}$ these (mixed norm) spaces were introduced by Hardy and Littlewood, see, for example, Duren's book "Theory of H^p Spaces". Hardy and Littlewood gave a sharp comparison of growth of means $M_p(u;r)$ for different indices p. In particular, if for a holomorphic function f in $\mathbb{D}$

$$M_p(f;r) = O\left((1-r)^{-\alpha}\right),\ \alpha \geq 0,$$

then

$$M_{p_0}(f;r) = O\left((1-r)^{-\alpha+1/p-1/p_0}\right),$$

as $r \to 1^-$ for any $p_0 > p$. By proving some continuous inclusions for mixed norm spaces $h(p,q,\alpha)$, we have extended the latter and some other estimates of Hardy and Littlewood to harmonic functions in the ball B_n.

Theorem. *For any* $\alpha > 0$, $0 < p, q \leq \infty$, *the following inclusions are continuous:*

$$(a)\quad h(p,q,\alpha) \subset h\left(p_0, q, \alpha + \frac{n-1}{p} - \frac{n-1}{p_0}\right), \qquad 0 < p < p_0 \leq \infty,$$

$$(b)\quad h^p \subset h\left(p_0, \infty, \frac{n-1}{p} - \frac{n-1}{p_0}\right), \qquad 0 < p < p_0 \leq \infty,$$

$$(c)\quad h^p \subset h\left(p_0, q, \frac{n-1}{p} - \frac{n-1}{p_0}\right), \qquad 1 < p < p_0 \leq \infty,\ p \leq q \leq \infty.$$

On the defect numbers of the boundary value problem for the properly elliptic equation

A. Babayan (State Engineering University of Armenia, Armenia)
barmenak@gmail.com

Let D be the unit disk of the complex plane with boundary $\Gamma = \partial D$. We consider in D the following elliptic equation

$$\frac{\partial}{\partial \bar{z}}\left(\frac{\partial}{\partial \bar{z}} - \mu\frac{\partial}{\partial z}\right)\left(\frac{\partial}{\partial z} - \nu_1\frac{\partial}{\partial \bar{z}}\right)\left(\frac{\partial}{\partial z} - \nu_2\frac{\partial}{\partial \bar{z}}\right) u = 0, \tag{1}$$

where μ, ν_1, ν_2 are complex constants such that $|\mu| < 1$, $|\nu_1| < 1$, $|\nu_2| < 1$, $\nu_2 = r\nu_1$, $0 < r < 1$. We seek the solution u of the equation (1), which belongs to the class $C^4(D) \bigcap C^{(1,\alpha)}(D \bigcup \Gamma)$ and on the boundary Γ satisfies the Dirichlet boundary conditions

$$\left. \frac{\partial^k u}{\partial r^k} \right|_{\Gamma} = f_k(x,y), \quad k = 0,1, \quad (x,y) \in \Gamma, \tag{2}$$

Here $f_k \in C^{(1-k,\alpha)}(\Gamma)$ $(k = 0,1)$ are prescribed functions on Γ, $\frac{\partial}{\partial r}$ is a derivative with respect to module of the complex number $(z = re^{i\varphi})$.

The problem (1), (2) is Fredholm, (see [1]). In the paper [2] there were found necessary and sufficient conditions for the unique solvability of this problem, and the formulas for the determination of the defect numbers of the problem (i.e. the number of the linearly independent solutions of the homogeneous problem (1), (2), when $f \equiv 0$, and the number of the linearly independent solvability conditions of the inhomogeneous problem), if the unique solvability failes. The similar result for the higher order properly elliptic equation was found in [3]. Further, in [4] it was proved, that in some cases the defect numbers of the problem (1), (2) are equal to one. In this talk we give a generalization of this result. Let $z = \mu\nu_1$. It was proved the following theorem.

Theorem. *The problem (1), (2) is uniquely solvable if and only if*

$$f_k(z) \equiv z^{k-2}(1-r^{k-1}) + z^{k-3}(1-r^{k-2}) + \ldots + (1-r) \neq 0, \quad k = 3, \ldots. \tag{3}$$

If for any coefficients μ, ν_1, ν_2 the conditions (3) do not hold, then the defect numbers of the problem (1), (2) are equal to one.

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On Jackson-Stechkin Type Inequalities for Best Approximation in Hilbert Spaces

V. Babenko (Dnepropetrovsk National University, Ukraine)
babenko.vladislav@gmail.com

We shall present new sharp Jackson-Stechkin type inequalities with generalized modulus of continuity for best approximations of elements of Hilbert space by subspaces, generated by given partition of unity.

Kolmogorov problem on the class of multiply monotone functions

Y. Babenko (Kennesaw State University, USA)
ybabenko@kennesaw.edu

Let G be the real line $\mathbf{R}$ or nonpositive half line $\mathbf{R}_-$. By $L_\infty(G)$ we will denote the space of all measurable essentially bounded functions $x\colon G \to \mathbf{R}$ with usual norm $\|\cdot\| = \|\cdot\|_{L_\infty(G)}$. For $r \in \mathbf{N}$ we will denote by $L^r_\infty(G)$ the space of all functions $x\colon G \to \mathbf{R}$ that have derivatives up to and including the order $r-1$ (in the case $G=\mathbf{R}_-$ we take, as usual, the one-sided derivative at the point $t=0$) such that derivatives $x^{(r-1)}$ are locally absolutely continuous and $x^{(r)}\in L_\infty(G)$. Define $L^r_{\infty,\infty}(G):=L^r_\infty(G)\bigcap L_\infty(G)$.

Kolmogorov formulated the following problem. Let some class $X\subset L^r_{\infty,\infty}(G)$ and the arbitrary system of integers

$$0\le k_0<k_1<...<k_d=r$$

be given. The problem is to find necessary and sufficient conditions for the system of positive numbers

$$M_{k_0},...,M_{k_d}$$

to guarantee the existence of the function $x\in X$, such that

$$\left\|x^{(k_i)}\right\|=M_{k_i},\ i=0,...,d.$$

All the previously known results are for the case of small fixed values of d.

In this talk we shall present a solution of Kolmogorov problem (for any d) on the class of multiply monotone functions $X=L^{r,r}_{\infty,\infty}(\mathbf{R}_-)$, which is the class of functions $x\in L^r_{\infty,\infty}(\mathbf{R}_-)$ that are nonnegative along with all their derivatives up to and including order r (derivative of order r must be nonnegative almost everywhere).

This is joint work with V. Babenko and O. Kovalenko.

Finite Difference Scheme for the Two-Phase Obstacle Problem†

R. Barkhudaryan (Institute of Mathematics of NAS, Armenia)
A. Arakelyan (Royal Institute of Technology, Sweden)
M. Poghosyan (Yerevan State University, Armenia)
rafayel@instmath.sci.am
avetik@kth.se, michael@ysu.am

We consider the two-phase obstacle problem: minimize the functional

$$\mathcal{J}(v) := \int_{\Omega} \left[\frac{1}{2}|\nabla v|^2 + \lambda^+ \max(v,0) + \lambda^- \max(-v,0) \right] dx$$

over the set $\mathbb{K} := \{v \in H^1(\Omega) : \ v - g \in H^1_0(\Omega)\}$.

Under some assumptions on given data we propose an algorithm for numerical solution of the two-phase obstacle problem based on the finite difference method with 5 point stencil and give an error estimate for this approximation.

Lieb-Thirring inequalities and their application to the spectral theory

D. Barseghyan (Nuclear Physics Institute, Czech Republic)
dianabar@bk.ru

In 1976, Lieb and Thirring ([1]) proved the following theorem:

Theorem 1. *For any orthonormal system $\Phi = \{\varphi_j\}_{j=1}^N \subset L^2(R^2)$ the following inequality holds*

$$\int_{R^2} {\rho_\Phi}^2 dxdy \leqslant C \sum_{j=1}^{N} \|\nabla \varphi_j\|^2_{L^2(R^2)},$$

†This project was partially supported by Göran Gustafsson foundation

where

$$\rho_\Phi \equiv \sum_{j=1}^{N} \varphi_j^2(x,y),$$

C is absolute constant and, as usual, $\nabla\varphi \equiv (\frac{\partial\varphi}{\partial x}, \frac{\partial\varphi}{\partial y})$.

Further the series of Lieb-Thirring type inequalities were established by many authors (see, in particular [2], [3]).

In this talk we consider finite orthonormal systems and present the following theorems proved for them:

Theorem 2. *There exists an absolute constant C, such that for any orthonormal system of real-valued functions* $\Phi = \{\varphi_j(x,y)\}_{j=1}^N \subset L^2(R^2)$, $N = 1,2,\ldots,$ *the following inequality takes place:*

$$\int_{R^2} \rho_\Phi^2 \, dx\, dy \le C(\ln N + 1) \sum_{j=1}^{N} \int_{R^2} |xy| |\hat{\varphi}_j|^2(x,y)\, dx\, dy.$$

Using the result of this theorem the following estimate for the spectrum of unbounded self-adjoint operator $L\psi = -\triangle\psi + |xy|\psi$ is proved:

Theorem 3. *The operator* $L\psi = -\triangle\psi + |xy|\psi$ *in* $L^2(R^2)$ *has a discrete spectrum* $\{\lambda_j\}_{j=1}^\infty$ *with* $0 \le \lambda_1 < \lambda_2 < \ldots,$ $\lambda_j \to \infty$ *at* $j \to \infty$ *and there is a positive absolute constant* C' *such that the following estimate is true:*

$$\sum_{j=1}^{N} \lambda_j \ge C' \frac{N^{\frac{3}{2}}}{(1+\ln N)^{\frac{1}{2}}},\ N = 1,2,\ldots$$

Also we have proved:

Theorem 4. *For any natural number p there is a constant* C_p *such that for any orthonormal system of real valued functions* $\Phi = \{\varphi_j\}_{j=1}^N$ *in* $L^2(R^2)$ *the following inequality takes place:*

$$\int_{\mathbb{R}^2} \rho_\Phi^{p+1} \, dx\, dy \le C_p(\ln^p N + 1) \sum_{j=1}^{N} \int_{R^2} |x|^p |y|^p |\hat{\varphi}_j|^2(x,y)\, dx\, dy.$$

Theorem 5. *For any natural number p the operator $L\psi = -\triangle\psi + |xy|^p\psi$ in $L^2(R^2)$ has a discrete spectrum $\{\lambda_j\}_{j=1}^{\infty}$ with $0 \leq \lambda_1 < \lambda_2 < \ldots$, $\lambda_j \to \infty$ at $j \to \infty$ and there is a positive absolute constant C'_p such that the following estimate is true:*

$$\sum_{j=1}^{N} \lambda_j \geq C'_p \frac{N^{\frac{2p+1}{p+1}}}{(1+\ln^p N)^{\frac{1}{p+1}}}, \quad N = 1, 2, \ldots$$

Theorem 6. *For any natural number $d \geq 2$ there is a constant C_d such that for any orthonormal system of real valued functions $\Phi = \{\varphi_j\}_{j=1}^{N}$ in $L^2(R^d)$,*

$$\int_{\mathbb{R}^d} \rho_\Phi^2 \, dx_1 \cdots dx_d \leq$$

$$\leq C_d(\ln^{d-1} N + 1) \sum_{j=1}^{N} \int_{R^d} |x_1 \cdots x_d| |\hat{\varphi}_j|^2 (x_1, \ldots, x_d) \, dx_1 \ldots dx_d.$$

Theorem 7. *The operator $L\psi = -\triangle\psi + |x_1 \ldots x_d|\psi$ in $L^2(R^d)$, $d \geq 2$ has a discrete spectrum $\{\lambda_j\}_{j=1}^{\infty}$ with $0 \leq \lambda_1 < \lambda_2 < \ldots$, $\lambda_j \to \infty$ at $j \to \infty$, and there is a positive absolute constant C'_d such that*

$$\sum_{j=1}^{N} \lambda_j \geq C'_d \frac{N^{\frac{4+d}{2+d}}}{(1+\ln N)^{\frac{2d-2}{2+d}}}, \quad N = 1, 2, \ldots$$

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Existence of the best n-term approximants for structured dictionaries †

P. Bechler (University of Warsaw, Poland)
pbechler@mimuw.edu.pl

This paper investigates the existence of the elements of the best n-term approximation in infinite dimensional Hilbert spaces. The notion of *uniform linear independence (ULI)* for a dictionary is introduced. It is shown that if the dictionary used for approximation satisfies the Bessel inequality and has the ULI property then for every element of the Hilbert space there exists an element of the best n-term approximation. It is also shown, that if a dictionary does not satisfy the ULI property, then there exists an arbitrarily small compact perturbation of this dictionary for which the elements of the best n-term approximation need not exist. The obtained results are applied to frames.

†This research was partially supported by the Polish Ministry of Science and Higher Education grant no. N N201 269335.

Bounded projections on L^p spaces in the unit disk

A. Beknazaryan and A. Petrosyan (Yerevan State University, Armenia)
petrosyan@instmath.sci.am

Some linear operators, which depend on normal pair of weighted functions $\{\varphi, \psi\}$, in the Banach spaces $L^p(D)$, where D is the unit disk in the complex plane, are considered. We investigate, for which p these operators are bounded.

The concept of a normal pair of weight functions was introduced by Shields and Williams [1], and it appeared to be convenient notion for a statement, bound with estimates of integrals and for construction of projectors in weighted spaces.

A function is called normal if there exist $k > \varepsilon > 0$ and $r_0 < 1$ such that

$$\frac{\varphi(r)}{(1-r)^{\varepsilon}} \searrow 0 \quad \textit{and} \quad \frac{\varphi(r)}{(1-r)^{k}} \nearrow \infty \quad (r_0 \leq r, \;\; r \to 1-0). \tag{1}$$

The functions $\{\varphi, \psi\}$ is called a normal pair if φ is normal and if, for some k satisfying (1), there exists $\alpha > k-1$ such that $\varphi(r)\psi(r) = (1-r^2)^{\alpha}, \quad 0 \leq r < 1$.

The following result is obtained:

Theorem 1. *For $p(k-\alpha) < 1$ the integral operator*

$$Tf(z) = \int_D \frac{\psi(z)\varphi(w)}{|1-z\bar{w}|^{2+\alpha}} f(w)\, dA(w)$$

is a bounded projector in $L^p(D, dA)$.

The case $p = 1$ is surveyed in [1]. Note that the case of power weight functions is considered in [2, Chapter 1, §2, Theorem 1.9].

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The Correction Theorem in Sobolev Spaces

E. Berezhnoi (Yaroslavl State University, Russia)
ber@uniyar.ac.ru

Let $S(\mu)$ be the space of Lebesgue measurable functions $x : R_+ \to R$.

Let Q be the unit cube in $\mathbf{R}^n$ with usual Lebesgue measure, $S(\mu)$ - be the space of Lebesgue measurable functions $f : Q \to \mathbf{R}$. For $f \in S(\mu)$, as usually, by $\lambda(f, \alpha)$ we denote the distribution function of f i.e. $\lambda(f, \gamma) = \mu(\{\tau : |f(\tau)| > \gamma\})$.

Let X be a symmetric space on Q and $\psi(X, \tau) = \{\|\chi(U)|X\| : \mu(U) = \tau\}$ be its fundamental function.

For any $f : Q \to R$ we shall use $Df(t) = (\frac{\partial f}{\partial x_1}(t), ..., \frac{\partial f}{\partial x_n}(t))$ for the weak partial derivative at the point $t \in Q$ if it exists.

As usually, by W^1_X we denote Sobolev space on Q, the norm in which is defined by formula

$$\|f|W^1_X\| = \|f|X\| + \|Df|X\|.$$

Theorem 1. *Let X be a symmetric space and let its fundamental function $\psi(\tau)$ be strictly monotone. Let also $f \in W^1_X$ with $\|f|W^1_X\| = 1$.*

Then there exists a sequence of open sets $U_{n+1} \subseteq U_n$, such that $\mu(U_n) \leq \psi^{-1}(\frac{c_0}{n})$ (the constant c_0 depends only on dimension) and for $t \in Q \setminus U_n$ we *have* $|f(t) - f(s)| \leq n|t - s|$.

Theorem 2. *Let* $\psi : [0, 1] \to R_+$ *be concave, strictly monotone function satisfying* $\lim_{\tau \to +0} \psi(\tau) = 0$. *Assume that for a given functions* f *one can find a sequence of open sets* $U(n+1) \supseteq U(n)$ *with* $\psi(\mu(U(n))) \leq \frac{c_0}{n}$ *(constant* c_0 *does not depend on* $n \in N$*) with the following property: for each* $t \in Q \setminus U(n)$ *there exists a number* $r(t) > 0$ *such that if* $s \in Q \setminus U(n)$ *and* $|t - s| \leq r(t)$*, then* $|f(t) - f(s)| \leq n|t - s|$.

Then $\sup_{n \in N} n\psi(\lambda(Df, n)) < \infty$.

From theorems 1 - 2 it's possible to get the characterization of functions $f \in W_X^1$ in terms of correction, when X is a Marzinkiewicz space.

The work is done under partial support of RFFI, project 11-01-00321.

Low-discrepancy sets and harmonic analysis

D. Bilyk (University of South Carolina, USA)
bilyk.dmitriy@gmail.com

Harmonic analysis plays an important role in constructions of low-discrepancy sets and cubature formulas. We shall illustrate the use of analytic methods by means of several examples, such as the van der Corput set, the Fibonacci lattice, and their higher dimensional analogs.

Growth and value distribution of rational approximants

H.-P. Blatt (Katholische Universitaet Eichstaett-Ingolstadt, Germany)
hans.blatt@ku-eichstaett.de

We investigate the growth and the distribution of a-values, $a \in \overline{\mathbb{C}}$, of rational approximants r_n to a function f on a compact set E in $\mathbb{C}$, where $r_n = r_{n,m_n}$ is a rational function with numerator degree $\leq n$ and denominator degree $\leq m_n$, as $n \to \infty$. Three different situations are considered:

(1) f is meromorphic on E and $\{r_n\}_{n\in\mathbb{N}}$ is a sequence of maximally convergent rational functions to f on E.

(2) E is a continuum in a region D, f continuous on E and $\{r_n\}_{n\in\mathbb{N}}$ converges geometrically to f on E.

(3) $f \in C[-1,1]$, but f is not holomorphic on $[-1,1]$ and $\{r_{n,m_n}\}_{n\in\mathbb{N}}$ is a sequence of rational best approximants in the upper half of the Walsh table, i.e.,

$$m_n \leq cn \text{ and } 0 \leq c < 1.$$

In case (1) we obtain a Jentzsch-Szegő type result, i.e., the zero distribution converges weakly to the equilibrium distribution of the maximal Green domain $E_{\rho(f)}$ of meromorphy of f if f has a singularity of multivalued character on the boundary of $E_{\rho(f)}$. In case (2), the geometric convergence on E implies the m_1-almost uniform convergence to a meromorphic function on D if the number of poles of r_n in D is bounded and if the number of zeros of r_n in any compact set $K \subset D$ is of type $o(n)$ as $n \to \infty$. In case (3), it is shown that any point z_0 of $[-1,1]$ is either a limit point of poles of r_n or z_0 is a limit point of a-values of r_n for any $a \in \mathbb{C}$.

The Grothendick Inequality Revisited

R. Blei (University of Connecticut, USA)
blei@math.uconn.edu

The classical Grothendieck inequality can be equivalently rephrased as the assertion: if η is a bounded bilinear functional on a Hilbert space H, then there exist bounded mappings ϕ_1 and ϕ_2 from the closed unit ball B_H into $L^\infty(\Omega, \mu)$, for some probability space (Ω, μ), such that

$$\eta(\mathbf{x}, \mathbf{y}) = \int_\Omega \phi_1(\mathbf{x})\phi_2(\mathbf{y})d\mu, \quad (\mathbf{x}, \mathbf{y}) \in B_H \times B_H.$$

But more can be said: ϕ_1 and ϕ_2 can be engineered to be also continuous with respect to the norm topologies on H and $L^2(\Omega, \mu)$.

Definition 1. *Let η be a bounded n-linear functional on a Hilbert space H.*
i. *η is said to be projectively bounded if there exist a probability space (Ω, μ) and bounded mappings $\phi_1, \ldots, \phi_n$ from B_H into $L^\infty(\Omega, \mu)$, such that*

$$\eta(\mathbf{x}_1, \ldots, \mathbf{x}_n) = \int_\Omega \phi_1(\mathbf{x}_1) \cdots \phi_n(\mathbf{x}_n)d\mu, \quad (\mathbf{x}_1, \ldots, \mathbf{x}_n) \in (B_H)^n. \qquad (1)$$

ii. *η is said to be projectively continuous if there exist a probability space (Ω, μ) and bounded mappings $\phi_1, \ldots, \phi_n$ from B_H into $L^\infty(\Omega, \mu)$ that are continuous with respect to the norm topologies on H and $L^2(\Omega, \mu)$, such that* (1) *holds.*

In the case $n = 1$, every bounded linear functional on a Hilbert space is projectively continuous. (Exercise.)

In the case $n = 2$, every bounded bilinear functional on a Hilbert space is projectively continuous. (Not quite an exercise; this is the upgraded Grothendieck inequality.)

In the case $n > 2$, there exist bounded trilinear functionals on a Hilbert space that are not projectively bounded, and *a fortiori* not projectively continuous, and thus

Question *For $n > 2$, which are the projectively bounded, and which are the projectively continuous n-linear functionals on an infinite-dimensional Hilbert space?*

In this talk we characterize projective continuity within a certain class of multilinear functionals that are naturally defined via fractional Cartesian products in a harmonic analysis framework.

On closed sets of approximation on Riemann surfaces

A. Boivin† (University of Western Ontario, Canada)

and

N. Askaripour (University of Toledo, USA)

boivin@uwo.ca, Nadya.Askaripour@utoledo.edu

A closed subset E of a (non-compact) Riemann surface R is called a set of holomorphic (resp. meromorphic) approximation if every function holomorphic on E can be approximated *uniformly on E* by functions holomorphic (resp. meromorphic) on R. The characterization of the sets of approximation (either holomorphic or meromorphic) is still open in general, though it is known in some cases, for example when E is compact (and R is arbitrary), or when R is of finite genus (and E is arbitrary), including results by N.U. Arakelian, A.H. Nersessian and A.G. Vitushkin. We will discuss some of these results and results obtained more recently with Nadya Askaripour.

†Research partially supported by NSERC grant.

Multidimensional Appell sequences via hypercomplex function theory

I. Cação† (University of Aveiro, Portugal)
isabel.cacao@ua.pt

As part of Clifford analysis, hypercomplex function theory provides a generalization to higher dimensions of the theory of holomorphic functions of one complex variable by using Clifford algebras. In this framework the analogue of holomorphic functions is obtained as the set of null-solutions to a generalized Cauchy-Riemann system and they are usually called monogenic. An essential difference between this approach and the classical generalization in terms of several complex variables relies on the underlying algebra, that remains commutative in the case of the function theory of several complex variables and is no longer commutative in the case of Clifford analysis. However, the lack of commutativity and the fact that the product of two monogenic functions is not in general monogenic is not a limitation to the construction of monogenic polynomials and, consequently, to the representation of monogenic functions as series of properly chosen polynomials. The aim of this talk is to present two different ways of constructing orthogonal bases of homogeneous monogenic polynomials that are multiples of their (hypercomplex) derivatives, i.e., that are Appell sequences. The first approach takes advantage of the Clifford algebra structure that allows the factorization of the Laplace operator into a product of two first-order differential operators of Cauchy-Riemann type and has its origin in classical (real) harmonic analysis. The second approach deals with an isomorphism between a monogenic hypercomplex variable and the holomorphic complex variable $z = x + i\,y$.

†Financial support from CIDMA-Centro de Investigacão e Desenvolvimento em Matemática e Aplicações of the University of Aveiro, through the Portuguese Foundation for Science and Technology (FCT), gratefully acknowledged.

Frequently hypercyclic entire functions†

D. Drasin (Purdue University, USA)
drasin@math.purdue.edu

We finish a question initiated by Blasco, Bonilla and Grosse-Erdmann. An entire function f is hypercyclic if given any polynomial (or entire function) g, there is a sequence $\{f^{(n_j)}\}$ or derivatives which tends to g in L^p-norm on compact sets of the plane. Here $1 \leq p \leq \infty$. This may be refined to speak of a function being 'frequently hypercyclic' if given $\varepsilon > 0, R < \infty$ one can take the $\{n_j\}$ of positive density (depending on g, ε and R) so that

$$|f^{(n_j)}(z) - g(z)| < \varepsilon \qquad (|z| < R).$$

The problem is to find the minimum rate of growth that such a function f, may have, and we present sharp bounds for all p. Several special cases were settled in 2010 by the above-mentioned authors.

The construction is direct with no functional analysis, but uses remarkable polynomials of Rudin-Shapiro and de la Valee Poussin.

Completely bounded L^p multipliers

S. Dutta (Indian Institute of Technology Kanpur, India)
sudipta@iitk.ac.in

Pisier considered the operator space structure on $L^p(G)$, for locally compact group G, by developing operator space complex interpolation. In this operator space structure one can define completely bounded $L^p(G)$ multipliers. It is natural to ask whether the space of completely bounded multipliers on $L^p(G)$ is strictly contained inside the space of multipliers on $L^p(G)$. For $p = 1$ and 2 it can be shown that these two spaces coincide.

†Joint with E. Saksman, Helsinki.

Pisier had shown that for compact abelian group the inclusion is strict for $1 < p < 2$. In this talk we will see that similar thing happens for any arbitrary locally compact abelian group. In the process we will see the *cb* version of Herz's famous multiplier homomorphism theorem.

On the series with monotonic coefficients by generalized Walsh system

S. Episkoposian (Yerevan State University, Armenia)
sergoep@ysu.am

Let a be a fixed integer, $a \geq 2$ and put $\omega_a = e^{\frac{2\pi i}{a}}$.

We give definitions of generalized Rademacher and Walsh systems.

Definition 1. *The Rademacher system of order a is defined by*

$$\varphi_0(x) = \omega_a^k \ \ if \ \ x \in \left[\frac{k}{a}, \frac{k+1}{a}\right), \ \ k = 0, 1, ..., a-1,$$

and for $n \geq 0$

$$\varphi_n(x+1) = \varphi_n(x) = \varphi_0(a^n x).$$

Definition 2. *The generalized Walsh system of order a is defined by*

$$\psi_0(x) = 1,$$

and if $n = \alpha_{n_1} a^{n_1} + ... + \alpha_{n_s} a^{n_s}$ *with* $n_1 > ... > n_s$, *then*

$$\psi_n(x) = \varphi_{n_1}^{\alpha_{n_1}}(x) \cdot ... \cdot \varphi_{n_s}^{\alpha_{n_s}}(x).$$

The basic properties of the generalized Walsh system of order a are obtained by H.E.Chrestenson, J. Fine, C. Vatari, N. Vilenkin and others. Denote the generalized Walsh system of order a by Ψ_a. Observe that Ψ_2 is the classical Walsh system.

In [1] it is proved the following

Theorem 1. *There exists a function $f(x) \in L^1[0,1)$ the Fourier series of which by Ψ_a does not converge to $f(x)$ in $L^1[0,1)$ norm.*

The following statements are true:

Theorem 2. *For any $\varepsilon > 0$ there exists a weight function $\mu(x)$ satisfying $0 < \mu(x) \leq 1, mes\{x \in [0,1] : \mu(x) \neq 1\} < \epsilon$ such that for every function $f(x) \in L^1_\mu[0,1]$ there exits a series by Ψ_a converging to $f(x)$ in the metric of $L^1_\mu[0,1]$ and such that nonzero coefficients of that series are decreasing in absolute value.*

Theorem 3. *For any $0 < \epsilon < 1$, $p > 2$ and every $f \in L^p[0,1]$ one can find a function $g \in L^p[0,1]$, $mes\{x \in [0,1] \; ; \; g \neq f\} < \epsilon$, such that the sequence $\{|c_k(g)|, \; k \in spec(g)\}$, is monotonically decreasing.*

The spectrum of f (denoted by $spec(f)$) is the support of $\{c_k(f)\}$, i.e. the set of integers k for which $c_k(f)$ is nonzero.

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On the n-widths for classes of analytic functions

Yu. Farkov (Russian State Geological Prospecting University, Russia)
farkov@list.ru

Let $BH^\infty(\Omega)$ be the class of functions f which are holomorphic in the domain $\Omega \subset \mathbb{C}^d$ and satisfy $|f| \leq 1$ therein. Let K be a compact subset of Ω. The history of the widths for the class $BH^\infty(\Omega)$ in $C(K)$ in the case $d = 1$ can be found in [1] (see also [2, p.276]).

Problem 1 [3, p.196]. Find conditions on K and Ω, such that

$$d_n(BH^\infty(\Omega), L^q(K)) \asymp \tilde{n}^{-1/q} \exp(-\tilde{n}\alpha),$$

with $1 \le q \le \infty$, $\alpha = 2\pi/(C(K,\Omega))^{1/d}$.

Further, let $U = \{z \in \mathbb{C} : |z| < 1\}$ be the open unit disk, $T = \partial U$, $U_R = RU$ $(R > 0)$, and let $d\sigma(e^{i\theta}) = d\theta/2\pi$ be the normalized Lebesgue measure on T, and let $d\nu(x+iy) = dxdy/\pi$ be the normalized area Lebesgue measure. Below s_n stands for any of the n-widths d_n, d^n, λ_n (see [1] or [2] for the definitions). The closed unit ball in the Hardy space $H^p(U_R)$ is denoted by $BH^p(U_R)$. For $1 \le q \le p \le \infty$, $R \ge 1$ we have

$$s_n(BH^p(U_R), L^q(\sigma)) = R^{-n} \quad \text{and} \quad s_n(BH^p(U_R), L^q(\nu)) = R^{-n}\left(\frac{qn}{2}+1\right)^{-\frac{1}{q}}$$

(see [4]). For any positive integer l, let $H_R(l,p)$ denote the class of those functions which have the l-th derivative belonging to $BH^p(U_R)$. Now, we state the following problems:

Problem 2. Prove that if $1 \le q \le p \le \infty$, $n \ge l$, $R \ge 1$, then

$$s_n(H_R(l,p), L^q(\sigma)) = \frac{(n-l)!}{n!} R^{l-n}$$

and

$$s_n(H_R(l,p), L^q(\nu)) = \frac{(n-l)!}{n!}\left(\frac{qn}{2}+1\right)^{-1/q} R^{l-n}.$$

In the case $p = q$ these equalities are well-known [2, Ch. 8]; see also [1,3,5].

Problem 3. Find the values of Bernstein n-widths $b_n(H_R(l,p), L^q(\sigma))$ for $1 \le p \le q \le \infty$.

Remark. It is known that if $R > 1$, then, for all $1 \le p, q \le \infty$,

$$s_n(H_R(l,p), L^q(\sigma)) \asymp n^{-l}R^{-n}, \quad s_n(H_R(l,p), L^q(\nu)) \asymp n^{-l-1/q}R^{-n},$$

$$s_n(A_R(l,p), L^q(\sigma)) \asymp n^{-l+\frac{1}{p}}R^{-n}, \quad s_n(A_R(l,p), L^q(\nu)) \asymp n^{-l+\frac{1}{p}-\frac{1}{q}}R^{-n},$$

where $A_R(l, p)$ are the Bergman-Sobolev classes (see [3] and the references therein).

PROBLEM 4. Find the values of n-widths $s_n(A_R(l, p), L^p(\sigma))$ for $1 \leq p < \infty$.

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On the generalized Harmonic-Cesaro summability of Fourier series

L. GALOYAN (Yerevan State University, Armenia)
lev_nik_galoyan@mail.ru

Let α and β be real numbers and let $A_n^{\alpha,\beta}$ be the sequence of constants defined by the series

$$(1-z)^{-1-\alpha} \cdot \left(\ln \frac{e}{1-z}\right)^{\beta} = \sum_{n=0}^{\infty} A_n^{\alpha,\beta} z^n \quad , \quad |z| < 1.$$

Let Σa_n be an infinite series and let S_n stands for the sequence of its partial sums. Then the sequence

$$t_n^{\alpha,\beta} = \frac{1}{A_n^{\alpha,\beta}} \sum_{k=0}^{n} A_{n-k}^{\alpha-1,\beta} S_k = \frac{1}{A_n^{\alpha,\beta}} \sum_{k=0}^{n} A_{n-k}^{\alpha,\beta} a_k$$

is called the (C,α,β) mean of the series Σa_n. The series Σa_n is said to be summable to s by a generalized Harmonic-Cesaro method ((C,α,β) summable to s) if $t_n^{\alpha,\beta} \to s$ as $n \to \infty$. This talk is devoted to the research on (C,α,β) summability of Fourier series by the trigonometric system. Let $t_n^{\alpha,\beta}(x,f)$ be (C,α,β) means of Fourier series of function $f \in L(-\pi,\pi)$ and let $\varphi_x(t) = f(x+t) + f(x-t) - 2S(x)$. We have proved the following theorems:

Theorem 1. *Let $f \in L(-\pi,\pi)$ be a periodic function with period 2π. If at the point x the following conditions hold:*

$$\int_0^h \varphi_x(t)dt = o(h),$$

$$\int_h^{\pi} |\varphi_x(t+h) - \varphi_x(t)| \cdot \frac{(\ln \frac{2\pi}{t})^{\beta}}{t^{1+\alpha}} dt = o\left(\frac{\left(\ln \frac{1}{h}\right)^{\beta}}{h^{\alpha}}\right) \qquad h \to +0$$

for some α, β with $\alpha \in (-1,0) \cup (0,1), \beta \in R$, or $\alpha = 0, \beta \geq 0$, then $t_n^{\alpha,\beta}(x,f) \to S$ as $n \to \infty$.

Note that for $\beta = 0$ this theorem was proved by L.Zhizhiashvili [1].

Theorem 2. *Let $f \in L(-\pi,\pi)$ be a periodic function with period 2π. If at the point x the following conditions hold for some $\beta > 1$:*

$$\int_0^h \varphi_x(t)dt = o\left(\frac{h}{\ln \frac{1}{h}}\right),$$

$$\int_h^{\pi} |\varphi_x(t+h) - \varphi_x(t)| \cdot \left(\ln \frac{2\pi}{t}\right)^{\beta} dt = o\left(h\left(\ln \frac{1}{h}\right)^{\beta-1}\right),$$

then $t_n^{-1,\beta}(x,f) \to S$ *as* $n \to \infty$.

Analogous results are valid for conjugate trigonometric Fourier series.

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Instability of the Riemann hypothesis

P. M. Gauthier (Université de Montréal, Canada)
gauthier@dms.umontreal.ca

The classical Riemann Hypothesis (RH) asserts that the non-trivial zeros of the Riemann zeta-function lie on the critical line. Variants of the RH have been formulated for other zeta-functions satisfying analogous functional equations, but the only case that has been resolved is the RH for zeta-functions over finite fields. Although the latter are defined algebraically, they turn out to be very explicit meromorphic functions. With approximation theory *à la Arakelian*, we show that the RH is unstable in the sense that if zeta-functions are slightly perturbed (while retaining the functional equation), the zeros of the perturbed functions may or may not all lie on the critical line. Both possibilities occur independently of the RH. This is joint work respectively with Zeron (Riemann zeta-function), Xarles (L-functions) and Tarkhanov (zeta-functions of curves over finite fields).

Euler-Frobenius numbers

W. Gawronski (University of Trier, Germany)
gawron@uni-trier.de

These numbers are defined as the coefficients of the Euler-Frobenius polynomials

$$P_{n,\lambda}(z) = \sum_{k=0}^{n} A_{n,k}(\lambda)\, z^k,$$

which usually are introduced via the rational function expansion

$$\sum_{\nu=0}^{\infty} (\nu+\lambda)^n\, z^\nu = \frac{P_{n,\lambda}(z)}{(1-z)^{n+1}},$$

n being a nonnegative integer and $\lambda \in [0,1)$. The special case $A_{n,k}(0)$ is known from combinatorics (Eulerian numbers) and the general one $A_{n,k}(\lambda)$ occurs e.g. in approximation theory, summability and rounding error analysis. Supplementing and extending known results on Eulerian numbers, various theorems for the Euler-Frobenius numbers $A_{n,k}(\lambda)$ and related quantities are established including asymptotic expansions, unimodality and monotonicity properties.

On finite dimensional approximations of infinite dimensional operators

L. Gevorgyan (State Engineering University of Armenia, Armenia)
levgev@hotmail.com

Let A be a linear operator, acting in a Hilbert space H. One of the oldest methods for solving infinite dimensional operator equations is the reduction of original equation to the finite dimensional case. Well known Ritz, Galerkin methods are based on this idea. Fix a set $\{f_k\}_1^n$ of linearly independent elements from H and denote by H_n their linear span. Let P_n be orthogonal projection of H onto H_n and denote by A_n be the restriction of the operator P_nA on H_n, i.e.

$$P_nAP_n = \begin{pmatrix} A_n & 0 \\ 0 & 0 \end{pmatrix}.$$

The main problem in this situation is the information which may be extracted from the knowledge available for the finite-dimensional approximation of the operator (e.g., how the spectra SpA_n of A_n may be used to define the spectrum of A or how a solution of the equation $Ax = b$ may be constructed from known solutions of $A_nx = P_nb$).

An example, connected with the Cantor ternary distribution shows that although the Hausdorff distance between SpA_n and SpA does not tend to zero, but the use of Christoffel numbers permits to rule out "insignificant" parts of SpA_n.

Paley function and A-integrity of series by general Franklin systems

G. Gevorkyan and A. Martirosyan
(Yerevan State University, Armenia)
ggg@ysu.am, anush.martirosyan@gmail.com

We consider the series

$$\sum_{n=0}^{\infty} a_n f_n(t) \tag{1}$$

where $\{f_n(t)\}_{n=0}^{\infty}$ is a general Franklin system corresponding to a strong regular quasi-dyadic partition of $[0, 1]$. Let

$$P(t) = \left(\sum_{n=0}^{\infty} a_n^2 f_n^2(t) \right)^{\frac{1}{2}}$$

be the Paley function of (1).

We have proved the following theorems.

Theorem 1. *There exists a constant* $C > 0$*, such that if* $g \in L_1(0,1)$ *and* $a_n = \int_0^1 g(t) f_n(t) dt$*, then*

$$\mu\{t : P(t) > y\} \leq \frac{C}{y}\|g\|_1.$$

Theorem 2. *If* $\lim_{y\to\infty} y \cdot \mu\{t : P(t) > y\} = 0$*, then for any bounded sequence* $\{\varepsilon_n\}_{n=0}^{\infty}$ *the series*

$$\sum_{n=0}^{\infty} \varepsilon_n a_n f_n(t) \tag{2}$$

is a Fourier-Franklin series of some A-integrable function in the sense of A-integrability.

Theorem 3. *Assume that the series* (1) *is a Fourier-Franklin series of some integrable function. Then for any bounded sequence* $\{\varepsilon_n\}_{n=0}^{\infty}$ *the series* (2) *is a Fourier-Franklin series of some A-integrable function in the sense of A-integrability.*

Theorem 4. *Any subseries of a Fourier-Franklin series of integrable function is a Fourier-Franklin series of some A-integrable function in the sense of A-integrability.*

Estimation of Spectral Parameters in Gaussian Stationary Models

M. Ginovyan (Boston University, USA)
ginovyan@math.bu.edu

In this talk we discuss a problem of statistical estimation of spectral parameters for continuous- or discrete-time, misspecified and correctly specified Gaussian stationary models.

Suppose we observe a finite realization $\mathbf{X}_T = \{X(t), 0 \leq t \leq T\}$ of a zero mean real-valued continuous- or discrete-time Gaussian stationary process $\{X(t), t \in \mathbb{U}\}$ with true spectral density function $g(\lambda)$, $\lambda \in \mathbb{Q}$. The time domain $\mathbb{U}$ is the real line $\mathbb{R} := (-\infty, \infty)$ in the c.t. case, and the set of integers $\mathbb{Z} = \{0, \pm 1, \pm 2, \ldots\}$ in the d.t. case. The frequency domain $\mathbb{Q}$ is $\mathbb{R}$ in the c.t. case, and $\mathbb{Q} = \mathbb{T} := [-\pi.\pi]$ in the d.t. case. In the c.t. case the process $X(u)$ is assumed measurable and mean-square continuous: $\mathbb{E}[X(t) - X(s)]^2 \to 0$ as $t \to s$.

We fit a certain parametric spectral model

$$\{f_\theta = f(\lambda, \theta), \theta \in \mathbf{\Theta} \subset \mathbb{R}^p\}$$

for this process, and consider the problem of estimation of unknown parameter θ.

We propose several minimum distance estimators $\widehat{\theta}_T = \widehat{\theta}(\mathbf{X}_T)$ defined by

$$\widehat{\theta}_T := \mathrm{argmin}_\theta D(f_\theta, \widehat{g}_T),$$

where $D(f_\theta, g)$ is an appropriate "distance" functional that measures the nearness of f_θ to g, while $\widehat{g}_T$ is either the periodogram of the process $X(t)$ or some non-parametric kernel estimator of the true unknown spectral density $g(\lambda)$, and describe the asymptotic distributions (as $T \to \infty$) of the suggested estimators both for misspecified and correctly specified models. In particular, we provide sufficient conditions for minimum distance statistics $\widehat{\theta}_T$ to be consistent and asymptotically normal estimators for unknown spectral parameter θ.

Almost Everywhere Summability of Walsh-Fourier series

U. Goginava (Tbilisi State University, Georgia)
zazagoginava@gmail.com

In my talk I will discuss a. e. convergence and convergence in measure of Fejér means, Reisz Logarithmic means, Norlund Logarithmic means of cubic and triangular partial sums of Walsh-Fourier series.

On the approximation of functions of several variables by the Fourier series

L. Gogoladze (Tbilisi State University, Georgia)
lgogoladze1@hotmail.com

The method of obtaining estimates of approximation of functions of several variables by linear means of their Fourier series is suggested, using the corresponding estimates for functions of a single variable.

On the coefficients of expansion of continuous function by the Faber–Schauder system

M. Grigoryan (Yerevan State University, Armenia)
gmarting@ysu.am

The Faber-Schauder system $\Phi = \{\varphi_n(x)\}_{n=0}^{\infty}$, $x \in [0,1]$ is a basis in $C[0,1]$ (the space of continuous on $[0,1]$ functions), i.e. for every $f(x) \in$

$C[0,1]$ there exists a unique series of the form

$$\sum_{n=0}^{\infty} A_n(f)\varphi_n(x)$$

converging to f in $C[0,1]$.

Definition. *Let $t \in (0,1]$. We say that coefficients $A_n(f)$ of expansion of $f \in C[0,1]$ by the Faber-Schauder system are **t-monotone** if*

$$|A_{n_k}(f)| \geq t \cdot \max\{|A_{n_j}(f)|, \forall j > k\}, \quad k = 1,2,...$$

where $\{n_k\}_{k=1}^{\infty}$ is the spectrum of $f(x)$, i.e. the set of integers where $A_n(f)$ is non zero.

In [1] the following Theorem is proved:

Theorem 1. *For every $\epsilon \in (0,1)$ there exists a measurable set $E \subset [0,1]$ with measure $|E| > 1-\epsilon$, such that for any $f \in C[0,1]$ there exists a function $g(x) \in C[0,1]$ coinciding with $f(x)$ on E, such that the coefficients of expansion of g by the Faber-Schauder system are t-monotone for all $t \in (0, \frac{1}{2}]$.*

Here we prove that *Theorem 1 is true for $t \in (\frac{1}{2}, 1]$ also.*

Theorem 2. For every $\epsilon \in (0,1)$ there exists a measurable set $E \subset [0,1]$ with measure $|E| > 1-\epsilon$, such that for any $f \in C[0,1]$ there exists a function $g(x) \in C[0,1]$ coinciding with $f(x)$ on E, such that the coefficients of expansion of g by the Faber-Schauder system are 1-monotone.

The following questions are open:

Question 1. *Does Theorem 2 or 1 hold for other bases in the space $C[0,1]$ (in particular, for the Franklin system)?*

Question 2. *Does Theorem 1 remain true for the trigonometric system for any $t \in (0,1]$?*

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Pseudohyperbolic equations of second order with degeneration

G. Hakobyan (Yerevan State University, Armenia)
S. Ghorbanian (Islamic Azad University Firoozkooh, Iran)
gurgenh@ysu.am, siavash_ghorbanian@yahoo.com

Let $\Omega \subset \mathbb{R}^n$ be a bounded domain with a smooth boundary in $x_n > 0$. We suppose that $\partial\Omega = \Gamma_1 \cup \Gamma_0$, where $\Gamma_0 = \partial\Omega \cap \{x_n = 0\}$ is a domain in the hyperplane $\{x_n = 0\}$ and $\Gamma_1 = \partial\Omega \backslash \Gamma_0$. In the infinite cylinder $Q = \mathbb{R}^+ \times \Omega$ we consider the following initial-boundary value problem for the degenerate pseudohyperbolic equation

$$\begin{cases} L\left(\frac{\partial^2 u}{\partial t^2}\right) + M(u) = 0, \\ u|_{t=0} = u_0(x),\ u_t|_{t=0} = u_1(x), \\ u|_{\Gamma^*} = 0, \end{cases} \tag{1}$$

where

$$L(u) = -\sum_{i,j=1}^{n-1} \frac{\partial}{\partial x_i}\left(b_{ij}(x,t)\frac{\partial u}{\partial x_j}\right) - \frac{\partial}{\partial x_n}\left(b_{nn}(x,t)\frac{\partial u}{\partial x_n}\right),$$

$$M(u) = -\sum_{i,j=1}^{n-1} \frac{\partial}{\partial x_i}\left(a_{ij}(x,t)\frac{\partial u}{\partial x_j}\right) - \frac{\partial}{\partial x_n}\left(a_{nn}(x,t)\frac{\partial u}{\partial x_n}\right),$$

$b_{ij}(x) = b_{ji}(x)$, $a_{ij}(x) = a_{ji}(x)$ $(i,j = 1,2,\dots,n)$, Γ^* is a part of the boundary $\partial\Omega$, which depends on the order of degeneration of the operator L and represents the whole boundary or coincides with its part Γ_1. We assume that the functions $a_{ij}(x,t)$ and $b_{ij}(x,t)$ are smooth and there exist numbers $\alpha \geq \beta \geq 0$ such that $x_n^{-\beta} b_{nn}(x,t)$ and $x_n^{-\alpha}|a_{nn}(x,t)|$ are bounded from above and below by positive numbers and for every point $x \in \bar{\Omega}$ and $t \geq 0$ the quadratic form $L_0(x,t;\hat{\xi}) = \sum\limits_{i,j=1}^{n-1} b_{ij}(x,t)\xi_i\xi_j$ is positive definite. Denote by H_{L_β} the Hilbert space which is the completion of $C_0^\infty(\Omega)$ in the metric generated by the following scalar product:

$$(u,v)_{L_\beta} = \int\limits_{\Omega} \left[\sum_{i=1}^{n-1} \frac{\partial u}{\partial x_i} \frac{\partial v}{\partial x_i} + x_n^\beta \frac{\partial u}{\partial x_n} \frac{\partial v}{\partial x_n} \right] dx.$$

Theorem. *For every initial functions* $u_0(x)$, $u_1(x) \in H_{L_\beta}$ *the generalized solution of the boundary value problem (1) exists and is unique, and* $\Gamma^* = \partial\Omega$ *for* $\beta < 1$, *while* $\Gamma^* = \partial\Omega \backslash \Gamma_0$ *for* $\beta \geq 1$.

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Modulus estimates in slit domains and applications

Hrant Hakobyan (Kansas State University, USA)
hakobyan@math.ksu.edu

Motivated by the work of Merenkov and MacKay-Tyson-Wildrick we obtain upper bounds for moduli of curve families in "slit domains" in $\mathbb{R}^n$. Some applications of these bounds are new examples of spaces which are Quasisymmetrically Co-Hopfian (e.g. homeomorphic to the Menger curve or a non porous Sierpinski carpet) as well as new examples of non-self similar Ahlfors 2-regular spaces where Poincare Inequality fails.

On classification of GC_2 sets in R^3

Hakop Hakopian (Yerevan State University, Armenia)
hakop@ysu.am

We characterize the GC_2 sets having 3 maximal planes in R^3 which in view of results of C. de Boor, A. Apozyan, G. Avagyan, and G. Ktryan (see [1-4]) completes the classification of all GC_2 sets in R^3.

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Holomorphic Besov spaces on the polydisk

A. Harutyunyan (Yerevan State University, Armenia)
anahit@ysu.am

This work is an introduction of ω- weighted Besov spaces of holomorphic functions on the polydisk. Let U^n be the unit polydisk in C^n and S be the space of functions of regular variation. Let $0 \leq p < \infty$, $\omega = (\omega_1, \ldots, \omega_n)$, $\omega_j \in S(1 \leq j \leq n)$ and $f \in H(U^n)$. The function f is said to be an element of the holomorphic Besov space $B_p(\omega)$ if

$$\|f\|^p_{B_p(\omega)} = \int_{U^n} |Df(z)|^p \prod_{j=1}^{n} \frac{\omega_j(1-|z_j|)}{(1-|z_j|^2)^{2-p}} dm_{2n}(z) < +\infty$$

where $dm_{2n}(z)$ is the $2n$-dimensional Lebesgue measure on U^n and D stands for a special fractional derivative of f. For example, if $n = 1$ then Df is the derivative of the function $zf(z)$.

We describe the holomorphic Besov space in terms of $L_p(\omega)$ space. Moreover projection theorems and theorems about bounded and right inverse operators on $B_p(\omega)$ ($p \geq 1$) are proved.

On the solution of an inverse Sturm-Liouville problem

T. Harutyunyan and A. Pahlevanyan
(Yerevan State University, Armenia)
hartigr@yahoo.co.uk, avetikpahlevanyan@gmail.com

Let $L(q, \alpha, \beta)$ denote the Sturm-Liouville problem

$$ly \equiv -y'' + q(x)y = \mu y,\ 0 < x < \pi,\ \mu \in C, \tag{1}$$

$$y(0)\cos\alpha + y'(0)\sin\alpha = 0,\ \alpha \in (0, \pi], \tag{2}$$

$$y(\pi)\cos\beta + y'(\pi)\sin\beta = 0,\ \beta \in [0, \pi), \tag{3}$$

where q is a real valued, summable on $[0, \pi]$ function (we write $q \in L^1_R[0, \pi]$).

In the famous work of Gelfand and Levitan [1] the necessary and sufficient conditions were found for sequences $\{\mu_n\}_{n=0}^{\infty}$ and $\{a_n\}_{n=0}^{\infty}$ to be the spectra and norming constants for the problem $L(q,\alpha,\beta)$ under conditions $\sin\alpha \neq 0$, $\sin\beta \neq 0$ (and some smoothness condition on q). We solve analogous problem without these restrictions.

Theorem. *For two sequences $\{\mu_n\}_{n=0}^{\infty}$ and $\{a_n\}_{n=0}^{\infty}$ to be the spectra and norming constants for $L(q,\pi,\beta)$ with $q \in L_R^1[0,\pi]$, it is necessary and sufficient that the relations*

$$\sqrt{\mu_n(q,\pi,\beta)} = \lambda_n(q,\pi,\beta) = n + \delta_n(\pi,\beta) + \frac{c}{2(n+\delta_n(\pi,\beta))} + l_n, \quad (4)$$

$$a_n(q,\pi,\beta) = \frac{\pi}{2(n+\delta_n(\pi,\beta))^2} + O\left(\frac{1}{n^4}\right) \quad (5)$$

hold, where

$$\delta_n(\alpha,\beta) = \frac{1}{\pi}\arccos\frac{\cos\alpha}{\sqrt{(n+\delta_n(\alpha,\beta))^2\sin^2\alpha + \cos^2\alpha}} -$$
$$\frac{1}{\pi}\arccos\frac{\cos\beta}{\sqrt{(n+\delta_n(\alpha,\beta))^2\sin^2\beta + \cos^2\beta}},$$

c is a constant and the reminders $l_n = l_n(q,\alpha,\beta)$ are such that the function

$$g(x) = \sum_{n=1}^{\infty} l_n \sin(n+\delta_n(\pi,\beta))\,x$$

is absolutely continuous on arbitrary segment $[a,b] \in (0,2\pi)$, uniformly in $\beta \in [0,\pi]$ and q from the bounded subsets of $L_R^1[0,\pi]$.

The similar results are obtained for the cases $\alpha \in (0,\pi)$, $\beta = 0$.

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Boundary Value Problems for Polyanalitic Functions over the Half-Space in Weighted Spaces

H. Hayrapetyan and A. Hayrapetyan
(State Engineering University of Armenia, Armenia)
hhairapet@seua.am

Let B_0 be the class of functions U, defined on $\Pi^+ = \{z; Imz > 0\}$ and satisfying $|U(z)| < A|z|^N, \quad Imz > y_0 > 0$, where A is a constants depending on y_0, N is an integer, depending only on U, and $L^1(\rho)$ is the space of real-valued functions over $(-\infty, \infty)$ with the norm

$$\|f\|_{L^1(\rho)} = \int_{-\infty}^{\infty} |f(x)||\rho(x)|dx < \infty,$$

where $\rho(x) = (1 + |x|)^\alpha$ and α is some real number. We consider the following Schwarz's type boundary value problem for the equation

$$\frac{\partial^n U(z)}{\partial \bar{z}^n} = 0, \quad z \in \Pi^+. \tag{1}$$

Problem P**.** Let $f_k, \ \ k=0,1,...,n-1$ be real-valued functions on $(-\infty, \infty)$, such that $f_k^{(n-k+1)} \in L^1(\rho)$. Find a function $U \in B_0$ satisfying (1) with boundary conditions

$$\lim_{y \to 0} \left\| \Re \frac{\partial^k U(z)}{\partial y^k} - f_k(x) \right\|_{L^1(\rho)} = 0, \quad k = 0, 1, ..., n-1.$$

In [1], [2] the Schwarz and Dirichlet problems for nonhomogeneous equation (1) were considered and necessary and sufficient conditions for solvability of this problems were obtained in the case when the boundary data is continuous.

In the present talk we consider the Schwarz's problem in the weighted spaces. We prove that Problem P has a solution for any f_k, when $\alpha < 1$. In the case of $\alpha \geq 1$ we obtain necessary and sufficient conditions for solvability of this problem.

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On one class of trigonometric series

R. Hovsepyan (Institute of Mathematics of NAS, Armenia)

We consider series

$$S \equiv a_0 + \sum_{n=1}^{\infty} a_n \cdot \cos nx + b_n \cdot \sin nx,$$

satisfying

$$\sum_{k=1}^{n} k\rho_k = o(n), \qquad \rho_k \equiv \sqrt{a_k^2 + b_k^2}.$$

We denote by $\mathcal{A}$ the class of all such series. Note that the class $\mathcal{A}$ is reach enough: there is a theorem about a.e. representation of measurable functions ([1]).

Zigmund proved ([2, p.76, 504-506]) that every series $S \in \mathcal{A}$ surely converges on some set, that has continuum cardinality in every interval δ.

On the other hand, there exists series (for general series, this is the result of Tandori, and for Fourier series - of S. Galstyan, see [3]) satisfying

$$\overline{\lim} S_n(x) = +\infty, \qquad \underline{\lim} S_n(x) = -\infty.$$

Here we give some propositions that complete the situation.

Theorem 1. *Let for a series $S \in \mathcal{A}$ the following condition hold: for every x, except a countable set, there exists a subsequence $n_k(x)$ such that*

$$S_{n_k(x)}(x) \to f(x), \quad k \to \infty \quad \text{and} \quad f \text{ is finite everywhere.}$$

Then the set of convergence of S has positive measure in every interval δ.

Theorem 2. *If under the conditions of Theorem 1 we have also $f \geq \varphi$ a.e., $\varphi \in \Delta$ (i.e., φ is a finite derivative), then the series converges a.e.. Moreover, it converges everywhere if $f \in \Delta$.*

Theorem 3. *If under the conditions of Theorem 1 f is integrable, then the series is a Fourier series.*

For comparison we have: there exists a series in $\mathcal{A}$ with positive partial sums (and, consequently, Fourier-Stieltjes series in fact), that converges everywhere to some $F \in \cap_{p<\infty} L^p$ and which is not a Fourier series.

Theorem 4. *If for $S \in \mathcal{A}$ we have $\underline{\lim} S_n(x) > -\infty$ on E, $\mu E > 0$, then the series converges a.e. on E.*

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Neumann problem for non-self-adjoint degenerate differential equations of fourth order

D. Kalvand (Azad Islamic University of Karraj, Iran)
Dariush_kalvand@yahoo.com

We consider the following differential equation

$$Lu \equiv (t^{\alpha}u'')'' + au''' + pu = f, \tag{1}$$

where $t \in [0,b], 0 \leq \alpha \leq 4,\ a, p = const, a > 0$ and $f \in L_2(0,b)$.

The completion of $C^2[0,b]$ in the norm

$$||u||^2_{W^2_\alpha} = \int_0^b \left(t^{\alpha}|u''(t)|^2 + |u(t)|^2 \right) dt$$

we denote by W^2_α. Note that for $0 \leq \alpha \leq 4$ there is a continuous embedding $W^2_\alpha \subset L_2(0,b)$, which is compact for $0 \leq \alpha < 4$.

Definition 1. *We say that $u \in W^2_\alpha$ is a generalized solution of equation (1), if for every $v \in W^2_\alpha$*

$$(t^{\alpha}u'', v''_h) - a(u'', v'_h) + p(u, v_h) = (f, v_h).$$

Here $v_h(t) = v(t)\psi_h(t)$, where $\psi_h(t) = 0$ for $t \in [0,h]$, $\psi_h(t) = 1$ for $t \in [2h,b]$ and $\psi_h(t) = h^{-3}(t-h)^2(5h-2t)$ for $t \in [h,2h]$. For the particular case of equation (1), when $p = 0$:

$$Mu \equiv (t^{\alpha}u'')'' + au''' = f \tag{2}$$

it is easy to see that $D(M) = D(L)$. For the solutions of equation (2) we have that the values of $u(0)$ and $u'(0)$ are finite. If $u \in W^2_\alpha$ is the generalized solution for equation (2), we can state that $(t^{\alpha}u''(t))_{|t=0} = (t^{\alpha}u''(t))'_{|t=0} = 0$, $u''(b) = u'''(b) = 0$. Therefore from Definition 1 we conclude that the generalized solution for the equation (1) is unique for $p > 0$ since for $u \in D(L)$ we have $\lim_{h\to 0}(u''(t), u'_h(t)) = -\frac{1}{2}|u'(0)|^2$. Thus we have that the generalized solution for the equation (1) exists and is unique for $p > 0$. Note also that the inverse operator $L^{-1} : L_2(0, b) \to L_2(0, b)$ is compact for $p > 0$.

To consider the case when $a < 0$, we pass to the adjoint equation

$$Nv \equiv (t^{\alpha} v'')'' + av''' + pv = g, \; a < 0, \;\; g \in L_2(0, b). \qquad (3)$$

Definition 2. *We say that $v \in L_2(0, b)$ is a generalized solution of the equation (3), if for every $u \in D(L)$ the equality $(Lu, v) = (u, g)$ holds.*

Using uniquely solvability of the equation (1) we conclude that for $p > 0$ the equation (3) is also uniquely solvable for every $g \in L_2(0, b)$. In addition, we can state that the inverse operator N^{-1} for $0 \le \alpha < 4$ is compact.

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On the unbounded divergence sets of series in Franklin system

D. Karagulyan (Yerevan State University, Armenia)
davidkar89@yahoo.com

We say that E is the unbounded divergence set for a sequence of functions $f_n(x)$, $x \in [0, 1]$, if $f_n(x)$ unboundedly diverges at any point $x \in E$

and converges if $x \notin E$. We have obtained a complete characterization of these sets for general series in Franklin system $\{F_n(x)\}$.

Theorem 1. *The set $E \subset [0,1]$ is a divergence set for a Franklin series*

$$\sum_{n=1}^{\infty} a_n F_n(x)$$

if and only if E is a G_δ-set.

Theorem 2. *The set $E \subset [0,1]$ is a divergence set for a series*

$$\sum_{n=1}^{\infty} |a_n F_n(x)|$$

if and only if E is a G_δ-set.

Analogous theorems for Haar system has been proved by M. A. Lunina [1] in 1976.

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On the divergence sets of some sequences of operators

G. Karagulyan (Institute of Mathematics of NAS, Armenia)
g.karagulyan@yahoo.com

We consider sequences of bounded linear operators $U_n : L^1[0,1] \to C[0,1]$ satisfying the properties

1. $\sup_n \|U_n\|_{L^\infty \to C} < \infty$,

2. if $f \in L^\infty[0,1]$ and $f(x) = c$ on (α, β), then we have

$$\lim_{n\to\infty} U_n(x, f) = c, \quad x \in (\alpha, \beta),$$

and the convergence is uniform on any closed subset of (α, β),

3. for any $f \in L^\infty[0,1]$ we have $U_n(x, f) \to f(x)$ as $n \to \infty$ almost everewhere.

We say E is the divergence set of a sequence $U_n(x, f)$ for some $f \in L^1[0,1]$, if $U_n(x, f)$ diverges at any point $x \in E$ and converges if $x \notin E$. We prove the following

Theorem. *Suppose that a sequences of operators $U_n : L^1[0,1] \to C[0,1]$ satisfies the properties (1)-(3). Then $E \subset [0,1]$ is a divergence set for $U_n(x, f)$ for some $f \in L^1[0,1]$ if and only if E is a $G_{\delta\sigma}$-nullset.*

Note that many sequences of operators in the theory of Fourier series have properties (1)-(3). So the theorem gives a complete characterization of divergence sets of some Fourier operators. This and some other theorems related to the subject are proved in the papers [1]-[3].

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Weighted integral representations of holomorphic functions in tube domains over real balls

A. Karapetyan (Institute of Mathematics of NAS, Armenia)
armankar2005@rambler.ru

Denote by $B_n = \{(y_1, y_2, \ldots, y_n) \in R^n : |y|^2 \equiv y_1^2 + y_2^2 + \cdots + y_n^2 < 1\}$ the real unit ball in R^n. For $1 \leq p \leq 2, 1/p \leq s < +\infty, \alpha > -1$ we consider the space of functions f, holomorphic in the tube domain $T_{B_n} = \{z = x + iy \in C^n : x \in R^n, y \in B_n\}$ and satisfying the condition

$$\int_{B_n} \left(\int_{R^n} |f(x+iy)|^p dx \right)^s (1 - |y|^2)^\alpha dy < +\infty.$$

For these classes the following integral representation is obtained ($z \in T_{B_n}$):

$$f(z) = \frac{1}{(2\pi)^n} \int_{T_{B_n}} f(w) \Phi(z,w) (1 - |v|^2)^\alpha dm(w) \quad (w = u + iv),$$

where the kernel $\Phi(z,w)$ is holomorphic in z, antiholomorphic in w and can be written in the following explicit form:

$$\Phi(z,w) = \int_{R^n} \frac{e^{i<z-\overline{w},t>}}{\gamma^*(2t)} dt, \quad z, w \in T_{B_n},$$

where

$$\gamma^*(t) \equiv 2 \cdot \int_0^1 ch(\tau|t|)(1-\tau^2)^{\alpha + \frac{n-1}{2}} d\tau, \quad t \in R^n.$$

Estimates of φ-approximation functions of several variables with step functions

I. Katkovskaya (Belorussian National Technical University, Belarus)
vkrotov@cosmostv.by

Let Φ be the class of increasing functions $\varphi : \mathbb{R}_+ \to \mathbb{R}_+$, $\varphi(0) = \varphi(+0) = 0$, $\varphi(t) > 0$ for $t > 0$. Let $I = \prod_{i=1}^{n}[a_i, b_i] \subset \mathbb{R}^n$ be n-dimensional interval and $\varphi(L)(I)$ be the set of classes of equivalence of measurable functions on I such that $\varphi \circ f \in L^1(I)$,

$$d_\varphi(f,g) = \int_I \varphi(|f(x) - g(x)|)\,dx,$$

The anisotropic modulus of continuity of function $f \in \varphi(L)(I)$ is defined as

$$\omega(\delta_1,\ldots,\delta_n; f)_\varphi = \sup_{0<h_i<\delta_i} \prod_{i=1}^{n} \int_{a_i}^{b_i - h_i} \varphi(|f(x+h) - f(x)|)\,dx_i$$

(the product of integrals we understand as repeating integration, $0 < \delta_i \le b_i - a_i$). Such characteristic is interesting only with doubling condition $\varphi(2t) = O(\varphi(t))$ ($t \to +\infty$) (see [1, theorem 6.3]).

For $l \in \mathbb{N}^n$ denote by $\mathcal{H}_l(I)$ the class of functions χ on I that are constant on any interval of the form

$$\prod_{i=1}^{n} \left(a_i + (b_i - a_i)\frac{k_i - 1}{l_i}, a_i + (b_i - a_i)\frac{k_i}{l_i} \right), \quad k_i \in \{1,\ldots,l_i\}^n.$$

Let also

$$\mathcal{H}_l^\lambda(I) = \{\chi \in \mathcal{H}_l(I) : |\chi(x)| \le \lambda\}.$$

The following theorem is an analogue of classic Jackson inequality for best approximations by step functions.

Theorem. *For any $\varphi \in \Phi$, $f \in \varphi(L)(I)$ and $l \in \mathbb{N}^n$ there exists a function $\chi \in \mathcal{H}_l(I)$, such that*

$$d_\varphi(f,\chi) \le 2^n \omega\left(\frac{b_1 - a_1}{l_1}, \ldots, \frac{b_n - a_n}{l_n}; f\right)_\varphi .$$

If $|f(x)| \leq \lambda$, then χ can be taken from $\mathcal{H}_l^\lambda(I)$.

In particular this theorem gives the estimates for best anisotropic approximations by multiple Haar and Walsh polynomials.

As an example we can take $\varphi(t) = t^p$, $p > 0$. The interesting special case is also $\varphi_0(t) = t(1+t)^{-1}$ (then $\varphi_0(L)(I)$ is the class of all measurable functions on I). For φ_0 the statement in isotropic case can be found in [2].

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On derivatives of multivalent maps

R. Khachatryan and R. Avetisyan (Yerevan State University, Armenia)
protectdrive@gmail.com

We prove that contingent and tangent cones of Clarke [1] for star-shaped sets are Boltyanskii tents [2]. Using this results we construct lower and upper differentials for multivalent maps with star-shaped graphs. We study also the question of existence of selections possessing directional derivatives.

Let $M \subseteq \mathbb{R}^n$. Denote

$$M^0 = \{x \in M / \forall y \in M, \lambda y + (1-\lambda)x \in M, 0 \leq \lambda \leq 1\}.$$

If $M^0 \neq \emptyset$, then the set M is called star-shaped. Assume $x \in M$.

Definition 1. *([1])* $v \in T_M(x)$ *iff*

$$\forall \varepsilon > 0,\ \ \forall \alpha > 0 \quad \exists u \in B_\varepsilon(v), \exists h \in (0, \alpha] : x + hu \in M$$

Definition 2. *Let* $a : \mathbb{R}^n \to 2^{\mathbb{R}^m}$ *be a multivalent map,* $(x_0, y_0) \in graf(a)$ *and* $K \subseteq T \equiv T_{graf(a)}(x_0, y_0)$ *be a closed convex cone. The multivalent map* $D^K a(x_0, y_0)(\overline{x}) \equiv \{\overline{y} \in \mathbb{R}^m / \ \ (\overline{x}, \overline{y}) \in K\}$ *is called a derivative of a at* (x_0, y_0).

Definition 3. *A positively homogeneous map* $\Delta : \mathbb{R}^n \to 2^{\mathbb{R}^m}$ *is called the lower differential for a at* $(x_0, y_0) \in graf(a)$, *if for every* $\epsilon > 0$ *there exists a* $\delta > 0$ *such that*

$$y_0 + \Delta(\overline{x}) \subseteq a(x_0 + \overline{x}) + \varepsilon \|\overline{x}\| B_1(0) \quad \textit{for all} \ \ \overline{x} \in B_\delta(0).$$

The following is true:

Theorem. *Let* $a : \mathbb{R}^n \to 2^{\mathbb{R}^n -}$ *be a multivalent map with closed star-shaped graph and* $(x_0, y_0) \in graf(a)$, *where* $x_0 \in int(dom(a^0))$ *and* $a(x_0)$ *is bounded. then for some* $\gamma > 0$ *the map* $\Delta(\overline{x}) \equiv D^T a(x_0, y_0)(\overline{x}) \bigcap \gamma \|\overline{x}\| B_1(0)$ *is lower differential for a at* (x_0, y_0).

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Convergence of greedy approximation a.e and in $L^1(0,1)$ with respect to general Haar system

A. Kobelyan (Yerevan State University, Armenia)
a_kobelyan@ysu.am

We consider the convergence of greedy approximation with respect to general Haar system. Let $\{h_n(x)\}_{n=1}^{\infty}$ be a general Haar system normalized in $L^1(0,1)$. For integrable function f the greedy approximant is defined by

$$G_m(f) := \sum_{k\in\Lambda_m} c_k(f)h_k(x),$$

where $\Lambda_m \in \mathbb{N}$ is the set of cardinality m, containing the indices of the first m biggest (in absolute value) Fourier-Haar coefficients $c_k(f)$ of f. In general, G_m isn't determined uniquely.

T. Tao showed that there exists an integrable function for which Greedy algorithm for the classical Haar system diverges a.e., and latter this result were generalized by G.Gevorgyan and A. Stepanyan for any 0-regular wavelet expansion.

Note that S.J. Dilworth, D. Kutzarova, and P. Wojtaszczyk showed the existence of integrable function $f(x)$ for which the Greedy algorithm for the classical Haar system diverges in L^1.

The following theorem are true for general Haar system:

Theorem 1. For each $\varepsilon > 0$ there exists a measurable set $E \subset [0,1]$ with measure $|E| > 1-\varepsilon$, such that for every $f \in L^1_{[0,1]}$ there exists an integrable function g coinciding with f on E, such that

1. $||G_m(g)||_{L^1} \le 4||g||_{L^1} \le 12||f||_{L^1}$;
2. $G_m(g)$is determined uniquely and $spec(g) = \mathbb{N}$;
3. $\lim_{m\to\infty} ||G_m(g) - g||_{L^1} = 0$;
4. $\sum\limits_{k=1}^{\infty} |c_k(g)h_k(x)| < \infty$ *a.e.*.

As to the statement 4 of Theorem 1 note that M.G Grigoryan proved for the classical Haar system $h^c = \{h_n^c\}_{n=1}^{\infty}$, that for any $\varepsilon > 0$ there exists a measurable set $E \subset [0,1]$ with measure $|E| > 1-\varepsilon$, such that for every $f \in L^1[0,1]$ one can find a function $g \in L^1[0,1]$, which coincides with f on E, such that $\{|c_n(g)| : n \in spec(g)\}$ is decreasing to 0, $\{c_n\}_{n=1}^{\infty} \in l^q, \forall q > 2$ and the statement 4 in Theorem 1 is true.

We consider subsystems $\{h_k\}_{k\in\mathcal{S}} = \{h_{n_k}(x)\}_{k=1}^{\infty} = \{\varphi_k(x)\}_{k=1}^{\infty}$ of general Haar system with

$$\mathcal{S} = \{n_k\}_{k=1}^{\infty} \qquad \left|\bigcap_{m=1}^{\infty} \bigcup_{k=m}^{\infty} \Delta_{n_k}\right| = 1. \tag{1}$$

Theorem 2. Let $\{\varphi_k\}_{k=1}^{\infty} = \{h_{n_k}\}_{k=1}^{\infty} = \{h_k\}_{k\in\mathcal{S}}$ be a subsystem of general Haar system satisfying to (1), and let ε be an arbitrary positive number. Then there exists a measurable set $E \subset [1,0]$ with $|E| > 1-\varepsilon$, such that for every $f \in L^1[0,1]$ there exists an integrable function g coinciding with f on E, such that

1. $||G_m(g)||_{L^1} \le 4||g||_{L^1} \le 12||f||_{L^1}$;
2. $G_m(g)$ is determined uniquely and $spec(g) = \mathcal{S}$;
3. $\lim_{m\to\infty} ||G_m(g) - g||_{L^1} = 0$.

M.G. Grigoryan and S. Gogyan proved Theorem 2 for those subsystems of classical Haar system, that include complete groups.

Note that Theorem 2 is new for both classical Haar system and for general Haar system.

On a Conjecture by Louis Brickman Concerning Complex Polynomials

J. Koch (University of Wuerzburg, Germany)
julia.d.koch@mathematik.uni-wuerzburg.de

The argument principle shows that a complex polynomial with a zero in 0 maps the unit circle on a closed curve that winds around the origin at least once. It is plausible that it does so in some sort of uniform way, especially, the image curve has to linger in any sector of the complex plane for some minimal while and cannot cross it too fast. This is stated more precisely in the following conjecture by L. Brickman:

Conjecture. *For every polynomial $P \not\equiv 0$ of degree n with $P(0) = 0$ and every $\alpha \in (0, \pi)$ there exists an arc Γ_α on the unit circle $\partial\mathbb{D}$ of length at least $\frac{2\pi}{n}$ such that $|\arg(P(z))| \le \alpha$ for each $z \in \Gamma_\alpha$.*

Obviously, for all monomials $z \mapsto az^n$ with any complex number $a \neq 0$, the conjecture is true and the given boundary for the length of Γ_α is sharp. Particularly the case of $n = 1$, i. e. linear functions P, is trivial.

Though this conjecture is not only intriguing by itself, but has also practical value in the context of a rootfinder algorithm by Ruscheweyh (1984), so far only a few special cases have been proven [Clunie, Ruscheweyh and Salinas (1987)].

In our talk, we report on these results and give an outlook on recent findings.

Valued distribution of geometrically converging rational functions

R. Kovacheva (Institute of Mathematics and Informatics, Bulgaria)
rkovach@math.bas.bg

Let D be a domain, $\{r_n\}_{n\in\mathbf{N}}$ a sequence of rational functions of degree at most n and let each r_n have at most m poles in D, m fixed. We prove that if $\{r_n\}$ converges geometrically to some function f, $f \not\equiv 0$ on some continuum $S \subset D$ and if the number of the zeros of r_n on any compact subset is of growth $o(n)$ an $n \to \infty$, then the sequence $\{r_n\}_{n\in\mathbf{N}}$ converges m_1- almost uniformly to a meromorphic function in D. In the talk, applications of this result about meromorphic continuation will be provided. Especially, we obtain Picard-type theorems for the value distribution of m_1- maximally convergent rational functions, especially for Padé approximants and Chebyshev rational approximants - the real case.

Part of the results presented in the present talk are joined with H.-P. Blatt and R. Grothmann.

On divergence of Steklov means of Sobolev functions

V. Krotov and M. Prokhorovich (Belorussian State University, Belarus)
krotov@bsu.by

Let $f \in L^1_{\text{loc}}(\mathbb{R}^N)$ and $\mathcal{L}(f)$ be complement of the set of points $x \in \mathbb{R}^N$ such that there exists

$$\lim_{r\to+0} \frac{1}{\mu(B(x,r))} \int_{B(x,r)} f\, d\mu. \tag{1}$$

The classical Lebesgue theorem states that $\mu(\mathcal{L}(f)) = 0$ for any $f \in L^1_{\text{loc}}(\mathbb{R}^N)$. For regular functions we can say more. Federer–Ziemer [1] gave estimates of capacity $\text{Cap}_{1,p}$ and Hausdorff dimension $\dim_{\mathbb{H}}$ of the set $\mathcal{L}(f)$ for functions from Sobolev classes $W_1^p(\mathbb{R}^N)$: *If* $1 < p < N$ *and* $f \in W_1^p(\mathbb{R}^N)$*, then*

$$\dim_{\mathbb{H}}(\mathcal{L}(f)) \le N - p,\ \text{Cap}_{1,p}(\mathcal{L}(f)) = 0.$$

Theorem 1. *If* $1 < p < N$*, then for any* $0 < \varepsilon < N - p$ *there exists a function* $f_\varepsilon \in W_1^p\left(\mathbb{R}^N\right)$*, such that*

$$\dim_{\mathbb{H}}(\mathcal{L}(f_\varepsilon)) > N - p - \varepsilon,\ \text{Cap}_{1,p+\varepsilon}(\mathcal{L}(f_\varepsilon)) > 0.$$

Similar result is true for Calderon classes

$$C^p_\alpha(\mathbb{R}^N) = \left\{ f \in L^p : \|f\|_{C^p_\alpha} = \|f\|_{L^p} + \|\mathcal{S}_\alpha f\|_{L^p} < \infty \right\}.$$

Here

$$\mathcal{S}_\alpha f(x) = \sup_{B \ni x} r_B^{-\alpha} \frac{1}{\mu(B)} \int_B |f(y) - f_B| \, d\mu(y),$$

(f_B is the mean value of f over ball B). Note that $C_1^p(\mathbb{R}^N) = W_1^p(\mathbb{R}^N)$ [3]. The corresponding capacities are

$$\mathrm{Cap}_{\alpha,p}(E) = \inf \left\{ \|f\|_{C_\alpha^p}^p : f \in C_\alpha^p(X), \ f \geq 1 \text{ in neighborhood } E \right\}.$$

Theorem 2. *If* $0 < \alpha \leq 1$ *and* $1 < p < N/\alpha$, *then for any* $0 < \varepsilon < N - \alpha p$ *there exist* $f_\varepsilon \in C_\alpha^p\left(\mathbb{R}^N\right)$, *such that*

$$\dim_{\mathbb{H}}(\mathcal{L}(f_\varepsilon)) > N - \alpha p - \varepsilon,$$

$$\mathrm{Cap}_{\alpha+\varepsilon,p}(\mathcal{L}(f_\varepsilon)) > 0, \ \mathrm{Cap}_{\alpha,p+\varepsilon}(\mathcal{L}(f_\varepsilon)) > 0.$$

The estimates of capacities and Hausdorff dimensions of $\mathcal{L}(f)$ for functions from Calderon classes C_α^p on arbitrary doubling metric measure spaces are given in [3].

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On the number of maximal planes of GC_2 sets in $\mathbb{R}^3$

G. Ktryan (Yerevan State University, Armenia)
ktryangagik@yahoo.com

The geometric characterization (GC), introduced by Chung and Yao, identifies sets of interpolation nodes whose corresponding Lagrange polynomials are products of first degree polynomials. A maximal hyperplane for a GC_n set (a GC set of degree n) in $\mathbb{R}^d$ contains $\binom{n+d-1}{d-1}$ points of that set. There exist several conjectures about the number of maximal hyperplanes for a GC set. In the bivariate case, Gasca and Maeztu in 1982 conjectured that for every GC_n set there exists at least a maximal line. This has been proved for $n \leq 4$. Later on Carnicer and Gasca proved that for $n \leq 4$, and for every GC_n set, there exist at least 3 maximal lines and conjectured that this holds for $n > 4$. De Boor extended these conjectures to $\mathbb{R}^d$: at least a maximal hyperplane as extension of the first one and at least $d + 1$ maximal hyperplanes as extension of the second one. The same author proved that the second conjecture does not hold for $d > 2$, showing a counterexample with $d = 3, n = 2$ and only 3 maximal hyperplanes. Recently, Apozyan et al. proved that for $d = 3, n = 2$, there exists at least one maximal hyperplane. We prove that, in fact, when $d = 3, n = 2$, there exist at least 3 maximal hyperplanes. We also get that in this case at most one node is not using a maximal hyperplane in its Lagrange polynomial.

On multiple trigonometric series

O. Kuznetsova
(Institute of Applied Mathematics and Mechanics of NAS, Ukraine)
kuznets@iamm.ac.donetsk.ua

Let λ_n be a sequence of real numbers, $\lambda_n \to 0$ as $n \to \infty$. We consider questions of integrability and convergence of the following multiple trigonometric series:

1. $\lambda_0 + \sum_{n=1}^{\infty} \lambda_n \sum_{k \in nV \setminus (n-1)V} e^{i(k,x)}$,

where V is polyhedron of set W_b (see [1]) in $R^m, k \in Z^m, nV = \{x \in R^m : x/n \in V\}$ is a homothetic of V;

2. trigonometric power series

$$\lambda_0 + \sum_{n=1}^{\infty} \lambda_n \sum_{\sum k_j = n,\ k_j \geq 0} e^{i(k,x)}$$

(see [2]).

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Periodic wavelets with good angular-frequency localization†

E. Lebedeva (Saint Petersburg State Polytechnical University, Russia)
ealebedeva2004@gmail.com

We focus on a property of good localization of both periodic wavelet functions and theirs Fourier coefficients. The quantitative characteristic of this property is an uncertainty constant (UC). It is introduced by Breitenberger using physical reasons as the product of frequency and angular variances. The smaller uncertainty constant corresponds to the better localization. But there exists a universal lower bound for the UC (1/2 for chosen normalization). This is the essence of the Breitenberger uncertainty principle. Furthermore, there is no extremal function for this principle, id est there is no function attending the lower bound. So, to find a sequence of functions having asymptotically minimal UC and some additional setup, for example, a wavelet structure is a natural question.

We construct scaling and wavelet sequences with good localization. The scaling sequence has an asymptotically optimal UC. The UC of the wavelet sequence tends to 3/2. We discuss the following hypothesis: 3/2 is the best possible UC for periodic wavelet functions.

On moduli of p-continuity

M. Lind (Karlstad University, Sweden)
martin.lind@kau.se

Moduli of p-continuity provide a measure of fractional smoothness of functions via p-variation. We will discuss sharp estimates of the modulus

†The work is supported by RFBR(09-01-00162)and by grant of President RF MK-7413.2010.1.

of p-continuity in terms of the modulus of q-continuity $(1 < p < q < \infty)$.

The talk is based on joint work with Viktor Kolyada (Karlstad).

Analytic properties of power series with radius of convergence zero

W. Luh (Trier University, Germany)
wolfgang.luh@gmx.de

We consider power series with radius of convergence zero and discuss the following questions:

- Overconvergence- and universal-phenomena,
- Summability properties of elongated partial sums.

Continuity of the maximal operator in Sobolev spaces

H. Luiro (University of Jyväskylä, Finland)
hannes.luiro@jyu.fi

The classical Hardy-Littlewood maximal operator and its many variants are central tools on many fields of mathematical analysis. In the study of these operators the focus is usually on the absolute size of the maximal function. However, the fundamental role of the maximal operators e.g. in PDE-theory gives a motivation to study also their regularity properties.

The first step to this direction was made by J.Kinnunen who observed that Hardy-Littlewood maximal operator M is bounded on Sobolev space $W^{1,p}(\mathbb{R}^n)$, when $1 < p \leq \infty$. However, since M is highly non-linear, the continuity remained as an open problem. This question turns out to be much more delicate problem than boundedness.

The main result of this presentation says that M is continuous on $W^{1,p}(\mathbb{R}^n)$ exactly when $1 < p < \infty$ (not if $p = 1$ or $p = \infty$). However, there does not exist any modulus of continuity. As a main auxiliary tool we prove an explicit formula for the derivative of the maximal function.

These results can also be generalized to the wide class of different maximal operators including the spherical maximal operator or fractional maximal operator.

Fuglede's Spectral Set Conjecture For Two and Three Intervals

Sh. Madan (Indian Institute of Technology Kanpur, India)
madan@iitk.ac.in

The spectral set conjecture of B.Fuglede states that a set Ω is a spectral set if and only if Ω tiles $\mathbb{R}^d$ by translations. When Ω is a union of two intervals, Laba proved that the conjecture is true. We will give an embedding of the spectrum in a suitable vector space, with a degenerate conjugate linear form and characterize the spectrum, when it exists, of a set which is union of n intervals. Using this we give an easy proof of Labaś result. Then for all but one situation we will prove that "Spectrum implies Tiling" holds for 3 intervals. The exceptional situation poses a challenging problem. The "Spectrum implies Tiling" for three intervals was proved by the authors earlier.

On Special Functions in Hypercomplex Function Theory and Their Applications

H. Malonek (University of Aveiro, Portugal)
hrmalon@ua.pt

Hypercomplex Function Theory started in the beginning of the 1930s, mainly developed by R. Fueter and his disciples, as generalization of the classical *Function Theory of one Complex Variable* to the case of one *quaternionic variable*. However, the interest in multivariate analysis using more general Clifford algebras (which suggested to use the name *Clifford Analysis* in analogy to *Complex Analysis*) only started to grow significantly in the 70s and has been treated by many authors almost exclusively as some type of refinement of Harmonic Analysis. Since then also many papers on Clifford Analysis referring different classes of Special Functions have appeared. But approaches to Special Functions by means of algebraic methods, either Lie algebras or through Lie groups and symmetric spaces gained by that time importance and influenced their treatment in Clifford Analysis. In our talk we will rely on the generalization of the classical approach to Special Functions through differential equations with respect to the hypercomplex derivative, which is a more recently developed tool in Clifford Analysis and stresses the function theoretic origins of Clifford Analysis. In this context special attention will be payed to the role of Special Functions as intermediator between continuous and discrete mathematics. This corresponds to a more recent trend in combinatorics, since it has been revealed that many algebraic structures have hidden combinatorial underpinnings. Applications to quasiconformal 3D-mappings will also be mentioned.

Extremely Amenable Locally Compact Borel Subsemigroups

H. P. Masiha (K. N. Toosi University of Technology, Iran)
masiha@kntu.ac.ir

In this paper, we shall investigate the relationship between extreme amenability of a locally compact semigroup and its Borel subsemigroups. In particular, we obtain analogues of some nice results of Mitchell [1],[2], and Wong [3].

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The Zero Level Set for a Certain Weak Solution, with Applications to the Bellman Equations†

H. Mikayelyan (Max Planck Institute for Mathematics, Germany)
hayk@mis.mpg.de

We prove the following partial regularity result:

†Joint work with John Andersson.

Theorem. *Let u be a solution to $div(B \cdot \nabla u) = 0$, where $B = B(u) = I + (A - I)\chi_{\{u<0\}}$, $\chi_{\{u<0\}}$ denotes the characteristic function of the set $\{x : u(x) < 0\}$ and A is a matrix with strictly positive and bounded eigenvalues.*

Then $\mu = \Delta u^+$ is a positive and locally finite measure whose $spt(\mu) \subset F$ and has σ-finite $(n-1)-$dimensional Hausdorff measure. Furthermore for $\mu-$a.e. $x \in F$ there is a ball $B_r(x)$ such that $F(u) \cap B_r(x)$ is a $C^{1,\alpha}$ graph.

The authors' main interest in studying these equations is that they are strongly related to the Bellman equation:

$$\inf(\Delta w, Lw) = 0,$$

where L is a constant coefficient uniformly elliptic equation.

A generalization of Banach fixed point principle

L. Mikayelyan (Yerevan State University, Armenia)
mikaelyanl@ysu.am

Let X be a Banach space and L be some closed subspace of X. As usual, we denote by X/L the factor space equipped with norm

$$\|[x]\|_{X/L} = \inf_{y \in L} \|x + y\|,$$

where $[x]$ is the equivalence class generated by $x \in X$.

Definition 1. *We say that the operator $A : X \to X/L$ is a contraction if there is $\alpha \in [0,1)$such that $\|[Ax_1] - [Ax_2]\|_{X/L} \leq \alpha \|x_1 - x_2\|_X$.*

Definition 2. *We say that the point $x_0 \in X$ is "fixed" point for the operator $A : X \to X/L$ if $Ax_0 = [x_0]$.*

The following result is true:

Theorem. *If $A : X \to X/L$ is a contraction, then it has a unique "fixed" point.*

Solvability of Jump Boundary Value Problem on a Countable Set of Closed Curves

S. Mironova, B. Katz and A. Pogodina
(Kazan State Technical University, Russia)
srmironova@yandex.ru

Let D be a plane domain with the boundary Γ. We call $S_\alpha(D) = \iint_D (\mathrm{dist}(z,\Gamma))^{-\alpha} dxdy$, $z = x + iy$ an α-size of D. Let $\Gamma = \bigcup_{j\geq 1} \Gamma_j$ be a countable set of closed rectifiable curves Γ_j, which bound disjoint finite domains D_j^+. We will assume that Γ_j converge to a point z_0 as $j \to \infty$. Denote by A the set of all such sets of curves. Let $H_\nu(\Gamma_j)$ be the set of functions f satisfying the Hölder condition on Γ_j with exponent $\nu \in (0,1]$:

$$h_\nu(f,\Gamma_j) = \sup\left\{\frac{f(t_1) - f(t_2)}{|t_1 - t_2|^\nu} : t_1, t_2 \in \Gamma_j, t_1 \neq t_2\right\} < \infty.$$

Let $\mathbf{H}_\nu(\Gamma)$ be the set of all functions on $\bigcup_{j\geq 1} \Gamma_j$ such that 1) $f|_{\Gamma_j} \in H_\nu(\Gamma_j)$; 2) $\sup\{|f(t)|; t \in \bigcup_{j\geq 1} \Gamma_j\} < \infty$; 3) $\sup\{h_\nu(f;\Gamma_j), j = 1,2,3,...\} < \infty$.

Consider the following boundary value problem. To find a holomorphic in $\overline{\mathbb{C}} \setminus \overline{\bigcup_{j=1}^\infty \Gamma_j}$ function $\Phi(z)$ vanishing at ∞ which has boundary values $\Phi^+(t)$ and $\Phi^-(t)$ from inside and outside of $\bigcup_{j=1}^\infty D_j^+$, $t \in \Gamma_j, j = 1,2,\ldots$, satisfying

$$\Phi^+(t) - \Phi^-(t) = f(t), \quad t \in \Gamma_j, \quad j = 1,2,\ldots \tag{1}$$

Theorem. *Let $\Gamma \in A$, $f \in \mathbf{H}_\nu(\Gamma)$ and $\nu > \frac{1}{2}$. If $\sum_{j=1}^\infty S_{1-\nu}(D_j) < \infty$, then the series of Cauchy type integrals*

$$\Phi(z) = \sum_{j=1}^\infty \frac{1}{2\pi i} \int_{\Gamma_j} \frac{f(t)dt}{t-z} \tag{2}$$

converges in $\overline{\mathbb{C}} \setminus \overline{\bigcup_{j=1}^\infty \Gamma_j}$ to a solution of (1). Moreover, if for some $p > 2$ the series $\sum_{j=1}^\infty S_{p(1-\nu)}(D_j) < \infty$ converges, then the solution is bounded in a neighborhood of z_0.

Corollary. *Let* $\Gamma \in A$ *and* $f \in \mathbf{H}_\nu(\Gamma)$ *and* $\nu > \frac{1}{2}$. *If* $\sum_{j=1}^{\infty} \lambda_j \omega_j^{\nu} < \infty$, *where* λ_j *and* ω_j *are the length of* Γ_j *and the diameter of the biggest disk lying in* $\overline{D_j}$, *then the series (2) converges in* $\overline{\mathbb{C}} \setminus \overline{\bigcup_{j=1}^{\infty} \Gamma_j}$ *to a solution of the jump problem (1). Moreover, if for some* $\mu < 2\nu - 1$ *we have* $\sum_{j=1}^{\infty} \lambda_j \omega_j^{\mu} < \infty$, *then the solution is bounded in a neighborhood of* z_0.

Vector Valued Bilinear Maximal Operator

P. Mohanty (Indian Institute of Technology Kanpur, India)
parasar@iitk.ac.in

For locally integrable functions f and g on $\mathbb{R}^n$ define the bilinear Hardy-Littlewood maximal function M as

$$M(f,g) = \sup_{r>0} \frac{1}{|B_r(0)|} \int_{B_r(0)} |f(x-y)g(x+y)| dy.$$

In a remarkable result of M.Lacey it was shown that

$$\|M(f,g)\|_{L^{p_3}(\mathbb{R})} \lesssim \|f\|_{L^{p_1}(\mathbb{R})} \|g\|_{L^{p_2}(\mathbb{R})}$$

where $\frac{1}{p_3} = \frac{1}{p_1} + \frac{1}{p_2}$ and $2/3 < p_3 \leq \infty$, $1 < p_1, p_2 \leq \infty$. In this talk we will present the bilinear analogue of Fefferman-Stein's vector valued inequality for classical Hardy-Littlewood maximal function. More precisely, we will see that

$$\|(\sum_j |M(f_j, g_j)|^{r_3})^{\frac{1}{r_3}}\|_{L^{p_3}(\mathbb{R})} \lesssim \|(\sum_j |f_j|^{r_1})^{\frac{1}{r_1}}\|_{L^{p_1}(\mathbb{R})} \|(\sum_j |g_j|^{r_2})^{\frac{1}{r_2}}\|_{L^{p_2}(\mathbb{R})}$$

for $1 < p_1, p_2 < \infty$, $1 < r_1, r_2 \leq \infty, 2/3 < p_3 < \infty$, and $2/3 < r_3 \leq \infty$ satisfy $\frac{1}{p_3} = \frac{1}{p_1} + \frac{1}{p_2}$ and $\frac{1}{r_3} = \frac{1}{r_1} + \frac{1}{r_2}$.

Continuous functions with universal Fourier series

J. Müller (University of Trier, Germany)
jmueller@uni-trier.de

Carleson's theorem shows that Fourier series of functions f continuous on the unit circle converge to f almost everywhere. Moreover, according to the Kahane and Katznelson theorem, for every set of vanishing measure there exists a continuous function so that the partial sums of the Fourier series diverge on the set.

We consider the question to what extend the divergence may be maximal on certain subsets of the circle in the sense that every reasonable function is the – pointwise or uniform – limit of a subsequence of the partial sums on the set.

On one representation spectrum

M. Nalbandyan (Yerevan State University, Armenia)
mikayeln@freenet.am

The set $A \subset \mathbb{N}$ is called a representation spectrum for Walsh system $\{w_k(x)\}_{k=1}^{\infty}$, if for every $f \in L^0[0,1]$ (the set of a.e. finite measurable functions) there exists a series $\sum\limits_{k \in A} a_k w_k(x)$ converging to f almost everywhere on $[0,1]$.

The set of natural numbers $\mathbb{N} = \{\sum_{i=0}^{\infty} \delta_i 2^i : \delta_i = 0,1\}$ is a representation spectrum, but if we replace every i by N_i in the exponents, where $\{N_i\}_{k=0}^{\infty}$ is an arbitrary increasing sequence of natural numbers, then it can be easily shown that the set $D = \{\sum_{i=0}^{\infty} \delta_i 2^{N_i} : \delta_i = 0,1\}$ in general is not an representation spectrum. But one can perturb this set into the set $\Lambda = \{k + o(\omega(k)) : k \in D\} = \{\lambda_n\}_{n=1}^{\infty}$ which is a representation spectrum. More precisely, the following result is obtained:

Theorem. *For arbitrary sequence of natural numbers* $N_0 < N_1 < \cdots < N_i < \ldots$ *and for every sequence* $\{\omega(k)\}_{k=1}^{\infty}$ *tending to infinity, the set* $D = \{\sum_{i=0}^{\infty} \delta_i 2^{N_i} : \delta_i = 0, 1\}$ *can be changed into the set* $\Lambda = \{k + o(\omega(k)) : k \in D\} = \{\lambda_n\}_{n=1}^{\infty}$, *which is a representation spectrum.*

Universal Pade approximants and generic approximation of functions by their Pade approximants

V. Nestoridis (University of Athens, Greece)
vnestor@math.uoa.gr

On transferring results from Universal Taylor series to Universal Padé approximants we obtain generic approximation on compact sets of arbitrary connectivity by Padé approximants of holomorphic functions defined on arbitrary open sets. This is impossible in the case of universal Taylor series where the approximation is realized by the partial sums which do not have any poles in C. Finally we strengthen a known result about generic approximation of entire functions by their Padé approximants.

Asymptotics for ménage Polynomials

Th. Neuschel (University of Trier, Germany)
neuschel@uni-trier.de

The polynomials defined by

$$U_n(t) = \sum_{k=0}^{n} (t-1)^k \frac{2n}{2n-k} \binom{2n-k}{k} (n-k)!$$

arise in combinatorics where occasionally they are termed as ménage hit polynomials. The special values $U_n(0)$ are the well known reduced ménage numbers. Using a connection formula involving certain ${}_3F_1$-polynomials strong and weak asymptotics for suitable normalized ménage polynomials are derived. The proofs rely on the asymptotic evaluation of parameter integrals combined with arguments from potential theory.

On Approximation of Functions by Product Means

H.K. Nigam
(Mody Institute of Technology and Science (Deemed University), India)
harekrishnan@yahoo.com

In this paper we introduce the concept of $(N, p_n)(C, 1)$ means and establish two theorems on degree of approximation of function $f \in Lip\,\alpha$ class and $f \in Lip(\xi(t), r)$ class by $(N, p_n)(C, 1)$ product summability means of its Fourier series.

We prove the following theorems:

Theorem 1. *Let (N, p_n) be a regular Nörlund method defined by a positive, monotonic, non-increasing sequence $\{p_n\}$. Let f be a 2π-periodic function, Lebesgue integrable on $[0, 2\pi]$ and belonging to $Lip\,\alpha$ class, then the degree of approximation of f by $N_n^p C_n^1$ means of its Fourier series is given by*

$$\left\| N_n^p C_n^1 - f \right\|_\infty = O\left(\frac{1}{(n+1)^\alpha}\right) \quad for \quad 0 < \alpha < 1$$

where $N_n^p C_n^1$ is the $(N, p_n)(C, 1)$ mean of series

$$f(x) \sim \frac{1}{2}a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx). \tag{1}$$

Theorem 2. *Let (N, p_n) be a regular Nörlund method defined by a positive, monotonic, non-increasing sequence $\{p_n\}$. Let f be a 2π-periodic function, Lebesgue integrable on $[0, 2\pi]$ and belonging to $Lip(\xi(t), r)$ class, $r \geq 1$, then the degree of approximation of f by $N_n^p C_n^1$ means of its Fourier series is given by*

$$\left\| N_n^p C_n^1 - f \right\|_r = O\left[(n+1)^{\frac{1}{r}} \xi\left(\frac{1}{n+1} \right) \right],$$

where $N_n^p C_n^1$ is the $(N, p_n)(C, 1)$ mean of series (1) *provided $\xi(t)$ satisfies the following conditions:*

$$\left\{ \int_0^{\frac{1}{n+1}} \left(\frac{t|\phi(t)|}{\xi(t)} \right)^r dt \right\}^{\frac{1}{r}} = O\left\{ \frac{1}{n+1} \right\} \tag{2}$$

and

$$\left\{ \int_{\frac{1}{n+1}}^{\pi} \left(\frac{t^{-\delta}|\phi(t)|}{\xi(t)} \right)^r dt \right\}^{\frac{1}{r}} = O\left\{ (n+1)^{\delta} \right\}, \tag{3}$$

where δ is an arbitrary number such that $s(1-\delta) - 1 > 0$, $\frac{1}{r} + \frac{1}{s} = 1$, $1 \leq r \leq \infty$, and conditions (2) *and* (3) *hold uniformly in x.*

Lagrangian Dynamics and a Weak KAM theorem for the infinite-dimensional torus with $d \geq 1$

L. Nurbekyan
(Instituto Superior Tecnico, Portugal and Yerevan State University, Armenia)
D. Gomes (Instituto Superior Tecnico, Portugal)
levonnurbekian@yahoo.com, dgomes@math.ist.utl.pt

Let $I := [0, 1]$ and $L^2(I) := \{M : I \to \mathbb{R}^d; \int_I |M(x)|^2 dx < \infty\}$. We consider certain class of Lagrangians $L(M, N)$ which are invariant under the measure preserving transformations and are periodic in the spatial variable M. For any $c \in \mathbb{R}$ denote

$$L_c(M,N) = L(M,N) - c\int_I N d\nu_0, \quad \tilde{L}(M,N) = L_c(M,-N),$$

and let $H(M,N)$ be the corresponding Hamiltonian,

$$H_c(M,N) = H(M,N+c), \quad \tilde{H}(M,N) = H_c(M,-N).$$

For fixed c and any $\varepsilon > 0$ we consider the so called discounted cost infinite horizon problem together with its value function

$$V_\varepsilon(M) := \inf_x \left\{ \int_0^\infty e^{-\varepsilon t}\tilde{L}(x,\dot{x})dt : x \in AC^2_{loc}((0,\infty);L^2(I)), x(0) = M \right\}.$$

We prove that for every $\varepsilon > 0$ the value function V_ε satisfies the equation

$$\varepsilon V_\varepsilon(M) + \tilde{H}(M, \nabla_{L^2} V_\varepsilon(M)) = 0$$

in a viscosity sense.

Out of the family of functions $\{V_\varepsilon\}$ we obtain a function $U : L^2(I) \to \mathbb{R}$, which is the value function of the minimization problem

$$U(M) = \inf\left\{ \int_0^T \tilde{L}(x(s),\dot{x}(s)) + \bar{H}(c)ds + U(x(T)); x(0) = M \right\}$$

and satisfies the Hamilton-Jacobi equation

$$H(M, c + \nabla_{L^2} U) = \bar{H}(c),$$

where $\bar{H}(c)$ is the so called effective Hamiltonian. Furthermore, for differentiability points of U we prove the existence of the minimizing trajectories and obtain asymptotics for them. In addition, we prove the existence of so called two-sided minimizers.

Presented results are extensions of the similar results proven by *W. Gangbo* (Georgia Institute of Technology) and *A. Tudorascu* (University of Wisconsin) for $d = 1$.

Wiener Conjecture on Cyclic Vectors

A. M. Olevskii (Tel-Aviv University, Israel)
olevskii@post.tau.ac.il

Wiener characterized cyclic vectors (with respect to translations) in $L^p(\mathbb{R})$, $p = 1, 2$, in terms of the zero set of the Fourier transform. He conjectured that a similar characterization should be true for $1 < p < 2$. I'll discuss this conjecture.

The talk is based on joint work with Nir Lev.

Maximal operators in variable Lebesgue spaces

G. Oniani (Akaki Tsereteli State University, Georgia)
oniani@atsu.edu.ge

We study the questions on the boundedness and the validity of modular inequality in variable Lebesgue spaces for maximal operators corresponding to homothety invariant and translation invariant bases. The obtained results extend the previous ones by T. Kopaliani and A.Lerner. The results concerning the boundedness question are joint with T.Kopaliani.

On Vinogradov series and Riemann – Schrodinger function

K.I. Oskolkov (University of South Carolina, USA)
oskolkov@math.sc.edu

In the talk, the results concerning trigonometric series

$$V[f](x_1,\ldots,x_r) := \sum_{n\in\mathbb{Z}} \widehat{f}_n e^{2\pi i(x_1 n + x_2 n^2 + \cdots + x_r n^r)}, \quad (x_1,\ldots,x_r) \in \mathbb{R}^r,$$

and their various applications will be discussed, where $\widehat{f}_n$ are Fourier coefficients of a certain function, while the real coefficients $(x_1,\ldots,x_r)$ of the algebraic polynomial in the exponent serve as r independent variables. Such series are called I.M. Vinogradov's series. In particular, recent results of the study of multi-fractal properties of Riemann – Schrödinger function

$$\Phi := \sum_{n\in\mathbb{Z}\setminus\{0\}} \frac{e^{\pi i(tn^2+2xn)}}{\pi i n^2}, \quad (t,x) \in \mathbb{R}^2.$$

will be presented.

Approximation of Poisson integrals of functions from the Lipschitz class by de la Valle Poussin sums

I. Ovsii and A. Serdyuk
(Institute of Mathematics of NAS of Ukraine, Ukraine)
ievgen.ovsii@gmail.com, serdyuk@imath.kiev.ua

Let C be the space of continuous 2π-periodic functions $f(t)$ with the norm $\|f\|_C = \max_t |f(t)|$ and let

$$H^\alpha = \{\varphi \in C : \ |\varphi(t') - \varphi(t'')| \leqslant |t' - t''|^\alpha \ \ \forall t', t'' \in \mathbb{R}\},$$

where $\alpha \in (0,1)$. Further, let $C^q_\beta H^\alpha$ be the set of the Poisson integrals of functions φ from H^α, i.e., functions f of the form

$$f(x) = A_0 + \frac{1}{\pi}\int_0^{2\pi} \varphi(x+t) P_{q,\beta}(t)\, dt, \quad A_0 \in \mathbb{R}, \quad x \in \mathbb{R}, \quad \varphi \in H^\alpha,$$

where $P_{q,\beta}(t) = \sum\limits_{k=1}^{\infty} q^k \cos(kt + \frac{\beta\pi}{2})$, $q \in (0,1)$, $\beta \in \mathbb{R}$, is the Poisson kernel with parameters q and β. Denoting by $S_k(f;x)$ the k-th partial sum of the Fourier series, we associate each function $f \in C^q_\beta H^\alpha$ with their de la Vallée Poussin sum

$$V_{n,p}(f;x) = \frac{1}{p}\sum_{k=n-p}^{n-1} S_k(f;x), \quad p, n \in \mathbb{N}, \quad p \leqslant n.$$

We consider the problem of obtaining asymptotic equalities as $n - p \to \infty$ for the quantities

$$\mathcal{E}(C^q_\beta H^\alpha; V_{n,p}) = \sup_{f \in C^q_\beta H^\alpha} \|f(\cdot) - V_{n,p}(f;\cdot)\|_C.$$

The following result is obtained.

Theorem. *Let $q \in (0,1)$, $\beta \in \mathbb{R}$, $n, p \in \mathbb{N}$, $p \leqslant n$ and $\alpha \in (0,1)$. Then the following asymptotic equality*

$$\mathcal{E}(C^q_\beta H^\alpha; V_{n,p}) = \frac{q^{n-p+1}}{p(n-p+1)^\alpha}\times$$

$$\times\left(\frac{2^{\alpha+2}}{\pi^2}\frac{1-q^{2p}}{1-q^2}\boldsymbol{K}(q^p)\int_0^{\pi/2} t^\alpha \sin t\, dt + \frac{O(1)}{(1-q)^{\delta(p)}(n-p+1)^{1-\alpha}}\right),$$

is true as $n - p \to \infty$, where $\boldsymbol{K}(\cdot)$ is the complete elliptic integral of the 1st kind,

$$\delta(p) := \begin{cases} 2, & p = 1, \\ 3, & p = 2, 3, \dots, \end{cases}$$

and $O(1)$ is a quantity uniformly bounded in n, p, q, α and β.

Stability of Projective Gabor Frames for Coorbit Spaces on Locally Compact Abelian Groups†

S. Pandey (R. D. University, India)
sheelpandeynew@hotmail.com

Using projective group representation ρ of the phase $\mathcal{G} \times \hat{\mathcal{G}}$ on $L^2(\mathcal{G})$, we define a coorbit space $\mathcal{C}_o\mathcal{Y}(\mathcal{G})$ associated with a solid Banach space $\mathcal{Y}(\mathcal{G} \times \hat{\mathcal{G}})$ satisfying some suitable conditions, where $\mathcal{G}$ is locally compact abelian group with dual group $\hat{\mathcal{G}}$. We prove a theorem to ascertain a frame for $\mathcal{C}_o\mathcal{Y}(\mathcal{G})$ and prove two theorems about the stability of this frame under perturbations of the window function and lattice point respectively.

Sharp Kolmogorov type inequalities for norms of Riesz derivatives of multivariate functions and their applications

N. Parfinovych, V. Babenko and S. Pichugov
(Dnepropetrovsk National University, Ukraine)
nparfinovich@yandex.ru , babenko.vladislav@gmail.com

Let L_∞ be the space of functions $f : \mathbb{R}^m \to \mathbb{R}$ with norm $\|f\|_\infty = \operatorname{ess\,sup}_{t\in\mathbb{R}^m} |f(t)|$. By $L^\Delta_{\infty,\infty}$ denote the collection of functions $f \in L_\infty$, such that $\Delta f \in L_\infty$.

The Riesz derivative of order α ($0 < \alpha < 2$) of function $f : \mathbb{R}^m \to \mathbb{R}$ is defined as follows:

$$(D^\alpha f)(x) := \frac{1}{d_{m,2}(\alpha)} \int_{\mathbb{R}^m} \frac{2f(x) - f(x-t) - f(x+t)}{|t|^{m+\alpha}}\, dt,$$

$$d_{m,2}(\alpha) = \frac{2^{1-\alpha}\pi^{1+\frac{m}{2}}}{\sin\left(\frac{\alpha\pi}{2}\right)\Gamma\left(1+\frac{\alpha}{2}\right)\Gamma\left(\frac{m+\alpha}{2}\right)}$$

† The author is thankful to DST for the financial support under the project SR/S4/MS/317/06.

Note, that the Riesz derivative D^α represents a fractional power of the Laplace operator $(-\triangle)^{\alpha/2}$.

For given $\rho \geq 0$, $h > 0$, and $\delta = \sqrt[m]{2}$ denote

$$\psi(\rho)=\begin{cases} \frac{1}{2m}\left(\rho^2 - \frac{1}{\delta^2} - \sigma_{m-1}G_1\left(\frac{1}{\delta}\right) + \frac{1}{2}\right), & 0 \leq \rho \leq \frac{1}{\delta}, \\ \frac{1}{2m}\left(-\rho^2 - 2\sigma_{m-1}G_1(\rho) + \frac{1}{\delta^2} + \sigma_{m-1}G_1\left(\frac{1}{\delta}\right) + \frac{1}{2}\right), & \frac{1}{\delta} < \rho \leq 1, \\ \psi(1), & \rho > 1, \end{cases} \tag{1}$$

where

$$G_h(\rho) = \begin{cases} \frac{1}{\sigma_{m-1}(m-2)}\left(\frac{1}{\rho^{m-2}} - \frac{1}{h^{m-2}}\right), & m \geq 3, \\ \frac{1}{2\pi}\ln\frac{h}{\rho}, & m = 2, \end{cases}$$

σ_{m-1} is the surface area of the unit sphere in the space $\mathbb{R}^m$.

For $h > 0$ and $x \in \mathbb{R}^m$ denote

$$\varphi_{h,2}(x) = h^{-2}\psi(h|x|). \tag{2}$$

New sharp Kolmogorov type inequality that estimates the uniform norm of Riesz derivative of a function $f \in L^{\Delta}_{\infty,\infty}$ by uniform norm of f and uniform norm of Δf is obtained.

Theorem. *Let $0 < \alpha < 2$. For all functions $f \in L^{\Delta}_{\infty,\infty}$ the following sharp inequality holds:*

$$\|D^\alpha f\|_\infty \leq \frac{\|D^\alpha \varphi_{1,2}\|_\infty}{\|\varphi_{1,2}\|_\infty^{1-\alpha/2}} \|\triangle f\|_\infty^{\frac{\alpha}{2}} \|f\|_\infty^{1-\alpha/2}. \tag{3}$$

The extremal functions in (3) are the functions of type $f(t) = a\varphi_{h,2}(t)$, $h > 0$, $a \in \mathbb{R}$.

The Stechkin problem about approximation of unbounded operators D^α by bounded ones on the class of functions $f \in L^{\Delta}_{\infty,\infty}$ such that $\|\Delta f\|_\infty \leq 1$ and the problem about the best recovery of the operator D^α on the elements of this class, with the prescribed error δ, are solved.

Operator moduli of continuity

V. Peller (Michigan State University, USA)
peller@math.msu.edu

For a closed subset F of the real line and a continuous function f on F, the operator modulus of continuity Ω_f is defined by

$$\Omega_f(\delta) = \sup \|f(A) - f(B)\|,$$

where the supremum is taken over all self-adjoint operators A and B such that their spectra are contained in F and $\|A - B\| \le \delta$.

It will be shown that our earlier estimates for general continuous functions can be significantly improved in certain special cases.

In particular, we improve estimates of Kato and show that

$$\big\| \, |S| - |T| \, \big\| \le C\|S - T\| \log\left(2 + \log\frac{\|S\| + \|T\|}{\|S - T\|}\right)$$

for every bounded operators S and T on Hilbert space.

We also obtain sharp estimates of the operator moduli of continuity for continuous concave functions on $[0, \infty)$.

Next, we study the problem of sharpness of our general estimates. We construct a C^∞ function f on the real line such that $\|f\|_{L^\infty} \le 1$, $\|f'\|_{L^\infty} \le 1$, and

$$\Omega_f(\delta) \ge \operatorname{const} \delta \sqrt{\log\frac{2}{\delta}}, \quad \delta \in (0, 1].$$

Finally, we obtain sharp estimates of $\|f(A) - f(B)\|$ in the case when the spectrum of A has n points. Moreover, we obtain a more general result in terms of the ε-entropy of the spectrum that also improves earlier estimates of the operator moduli of continuity of Lipschitz functions on finite intervals.

The talk is based on joint work with A.B. Aleksandrov.

On bounded projections in the spaces $L^1(B)$ and $L^\infty(B)$

Albert Petrosyan (Yerevan State University, Armenia)
S. Petrosyan (Artsakh State University, Nagorno-Karabakh Republic)
apetrosyan@ysu.am, petrosyan-79@mail.ru

The bounded operators in Banach spaces $L^\infty(B)$ and $L^1(B)$ are considered, where B is the unit ball in $\mathbb{C}^n$. The range of values of these operators are, accordingly, the spaces $A_\infty(\varphi)$ and $A^1(\psi)$ of holomorphic functions which depend on normal pair of weight functions $\{\varphi, \psi\}$:

$$A^1(\psi) = \Big\{ f \in H(B) \colon \ \|f\|_\psi = \int_B |f(z)|\psi(z) d\nu(z) < \infty \Big\};$$
$$A_\infty(\varphi) = \Big\{ f \in H(B) \colon \ \|f\|_\varphi = \sup_{z \in B} |f(z)|\varphi(z) < \infty \Big\}.$$

Let
$$K_\alpha(z,w) = \frac{\Gamma(n+\alpha+1)}{\Gamma(n+1)\Gamma(\alpha+1)} \cdot \frac{1}{(1-\langle w,z\rangle)^{n+1+\alpha}}$$

is the weighted kernel function of the unit ball. The following result is obtained:

Theorem 1. (i) *The transformation Q defined by*

$$(Qf)(z) = \int_B K_\alpha(z,w) f(w)\varphi(w)\, d\nu(w), \quad f \in L^1(B),\ z \in B,$$

is a bounded operator mapping $L^1(B)$ onto $A^1(\psi)$; the operator ψQ is a bounded projection of $L^1(B)$ onto the subspace $\psi A^1(\psi)$.

(ii) *The transformation P defined by*

$$(Ph)(z) = \int_B K_\alpha(z,w) h(w)\psi(w)\, d\nu(w), \quad h \in L^\infty(B),\ z \in B,$$

is a bounded operator mapping $L^\infty(B)$ onto $A_\infty(\varphi)$; the operator φP is a bounded projection of $L^\infty(B)$ onto the subspace $\varphi A_\infty(\varphi)$.

Fourier expansion of differential operator by means of eigenfunctions: special case

Anush Petrosyan (Yerevan State University, Armenia)
petrosyan-an@rambler.ru

We consider an ordinary linear self-adjoint differential operator L in $L^2(R)$ of m-th order ($m \geq 2$), generated by the following operation l:

$$l(y) = \frac{1}{i^m} y^{(m)} + \sum_{k=0}^{n'-1} \frac{1}{i^{2k}} (p_{2k} y^{(k)})^{(k)} +$$

$$+ \sum_{k=0}^{n-1} \frac{1}{2i^{2k+1}} \left\{ (p_{2k+1} y^{(k)})^{(k+1)} + (p_{2k+1} y^{(k+1)})^{(k)} \right\},$$

where y is a function defined on R, i is the imaginary unit, $n = \left[\frac{m}{2}\right], n' = \left[\frac{m-1}{2}\right]$ and p_k is a real measurable function on R, satisfying $\int_{-\infty}^{\infty} |p_k(x) - a_k| < \infty$, $k = 0, 1, ..., m-2$ for some real numbers a_k. We consider the polynomial $Q(\lambda) = \lambda^m + \sum_{k=0}^{m-2} a_k \lambda^k$, $\lambda \in C$. We denote by $\aleph$ the number of those numbers $\mu \in R$ for which the equation $Q(\lambda) = \mu$ has a multiple real root. For $\aleph \neq 0$ we denote these values by $\mu_1 < \mu_2 < ... < \mu_\aleph$. Let also $\mu_0 = -\infty, \mu_{\aleph+1} = \infty$ and $\theta = 0$ for odd m, and $\theta = 1$ for even m. For each $k = \theta, \theta+1, ..., \aleph$ and for $\mu \in (\mu_k, \mu_{k+1})$ the number of real roots of the equation $Q(\lambda) = \mu$ is constant and is denoted by $2r_k + 1 - \theta$. We prove that if $k = \theta, \theta+1, ..., \aleph$ is not an eigenvalue of L, then the differential equation $l(y) = \mu y$ has $2r_k + 1 - \theta$ linearly independent bounded solutions $\varphi_j(x, \mu)$, $j = \theta, \theta+1, ..., 2r_k$, and under some normalization the following theorem is true:

Theorem. *Every $f \in L^2(R)$ can be represented by the following Fourier expansion:*

$$f(x) = \sum_j \psi_j(x) \int_{-\infty}^{\infty} f(t) \overline{\psi_j(t)} dt + \sum_{k=\theta}^{\aleph} \sum_{j=\theta}^{2r_k} \int_{\mu_k}^{\mu_{k+1}} \Phi_j(\mu) \varphi_j(x, \mu) d\mu, \quad x \in R, \tag{1}$$

and the Parseval's equality holds:

$$\int_{-\infty}^{\infty} |f(x)|^2 dx = \sum_j \left| \int_{-\infty}^{\infty} f(t)\overline{\psi_j(t)} dt \right|^2 + \sum_{k=\theta}^{\aleph} \sum_{j=\theta}^{2r_k} \int_{\mu_k}^{\mu_{k+1}} |\Phi_j(\mu)|^2 d\mu, \quad (2)$$

where $\{\psi_j(x)\}$ is the orthonormal system of all eigenfunctions of the operator L and $\Phi_j(\mu) = \int_{-\infty}^{\infty} f(t)\overline{\varphi_j(t,\mu)}dt$, $\mu_k < \mu < \mu_{k+1}$, $\theta \leq j \leq 2r_k$.

By the change of the variable $\mu = Q(\lambda)$ it's possible to simplify formulas (1) and (2):

$$f(x) = \sum_j \psi_j(x) \int_{-\infty}^{\infty} f(t)\overline{\psi_j(t)} dt + \int_{-\infty}^{\infty} F(\lambda) u(x,\lambda) d\lambda \quad (3)$$

$$\int_{-\infty}^{\infty} |f(x)|^2 dx = \sum_j \left| \int_{-\infty}^{\infty} f(t)\overline{\psi_j(t)} dt \right|^2 + \int_{-\infty}^{\infty} |F(\lambda)|^2 d\lambda \quad (4)$$

where $F(\lambda) = \int_{-\infty}^{\infty} f(t)\overline{U(t,\lambda)}dt$ and $U(x,\lambda)$ is the unique bounded solution of the equation $l(U) = Q(\lambda)U$. Moreover, the second integral in (3) and the integral (4) converge in the norm of the space $L^2(R)$.

Discrete Mellin transform

S. Petrosyan (Artsakh State University, Nagorno-Karabakh Republic)
petrosyan-79@mail.ru

The discrete analogue of Fourier transform is well-known. However, such analogue for Mellin's transform was absent. We introduce the discrete analogue of Mellin's transform, in the following form:

$$X(z_k) = \sum_{n=0}^{N-1} x(n) q^n t_n^k \, .$$

Also the formula of its inversion is proved:

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(z_k) q^{-n} t_k^{-n} .$$

Polynomial approximation via subanalytic geometry methods

Wiesław Pleśniak (Jagiellonian University, Poland)
Wieslaw.Plesniak@im.uj.edu.pl

In multivariate polynomial approximation one has to manage with the complicated geometry of sets. In particular, serious obstacles are caused by cuspidal sets. It appears that a lot of such problems may be surmounted by using tools provided by the Hironaka-Łojasiewicz-Gabrielov subanalytic geometry, and more general by its far-going generalizations that are polynomially bounded *o*-minimal structures. The goal of my talk will be to demonstrate the effectiveness of such an approach by presenting some important results concerning approximation of analytic and/or infinitely differentiable functions on compact sets of the space $\mathbb{C}^N$ or $\mathbb{R}^N$.

Fast Convergence of the Fourier-Pade Approximation for Smooth Functions

A. Poghosyan (Institute of Mathematics of NAS, Armenia)
arnak@instmath.sci.am

The problem of approximating a function using a finite number of its Fourier coefficients $\{f_n\}_{n=-N}^{N}$ is considered. Investigations are performed in the framework of the Fourier-Pade approximation.

A class of approximations $S_{N,p,q}(f)$ is studied based on the Fourier series representation of f. Fast convergence of such approximations is proved for smooth on a finite interval functions compared with the truncated Fourier series approximation. Exact constants of the asymptotic errors are derived. Numerical illustrations clarify theoretical estimates.

Almost everywhere convergence of spherical partial integrals and extensions to Lie groups

E. Prestini (Universita' di Roma "Tor Vergata", Italy)
prestini@mat.uniroma2.it

We provide a much shorter proof of the result, by Carbery and Soria, of a.e. convergence of spherical Fourier partial integrals on $\mathbb{R}^n$ for functions in a suitable Sobolev space. Our result, being independent of the geometry of the ball, is more general as well. As such it has been extended to all connected Lie groups.

Estimates for K-functionals via smoothness moduli

E. Radzievskaya (National University of Food Technology, Ukraine)
radzl58@mail.ru

Let $r \in \mathbb{N}$, f be a complex-valued function defined on the interval of real line, and let $\omega^{[r]}(\delta, f)_{L_p[a;b]}$ be the r-th smoothness modulus. In the case of $f \in C[a;b]$ we denote $\omega^{[r]}(\delta, f)_{C[a;b]} := \omega^{[r]}(\delta, f)_{L_\infty[a;b]}$, and for $r = 1$ we set $\omega(\delta, f)_C := \omega^{[1]}(\delta, f)_C$.

Consider a finite family of functionals $\{U_j\}$ of the form

$$U_j(g) := \int_0^1 g^{(K_j)}(\tau)d\sigma_j(\tau) + \sum_{l=0}^{K_j-1} C_{j,l}g^{(l)}(0), \tag{1}$$

where σ_j belongs to the set of functions of bounded variation on $[0;1]$. The number K_j is called an order of functional U_j and is denoted by *ord* U_j.

Let X be one of the spaces L_p or l. Then for $\delta > 0$ and $f \in X$ define the following two K-functionals

$$K(\delta, f; X, W_U^r(X)) := \inf_{g \in W_U^r(X)} (\|f-g\|_X + \delta\|g\|_{W^r(X)}),$$

$$K(\delta, f; X, W_U^r(X)) := \inf_{g \in \widetilde{W}_U^r(X)} (\|f-g\|_X + \delta\|g\|_{W^r(X)}),$$

where $W_U^r(X) := \{g \in W^r(X) : U_j(g) = 0, ord\ U_j \leqslant r\}$ and $\widetilde{W}_U^r(L_p) := W_U^r(L_p)$

$$\widetilde{W}_U^r(l) := \{g \in W^r(l) : U_j(g) = 0, ord\ U_j \leqslant r\}.$$

The following propositions are true:

Theorem 1. *Let $\{U_j\}$ be a finite family (possibly empty) of functionals of the form (1). If the system $\{U_j\}$ is not empty, then we assume that the functions σ_j from (1) share the following property: every σ_j has at least one jump point, and moreover, functionals U_j with the same order can be associated with different jump points of σ_j.*

Then for every $n \in \mathbb{N}$ there exists a positive constant $c(n, U)$, independent of $X = L_p$ or $X = C$ and of $r = 1, \ldots, n$, such that

$$K(\delta^n, f; X, \widetilde{W}_U^r(X)) \leqslant c(n, U)\delta^r \|f\|_{W^r(X)},\ 0 < \delta \leqslant 1,\ \delta \in W_U^r(X).$$

Theorem 2. *Let $n, r \in \mathbb{N}$, $r \leqslant n$ and the system of functionals $\{U_j\}$ satisfies the conditions of Theorem 1 and do not contain functionals with orders less than r. Then there exists a positive constant $c(n; U)$, independent of $X = L_p$ or $X = C$, such that*

$$K(\delta^n, f; X, \widetilde{W}_U^r(X)) \leqslant c(n, U)(\omega^{[r]}(\delta, f)_X + \delta^r\|f\|_X),\ \ 0 < \delta \leqslant 1,\ f \in X.$$

On extension of Chang-Yao natural lattice

L. Rafayelyan (Russian - Armenian (Slavonic) University, Armenia)
levon.rafayelyan@googlemail.com

We study constructions of poised nodes for bivariate Lagrange interpolation. Denote the space of all polynomials $p(x, y)$ in two variables of total degree $\leq n$ by Π_n. For a Π_n-poised set X we call maximal a line containing $n+1$ points, a conic containing $2n+1$ points, and an irreducible algebraic curve of degree 3 containing $3n$ Π_3-independent points of X.

Chung and Yao introduced the natural lattice (NL_n), which can be characterized as having $n+2$ maximal lines, which is the maximum possible number.

Bellow we extend the NL_n lattice to the case of lines, conics, and irreducible algebraic curves of degree 3.

Consider m lines, conics, and irreducible algebraic curves of degree 3 $g_1, ..., g_m$ with $\sum_{i=1}^{m} \sigma_i = n+2$ and $\sigma_i := deg(g_i)$ such that

1. Any g_i and g_j intersect at $\sigma_i \sigma_j$ points, where $i \neq j$, $1 \leq i, j \leq m$,

2. For any distinct indices i, j, k we have $g_i \cap g_j \cap g_k = \emptyset$, where $1 \leq i, j, k \leq m$.

All above intersection points we call black. Let us add one point on each conic and three points on each curve of degree 3 distinct from black points and call them white. Also the three white points on each curve of degree 3 are not collinear. We call the set consisting of all these black and white points $NL_n(3)$ lattice.

Theorem 1. *The lattice $NL_n(3)$ is Π_n-poised. Moreover a Π_n-poised set has maximal lines, conics, and/or irreducible algebraic curves of degree 3 with total degree $n+2$ if and only if it is the set $NL_n(3)$.*

Approximation of operators in dual spaces by adjoints

O. Reynov (Saint Petersburg State University, Russia)
orein51@mail.ru

Let X, Y be Banach spaces, $T : Y^* \to X^*$ be a linear continuous operator. Is it possible to approximate T by operators of the kind $S^* : Y^* \to X^*$ (adjoint to the operators from X to Y) in the topologies of compact convergence in $L(Y^*, X^*)$ and pointwise $X \times Y^*$-convergence in $L(Y^*, X^*)$, as well as to approximate the operator $T^*|_X : X \to Y^{**}$ in corresponding topologies by operators acting from X to Y? The properties of (C, k)-metric approximation are introduced. Some sufficient (and, in a sense, necessary) conditions are given for the approximation of T (or $T^*|_X$) on all k-dimensional subspaces of corresponding spaces. Some examples are considered of the operators which can not be approximated (e.g., $X \times Y^*$-pointwise), — for Banach spaces with AP, but without MAP; for the spaces with AP and MAP; for the spaces without AP.

The Maximal Range Problem for Polynomials in the Unit Disk

S. Ruscheweyh (University of Wuerzburg, Germany)
ruscheweyh@mathematik.uni-wuerzburg.de

Let Ω be some domain in the complex plane, with $0 \in \Omega$, and let $\mathbb{D}$ be the unit disk. For $n \in \mathbb{N}$ we set

$$\mathcal{P}_n(\Omega) := \{P \in \mathcal{P}_n \,:\, P(0) = 0,\, P(\mathbb{D}) \subset \Omega\},$$

where $\mathcal{P}_n$ denotes the set of complex polynomials of degree $\leq n$. The *maximal range* of degree n with respect to Ω is then defined as

$$\Omega_n := \bigcup_{P \in \mathcal{P}_n(\Omega)} P(\mathbb{D}).$$

We give a complete description of the extremal polynomials in $\mathcal{P}_n(\Omega)$, i.e. such P with $P(\partial\mathbb{D}) \cap (\partial\Omega_n \setminus \partial\Omega) \neq \emptyset$. This identification leads in many cases to a unified approach to new and old estimates for polynomials with certain range restrictions (most elementary example: polynomials with positive real part in $\mathbb{D}$). Furthermore, if Ω is simply connected, the extremal polynomials are very sensible candidates for a polynomial approximation to the conformal mappings of $\mathbb{D}$ onto Ω. We discuss some special cases and an intriguing 'arc'-conjecture.

Partial generalized variation and multivariate Fourier series

A. Sahakian (Yerevan State University, Armenia)
and U. Goginava (Tbilisi State University, Georgia)
sart@ysu.am, zazagoginava@gmail.com

We consider Fourier series of functions of bounded partial Λ-variation on the d-dimensional torus $[-\pi, \pi]^d$. The sufficient and necessary conditions on the sequence $\Lambda = \{\lambda_n\}$ are found for the convergence of Fourier series of functions of bounded partial Λ-variation.

The convergence of Cesàro means of negative order is also investigated.

On the absolute convergence of series in the Faber-Schauder system

A. Sargsyan and T. Grigoryan (Yerevan State University, Armenia)
asargsyan@ysu.am

It is known that the Faber-Schauder system is basis in $C[0,1]$, and is not an unconditional basis in $C[0,1]$ (it is known that $C[0,1]$ cannot have unconditional bases).

There exists a function $f \in C[0,1]$, the expansion of which by the Faber-Schauder system can be rearranged to become divergent in $C[0,1]$. Moreover, in [1] a function $f \in C[0,1]$ is constructed, the expansion of which by the Faber-Schauder system can be rearranged to become divergent in measure on $[0,1]$.

In [2] the following theorem is proved.

Theorem 1. *For each $0 < \epsilon < 1$ and for each function $f(x) \in C[0,1]$ one can find a function $g(x) \in C[0,1]$, $mes\{x \ , \ g(x) \neq f(x) \ , \ x \in [0,1]\} < \epsilon$, with the expansion by the Faber-Schauder system unconditionally (absolutely) convergent in $C[0,1]$.*

Note that in this direction D. Men'shov proved the following fundamental theorem (see [3]).

Theorem (Men'shov). *Let $f(x)$ be an a.e. finite measurable function on $[0,2\pi]$. Then for each $\epsilon > 0$ one can find a continuous function $g(x)$ coinciding with $f(x)$ on a subset E of measure $|E| > 2\pi - \epsilon$, such that its Fourier series with respect to the trigonometric system converges uniformly on $[0,2\pi]$.*

In [4] Katznelson proved that in Men'shov's theorem it is impossible to get absolute convergence.

It must be pointed out that in Men'shovs theorem and in Theorem 1 the exceptional set on which the function $f(x)$ is modified depends on this function, while in Theorem 2 it is universal, independent of the function.

Theorem 2. *For every $\epsilon > 0$, there exists a measurable set $E_\epsilon \subset [0,1]$ with $|E_\epsilon| > 1 - \varepsilon$, such that for every function $f(x) \in C[0,1]$ one can find a function*

$g(x) \in C[0,1]$, $g(x){=}f(x)$, $x \in E_\epsilon$, with the expansion by the Faber-Schauder system unconditionally (absolutely) convergent in $C[0,1]$.

Corollary. *For every $\epsilon > 0$, there exists a measurable set $E_\epsilon \subset [0,1]$ with $|E_\epsilon| > 1-\varepsilon$, such that for every function $f \in C(E_\epsilon)$ there exists a series by the Faber- Schauder system, which unconditional converges to f in $C(E_\varepsilon)$- norm.*

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Some properties of Vilenkin system

S. Sargsyan and M. Grigoryan (Yerevan State University, Armenia)
stepansargsyan@mail.ru, gmarting@ysu.am

Let P be an arbitrary set of natural numbers $P \equiv \{p_1, p_2, \dots, p_k, \dots\}$ ($p_j \geq 2, \forall j \in \mathbb{N}$). Denote $m_k = \prod_{j=1}^{k} p_j$. For every $n \in \mathbb{N}$, and $x \in [0,1]$ the following (unique) expansions are true:

$$n = \sum_{j=1}^{k} \alpha_j m_{j-1} \text{ and } x = \sum_{j=1}^{\infty} \frac{x_j}{m_j},$$

where $0 \leq \alpha_j \leq p_j - 1$; $0 \leq x_j \leq p_j - 1$, $\forall j \in \mathbb{N}$, and $m_{k-1} \leq n < m_k$.

The Vilenkin's system is defined as follows:

$$W_0(x) \equiv 1; W_{m_{j-1}}(x) = exp\left(2\pi i \frac{x_j}{p_j}\right); W_n(x) = \prod_{j=1}^{k} (W_{m_{j-1}}(x))^{\alpha_j}$$

Particularly, for $P \equiv \{2, 2, \ldots, 2, \ldots\}$ the Vilenkin's system coincides with the Walsh system.

The following theorems are true:

Theorem 1. *For any $p \geq 1, 0 < \epsilon < 1$, and each function $f \in L^p[0, 1)$ one can find a function $g \in L^p[0, 1)$ with $mes\{x \in [0, 1) : g \neq f\} < \epsilon$ such that the sequence $\{|c_k(g)| : k \in spec(g)\}$ is monotonically decreasing, where $\{c_k(g)\}$ is the sequence of Fourier coefficients of g with respect to the Vilenkin's system.*

Theorem 2. *For any $0 < \varepsilon < 1$ there exists a measurable set $E \subset [0, 1]$ with measure $|E| > 1 - \varepsilon$ such that for each function $f(x) \in L^1(0, 1)$ one can find a function $g(x) \in L^1(0, 1)$, which coincides with $f(x)$ on E, and that Fourier-Vilenkin series of g converges to g in $L^1(0, 1)$-norm, and all nonzero elements of the sequence $\{|c_n(g)|\}$ are monotonically decreasing, where $\{c_n(g)\}$ are the Fourier-Vilenkin coefficients of g.*

Remark . *Note that the greedy algorithm for the modified function g with respect to the Vilenkin's system converges in $L^p[0, 1)$-norm, $p \geq 1$. Note also that in [1] - [3] the Theorems 1,2 are proved for the Walsh system.*

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Orthogonal spline-projector and related questions

A. Shadrin (University of Cambridge, UK)
A.Shadrin@damtp.cam.ac.uk

We will make an overview of some problems related to de Boor's 1972 conjecture (and its subsequent proof in 2001) on the max-norm boundedness of the orthogonal spline-projector of degree k. We start with the problems which were initial points of interest, namely construction of orthogonal bases in the spaces of continuous and r times differentiable functions (Ciesielski, Domsta), error of spline interpolation (de Boor, Subbotin), Galerkin approximation to solutions of boundary value problems (Douglas, Dupont, Wahlbin). We then describe several alternative approaches that were used in various particular cases (e.g., quadratic and cubic splines), as well as the key ideas of the general proof. Finally, we discuss developments that happened after the proof. They include: boundedness of the spline interpolation operator in C^{k-1} (Volkov, note that de Boor's conjecture is equivalent to C^k-boundedness), $\sqrt{k}$-bound for the actual value of the max-norm for splines with low smoothness (Foucart, Kayumov), rigorous extension to splines on infinite and periodic knot-sequences (de Boor), a counterexample for the linear splines on triangulations (Oswald), and some other.

Biorthogonal p-wavelet packets on a half-line

F. Shah (University of Kashmir, India)
fashah79@gmail.com

This paper deals with a construction of biorthogonal p-wavelet packets on a positive half-line $\mathbb{R}^+$ and their properties are characterized by means of Walsh-Fourier transform. Three biorthogonal formulas regarding these p-wavelet packets are derived. Moreover, it is shown how to obtain several new Riesz bases of the space $L^2(\mathbb{R}^+)$ by constructing a series of subspaces of these p-wavelet packets.

A weak invertibility criterion in the weighted space of holomorphic function

F. Shamoyan (Bryansk State University, Russia)
shamoyanfa@yandex.ru

We study the question of weak invertibility in weighted L_p spaces of holomorphic functions in a polydisc. We obtain a necessary and sufficient condition on the weight function under which every nowhere vanishing holomorphic function in the polydisc in the weighted L_p-spaces is weakly invertible in the corresponding L_q-space for all q.

On some new holomorphic area Nevanlinna-type spaces in circular rings on the complex plane

R. Shamoyan (Bryansk State Technical University, Russia)
rsham@mail.ru

In [1-4] we have introduced several new holomorphic area Nevanlinna-type spaces in the unit disk, unit polydisk and halfplane on a complex plane $\mathbb{C}$. In those paper we considered and solved various problems in those classes of functions such as descriptions of zero sets, parametric representations, descriptions of closed ideals, the action of differentiation operator.

The goal of this work will be to extend some results from these papers to the case of area Nevanlinna classes in circular rings on the complex plane.

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Strong proximinality of closed convex subsets

P. Shunmugaraj (Indian Institute of Technology Kanpur, India)
psraj@iitk.ac.in

We show that in a Banach space X every closed convex subset is strongly proximinal if and only if the dual norm is strongly sub- differentiable and for each norm one functional f in the dual space X^*, $J_X(f)$ - the set of norm one elements in X where f attains its norm, is compact. As a consequence, we prove that every closed convex subset is strongly proximinal in a Banach space X if and only if X is reflexive and the relative weak and norm topologies coincide on the unit sphere S_X of X. As an application of these results, it is observed that if the dual norm is strongly sub- differentiable, then every closed convex subset of X is strongly proximinal if and only if the metric projection onto every closed convex subset of X is upper semi-continuous.

Best m-Term Approximation and Tensor Products of Sobolev and Besov Spaces

W. Sickel (Friedrich-Schiller-University Jena, Germany)
winfried.sickel@uni-jena.de

The main topic of my lecture will be the asymptotic behaviour of the widths of best m-term approximation with respect to embeddings of tensor products of Sobolev as well as Besov spaces into L_p spaces. However, a part of the lecture will be also devoted to the description of those tensor products. In almost all cases our approach leads to final results.

Multivariate wavelet frames and frame-like systems

M. Skopina (St. Petersburg State University, Russia)
skopina@MS1167.spb.edu

In order to construct a wavelet frame with a desirable approximation order, it is necessary to provide vanishing moments for the generating wavelet functions. In the multi-dimensional case this problem is much more complicated than its one-dimensional version. In particular, two open algebraic problems are obstacles for the construction of compactly supported multivariate tight wavelet frames. Namely, first, it is not known if any appropriate row can be extended to a unitary matrix whose entries are trigonometric polynomials. Second, it is not known if any nonnegative trigonometric polynomial can be represented as a finite sum of squared magnitude of trigonometric polynomials. We suggest a way to get around these obstacles and give a constructive method for the improvement an arbitrary appropriate mask to obtain a scaling mask generating a compactly supported tight wavelet frame with a required approximation order. A method for the construction of dual wavelet frames is also developed. It appears that frame-type decompositions hold for some MRA-based wavelet systems which are not frames in L_2. We study frame-type decompositions and their approximation order in a more general situation.

Coxeter system of lines are sets of injectivity for twisted spherical means on $\mathbb{C}$

R. Srivastava (Harish-Chandra Research Institute, India)
rksri@hri.res.in

It is well known that any line in $\mathbb{R}^2$ is not a set of injectivity for the spherical means for the odd functions about that line. We prove that any line passing through the origin is a set of injectivity for the twisted spherical means (TSM) for functions $f \in L^2(\mathbb{C})$, whose each spectral projection $e^{\frac{1}{4}|z|^2} f \times \varphi_k$ is a polynomial. Then, we prove that any Coxeter system of even number of lines is a set of injectivity for the TSM for $L^q(\mathbb{C})$, $1 \leq q \leq 2$. The problem that any Coxeter system of odd number of lines can be a set of injectivity for the TSM for $L^q(\mathbb{C})$, $1 \leq q \leq 2$ is still open.

These results are quite explicit and adverse to the known result for the spherical means on $\mathbb{R}^2$, due to Agranovsky and Quinto (1996). Our result reveals that the Agranovsky-Quinto conjecture, “the sets of non-injectivity for the spherical means on $\mathbb{R}^n$ $(n \geq 2)$ are contained in a certain algebraic variety” does not continue to hold for the spherical means on the Heisenberg group $\mathbb{H}^1 = \mathbb{C} \times \mathbb{R}$.

Trigonometric Pade approximants for functions with regularly decreasing Fourier coefficients

A. P. Starovoitov and Yu. A. Labych (Gomel State University, Belarus)
svoitov@gsu.by, jlabych@yandex.ru

We consider real, continuous, 2π-periodic functions f expanded in a convergent Fourier series:

$$f(x) = \frac{a_0}{2} + \sum_{k=1}^{\infty}(a_k \cos kx + b_k \sin kx), \tag{1}$$

where the Fourier coefficients a_k and b_k are real numbers.

Let $\mathcal{R}^t_{n,m}$ be the class of trigonometric rational functions $r(x) = \frac{p_n(x)}{q_m(x)}$ where the numerator $p_n(x)$ and denominator $q_m(x)$ are trigonometric polynomials with real coefficients such that $\deg p_n \leqslant n$, $\deg q_m \leqslant m$. We define the *best uniform trigonometric rational approximations* to f to be:

$$\mathrm{R}^t_{n,m}(f) := \inf\{||f - r|| : r \in \mathcal{R}^t_{n,m}\},$$

where $||g|| = \max\{|g(x)| : x \in [0, 2\pi]\}$.

By a *trigonometric Padé approximant* to a function f we shall mean a rational function $\pi^t_{n,m}(x; f) = p^t_n(x)/q^t_m(x)$ in $\mathcal{R}^t_{n,m}$, where the numerator and denominator satisfy the condition

$$q^t_m(x)f(x) - p^t_n(x) = \sum_{k=n+m+1}^{\infty}(\widetilde{a}_k \cos kx + \widetilde{b}_k \sin kx).$$

Sufficient conditions describing the regular decrease of the coefficients of a Fourier series (1) are found which ensure that the trigonometric Padé approximants $\pi^t_{n,m}(x; f)$ converge to the function f in the uniform norm at a rate which coincides asymptotically with the highest possible one (see [1]):

Theorem. *Let f be a function in $T^{\alpha}_{\beta}(q)$, $\alpha \in \mathbb{N}$, $\beta \geqslant 1$, $q \in \mathbb{R}$. If*

$$\lim_{n\to\infty}(m(n))^{2+\beta}/n = 0,$$

then for all $x \in \mathbb{R}$, uniformly for all $x \in \mathbb{R}$ uniformly for m, $0 \leqslant m \leqslant m(n)$,

$$\mathrm{R}^t_{n,m}(f) \sim \|f - \pi^t_{n,m}(\cdot; f)\| \sim m!\, |a_{n+1}| \left(\frac{\alpha |q|}{n^{\alpha+1}} \right)^m$$

as $n \to \infty$.

The results obtained are applied to problems dealing with finding sharp constants for rational approximations.

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On recovery problems

N. Temirgaliyev (L.N. Gumilyov Eurasian National University, Institute of theoretical mathematics and scientific research, Kazakhstan)
temirgaliyev_nt@enu.kz

The following is the general definition in the statement of the recovery problem of the operator Tf by fuzzy information about $f = (f_1, ..., f_n) \in F$:

$$\delta_{\mathrm{N}}\left(\varepsilon^{(N)}\right) = \inf_{\left(l^{(N)}, \varphi_N\right) \in D_N} \sup \Big\{ \|Tf - \varphi_N(z_1, ..., z_k; \cdot)\|_Y ;$$

$$(z_1, ..., z_k) : \left| l^{(i)}_j(f) - z^{(i)}_j \right| \leq \varepsilon^{(i)}_j,\ j = 1, .., k; i = 1, ..., N_j \Big\} \tag{1}$$

where $N = N_1 + ... + N_k$ ($k, N_1, ..., N_k$ are positive integers), D_N is a given set of pairs $\left(l^{(N)}, \varphi_N\right)$, consisting of N informational functionals $l^{(N)} =$

$\left(l_1^{(1)}, ..., l_1^{(N_1)}, ..., l_k^{(1)}, ..., l_k^{(N_k)}\right)$ and of an algorithm

$$\varphi_N \left(z_1^{(1)}, ..., z_1^{(N_1)}, ..., z_k^{(1)}, ..., z_k^{(N_k)}; \cdot\right),$$

which processes the fuzzy information into a function of the same variable, as the approximable operator Tf (for details see [1]).

Further, the optimization problems in [1-5] are distinguished by their statements, and, interestingly, by notations for essentially the same mathematical objects, and also by formulation of the results.

A great deal of the work on this theme has been accomplished by J. Traub, H. Wozniakowski, L. Plaskota (see [2,3] and references therein), their coauthors and successors, where the main focus of the research was on the problem of minimization of the total *cost* of determination of approximate values $z_j^{(i)}$ (information "noise") in (1). Also, a big deal of attention is paid to interrelations between different concretizations of the general recovery problem (1).

Another direction of research is represented in the works of V.M. Tikhomirov, G. G. Magaril-Ilyaev, K. Yu. Osipenko and A. G. Marchuk (cf. [4]), where *exact solutions* for (1) are given.

And, finally, the approach to the solution of the problem of optimal recovery by fuzzy information, formulated in the form of "Computer (computational) diameter", consists of consecutive realization of the following three operations (cf. [1] and [5] and references therein):

1^0. Determination of $\succ\prec\ \delta_N(0)$; 2^0. Determination of $\{\tilde{\varepsilon}_N\}$ such that $\delta_N(0) \succ\prec \delta_N(\tilde{\varepsilon}_N)$, with simultaneously realization of 3^0. $\forall \eta_N \uparrow +\infty$: $\overline{\lim\limits_{N\to+\infty}} \frac{\delta_N(\tilde{\varepsilon}_N\eta_N)}{\delta_N(\tilde{\varepsilon}_N)} = +\infty$.

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Lebesgue-type inequalities for greedy approximation with respect to bases

V. Temlyakov (University of South Carolina, USA)
temlyak@math.sc.edu

I will discuss Lebesgue-type inequalities for greedy algorithms in Banach spaces. By the Lebesgue-type inequality I mean an inequality that provides an upper estimate for the error of a particular method of approximation of f (greedy algorithm in our case) by elements of a special form, say, form $\mathcal{A}$ (m-term approximant in our case), by the best-possible approximation of f by elements of the form $\mathcal{A}$.

I will begin with Lebesgue-type inequalities for greedy approximation with respect to specific classical systems: trigonometric, univariate Haar, multivariate Haar. Then I will consider unconditional bases and quasi-greedy bases. Some open problems will be discussed.

Metric and structural properties of harmonic measures

V. Totik
(University of Szeged, Hungary and University of South Florida, USA)
totik@mail.usf.edu

The talk discusses some properties of harmonic measures on infinitely connected domains. Sharp estimates will be given along with their applications to polynomial inequalities and polynomial approximation of $|x|$ on compact subsets of $\mathbb{R}$. Some further structural properties like convexity of harmonic densities will also be discussed.

Mean value theorem for polyanalytic polynomials

O. Trofymenko (Donetsk National University, Ukraine)
odtrofimenko@gmail.com

In my talk I will discuss about polyanalytic functions and special mean value equations.

On Riemann's mapping theorem

A. Vagharshakyan (Institute of Mathematics of NAS, Armenia)
vagharshakyan@yahoo.com

We introduce a new family of three-dimensional mappings, named weak - conformal and obtain a more natural generalization of Riemann's mapping theorem than the existing one for quasiconformal mappings. Note that conformal mappings in three dimensions form a rather narrow family (Liouville, see [1]).

When specified for two dimensions, our result provides a new proof of Riemann's mapping theorem. Recall that (see [2]):

Definition 1. A quasi-conformal mapping is a continuously differentiable homeomorphism

$$\varphi : \Omega_1 \to \Omega_2$$

for which an infinitesimal ball is mapped to an infinitely small ellipsoid the ratios of the main diagonals of which are uniformly bounded.

Definition 2. A weak-conformal mapping is a continuously differentiable homeomorphism

$$\varphi : \Omega_1 \to \Omega_2$$

for which an infinitesimal ball is mapped to an infinitely small ellipsoid the main diagonals of which form a geometric progression.

Definition 3. We say that a domain $\Omega \subset R^3$ is simply connected if

1. for an arbitrary bounded domain $\Omega_1 \subset R^3$, if $\partial\Omega_1 \subset \Omega$ then $\Omega_1 \subset \Omega$,

2. an arbitrary closed curve laying in domain Ω permits continuous deformation in domain Ω to the point.

Theorem. *Let Ω_1 and Ω_2 be simply connected, bounded domains in R^n, where $n = 2, 3$, with the smooth boundaries. Let $x_1 \in \Omega_1$ and $x_2 \in \Omega_2$ be some points.*

Then there is an one-to-one weak - conformal mapping

$$\varphi : \Omega_1 \to \Omega_2$$

of the domain Ω_1 onto the Ω_2 for which $\varphi(x_1) = x_2$.

Remark. Let $G_1(x, x_1)$ and $G_2(x, x_2)$ be Green's functions for those domains. Then for the constructed mapping we have

$$q(\vec{x}) = \lim_{r\to 0} \max_{y,z} \left\{ \frac{||\varphi(\vec{y}) - \varphi(\vec{x})||}{||\varphi(\vec{z}) - \varphi(\vec{x})||};\ ||x-y|| = ||z-x|| = r \right\} \leq$$

$$\leq \max_{y,z} \left\{ \left(\frac{||\nabla G_2(z, x_2)||}{||\nabla G_1(y, x_1)||} \right)^{\frac{n-2}{n-1}};\ G_1(y, x_1) = G_2(z, x_2) \right\},\ \ x \in \Omega_1.$$

Note that if $n = 2$ then $q \equiv 1$, which means that our mapping is conform.

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Bellman functions for extremal problems on *BMO*

V. Vasyunin
(St.Petersburg Department of the Steklov Mathematical Institute, Russia)
vasyunin@pdmi.ras.ru

There are few classical *BMO*-problems that are solved by Bellman function method (integral and weak forms of the John–Nirenberg inequality,

L^p-estimates of the BMO-functions). From these works (made mostly together with L. Slavin) it becomes clear that it is possible to formulate rather general method of solving a big class of extremal problems for integral functionals on BMO. The work is still in progress, but I would like to describe some results in that direction.

Sparse functions

P. Wojtaszczyk (University of Warsaw, Poland)
wojtaszczyk@mimuw.edu.pl

By a *sparse function* we understand a function f of N variables which effectively depends only on $k << N$ variables. More formally it means that either $f(x_1, \dots, x_N) = g(x_{j_1}, \dots, x_{j_k})$ for an unknown function g of k variables and an unknown sequence $j_1, \dots, j_k$ or $f(x_1, \dots, x_N) = g(A * (x_1, \dots, x_N))$ for an unknown function g of k variables and an unknown $N \times k$ matrix A. The problem is to effectively approximate such function f (under suitable assumptions) using point values.

In the talk I will review recent results on this problem and indicate possible open questions.

Mixed problem for non-self-adjoint degenerate differential equations of fourth order

E. Yousefi (Azad Islamic University of Karraj, Iran)
u3phi@kiau.ac.ir

We consider the following differential equation

$$Lu \equiv (t^\alpha u'')'' + au''' + pu = f, \tag{1}$$

where $t \in [0, b], 0 \leq \alpha \leq 4,\ a, p = const, a > 0$ and $f \in L_2(0, b)$.

Denote by $W_\alpha^2(0)$ the completion of

$$\dot{C}^2 = \{u \in C^2[0, b], u(0) = u'(0) = 0\}$$

in the norm $||u||^2_{W_\alpha^2(0)} = \int_0^b \left(t^\alpha |u''(t)|^2 + |u(t)|^2\right) dt$. Note that for $0 \leq \alpha \leq 4$ there is a continuous embedding $W_\alpha^2(0) \subset L_2(0, b)$, which is compact for $0 \leq \alpha < 4$.

Definition 1. *We say that $u \in W_\alpha^2(0)$ is a generalized solution of the equation (1), if for every $v \in W_\alpha^2(0)$ the following equality is valid:*

$$(t^\alpha u'', v_h'') - a(u'', v_h') + p(u, v_h) = (f, v_h).$$

Here $v_h(t) = v(t)\psi_h(t)$, where $\psi_h(t) = 0$ for $t \in [0, h]$, $\psi_h(t) = 1$ for $t \in [2h, b]$ and $\psi_h(t) = h^{-3}(t - h)^2(5h - 2t)$ for $t \in [h, 2h]$.

We prove that for $p = 0$ the generalized solution of the mixed problem (1) in case $0 \leq \alpha < 3$ exists and is unique for every $f \in L_2(0, b)$ and the inverse operator $L^{-1} : L_2(0, b) \to L_2(0, b)$ is compact. The number p in (1) we can consider as spectral parameter for the operator L. The domain of definition $D(L)$ of the operator L consists of the functions $u(t)$, for which $u(0) = 0$, $u'(0)$ is finite and is defined by f, but can not be given arbitrarily. Note that for $3 \leq \alpha \leq 4$ the point spectrum $\sigma(L) = \sigma_p(L) = \mathbb{C}$.

Now we consider the differential equation

$$Nv \equiv (t^\alpha v'')'' + av''' + pv = g, a < 0, g \in L_2(0, b). \tag{2}$$

Definition 2. *We say that $v \in L_2(0, b)$ is a generalized solution of the equation (2), if for every $u \in D(L)$ the equality $(Lu, v) = (u, g)$ is valid.*

We prove that the generalized solution of the mixed problem for the equation (2) for $0 \leq \alpha < 3$ exists and is unique for every $g \in L_2(0, b)$, but, in contrast to the equation (1) (where $a > 0$), the domain of definition $D(N)$ consists of the functions $u(t)$ for which $u(0) = u'(0) = 0$. Thus, the statement of the mixed problem for the equation (1) depends on the sign of the coefficient of the third derivative (Keldysh's effect). Note that for $0 \le \alpha < 3$ the inverse operator N^{-1} is compact. For $3 \le \alpha \le 4$ the residual spectrum of the operator N coincides with the whole complex plane $\mathbb{C}$.

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