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C O N T E N T S

ABSTRACTS

Exotic options under Levy processes

Rossella Agliardi (University of Bologna, Italy)
rossella.agliardi@unibo.it

In this work some pricing formulas are obtained for several exotic options when regular Lévy processes of exponential type (RLPE) are adopted as the driving processes. The development of pricing models replacing the traditional underlying source of randomness, the Brownian motion, by a Lévy process has fostered a good deal of work on exotic option pricing which parallels the existing results in the Gaussian framework. In our work the focus is on the pricing of European exotics and the aim is to present a valuation formula which is the most comprehensive as possible, in that several types of options can be priced directly and no specific method has to be devised for each of them. The formula is tailored to valuate discretely monitored options, which are the most popular ones in view of the regulatory issues and the trading practice. However, the continuous counterpart can be derived in some cases. This work employs a general framework and provides new pricing formulas for exotic option prices. Since a Brownian motion is a RLPE of order 2 and any exponential type - and thus is captured in this framework - each example includes the known pricing expressions of the classical Gaussian modeling. An extension is also provided to more general Feller processes and an approximate pricing formula is obtained by employing the fundamental solution to the Cauchy problem for some pseudo-differential operators.

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A Coherent Measure of Risk for Multivariate Portfolios

Rafik Aramyan
(Institute of Mathematics NAS and
Russian-Armenian State University, Armenia)
rafikaramyan@yahoo.com

Measures of risk are widely used in financial engineering to assess the risk of investments and to optimise the capital allocation. The modern theory of coherent risk measures (see [1],[2],[3]) aims to derive properties of risk measures from several basic axioms: translation-invariance, monotonicity, homogeneity, and convexity. The risk measures are mostly considered in the univariate case. When assessing risks of multivariate portfolios, the situation becomes more complicated.

Let X be a random variable in $\mathbb{R}^1$ that is the changes of values between two dates $X = \Delta P_i$.

Definition. *A measure of risk is a function $R(\Delta P_i)$. A measure of risk satisfying the following axioms is called coherent (consistent):*

1. *Translation invariance:* $R(\Delta P_i + \Theta) = R(\Delta P_i) - \Theta$.

2. *Subadditivity:* $R(\Delta P_{i1} + \Delta P_{i2}) \leq R(\Delta P_{i1}) + R(\Delta P_{i2})$.

3. *Positive Homogeneity:* $R(\lambda \Delta P_i) = \lambda R(\Delta P_i)$ *if* $\lambda > 0$.

4. *Monotonicity: If* $\Delta P_{i1} \leq \Delta P_{i2}$ *a.s., then* $R(\Delta P_{i2}) \leq R(\Delta P_{i1})$.

In [1] was shown that the function (expected shortfall)

$$R_\alpha(X) = -\frac{1}{\alpha}\int_0^\alpha Q(t)dt$$

is a coherent measure of risk. Here Q signifies the quantile function of ΔP_i, $0 \leq \alpha \leq 1$.

Now let X be a random vector in $\mathbb{R}^n$ that represents a financial gain for multivariate portfolios. We define (see [2])

$$H(p) = -\frac{1}{\alpha}\int_0^\alpha Q_{(p,X)}(t)dt, \quad p \in R^n, \tag{1}$$

here $Q_{(p,X)}$ signifies the quantile function of scalar product (p, X). By $\mathbf{B} \subset \mathbb{R}^n$ we denote the unique convex body (well known fact from convex geometry) whose support function is given by (1). By $d(\mathbf{B})$ we denote the diameter of $\mathbf{B}$. Now we consider the function $R_\alpha(X) = d(\mathbf{B})$ as a measure of risk of X.

Theorem. *The measure of risk $d(\mathbf{B})$ is subadditive, positive homogeneous and monotonic.*

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Markovian Projection of Stochastic Processes

Amel Bentata (Universität Zürich, Switzerland)
amel.bentata@math.uzh.ch

We give conditions under which the flow of marginal distributions of a discontinuous semimartingale can be matched by a Markov process whose infinitesimal generator can be expressed in terms of its local characteristics, generalizing a result of Gyongy (1986) to the discontinuous case. Our construction preserves the martingale property and allows to derive a partial integro-differential equation for the one-dimensional distribution of discontinuous semimartingales, extending the Kolmogorov forward equation (Fokker Planck equation) to a non-Markovian setting.

Dynamic quasi concave performance measures

Sara Biagini (University of Pisa, Italy)
sara.biagini@gmail.com

We define conditional quasi concave performance measures (CPMs), to accommodate for additional information. A particular care is put in the selection of the continuity property that CPMs must satisfy. Our notion encompasses a wide variety of cases, among which conditional acceptability indexes. We provide the characterization of a CPM in terms of a minimal family of conditional convex risk measures. In the case of indexes these risk measures are coherent. Then, dynamic performance measures (DPMs) are introduced and the problem of time consistency is addressed. The definition of time consistency chosen here ensures that the positions which are considered good tomorrow are already considered good today. Finally, we prove the equivalence between time consistency for a DPM and a type of weak acceptance consistency for the associated minimal families of risk measures. This is joint work with J. Bion-Nadal.

Dynamic Risk Measures and PDE
Application to a Stochastic Volatility Model

Jocelyne Bion-Nadal (CMAP, Ecole Polytechnique, France)
jocelyne.bion-nadal@cmap.polytechnique.fr

We show how the theory of dynamic risk measures provides viscosity solutions to a family of second-order parabolic partial differential equations, even in the degenerate case. Motivated by the martingale problem approach of Stroock and Varadhan, we make use of the time consistency characterization for dynamic risk measures, to construct time consistent convex Markov processes. This is done in a general setting in which compacity arguments cannot be used, and for which there does not always exist an optimal control. We prove that these processes lead to viscosity supersolutions, and even to viscosity solutions under some conditions, for second-order partial differential equations with convex generator.

We give an application to mathematical finance: We show that our results allow for the construction of a No Arbitrage Pricing Procedure in the context of stochastic volatility. Within this approach, convexity takes into account liquidity risk.

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Wealth distribution in Asset Exchange Models

Bruce M. Boghosian (American University of Armenia, Armenia)
Bruce.Boghosian@aua.am

Asset Exchange Models (AEMs) are possibly the simplest mathematical models of free-market economies imaginable. In spite of their simplicity, they are capable of explaining many interesting features of real economies, including Pareto's law of wealth distribution. This talk will describe AEMs as a model of exchange amongst discrete agents, and derive integrodifferential and differential equations governing wealth distribution in AEMs.

Functional Kolmogorov equations

Rama Cont (CNRS - Université de Paris VI, France)
rama.cont@gmail.com

We introduce a new class of infinite dimensional partial differential equations on the space C_0 of continuous functions which have properties analogous to finite dimensional parabolic PDEs and extend the Kolmogorov backward equation to a non-Markovian, path-dependent setting.

We prove a comparison principle for these infinite dimensional equations and show that they bear a close relation to Forward-Backward stochastic differential equations.

The proofs are based on the Functional Itô calculus (Dupire 2009, Cont & Fournie 2009, 2010).

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Small Noise Limit for Mild Solution of Stochastic Evolution Equations with Monotone Nonlinearity and Multiplicative Levy Noise

Hassan Dadashi
(Institute for Advanced Studies in Basic Sciences, Iran)
dadashi@iasbs.ac.ir

The study of stochastic partial differential equations is ubiquitous in many fields such as physics, financial mathematics, biology, etc. Among the qualitative properties of their solution, large deviation principle (LDP) has an important consideration especially in computations of rare events and studying the phase transition in physical systems. Explicitly, LDP notifies the convergence of solutions, when the intensity of noise converges to zero.

In this work, we study the LDP of the mild solution of a class of SPDEs with multiplicative Lévy noise and monotone nonlinearity. The traditional method in proving the LDP, using the contraction principle, involves many complicated functional inequalities. We use a recently developed method, the weak convergence method, which is based on the variational representation of moments of positive functional of the noise. To establish the LDP in this method, we must prove some sort of stability when a sequence of control functions is added to the noise. An Itô-type and the Burkholder-Davis-Gundy inequality are the essential tools in the arguments.

A characterization of subperfect Nash equilibria and application to portfolio choice

Boualem Djehiche (KTH, Sweden)
boualem@math.kth.se

I will review a simple characterization of subperfect Nash equilibria associated with dynamics driven by SDEs of mean-field type.

Infinite Horizon FBSDEs: Rational Expectations, Control and Stochastic Viscosity Solutions

Nikolaos Englezos (University of Piraeus, Greece)
englezos@unipi.gr

Motivated by a continuous time rational expectations model with random coefficients, we study a class of fully coupled forward backward stochastic differential equations (FBSDEs) with infinite horizon. Under standard Lipschitz and monotonicity conditions, and by means of the contraction mapping principle, we establish existence, uniqueness and dependence on a parameter of adapted solutions. Making further the connection with quasilinear backward stochastic partial differential equations (BSPDEs), we are led to the notion of stochastic viscosity solutions. A stochastic maximum principle for the optimal control problem of such FBSDEs is also provided as an application to this framework.

Existence, uniqueness, and optimal regularity for degenerate obstacle problems

Paul Feehan (Rutgers University, USA)
feehan@rci.rutgers.edu

Degenerate diffusion processes are widely used as asset price models in mathematical finance. Their generators are second-order degenerate partial differential operators. With the aid of weighted Sobolev and Hölder spaces, we prove existence, uniqueness, and global regularity of solutions to variational inequalities and obstacle problems for degenerate partial differential operators on unbounded domains. In mathematical finance, solutions to obstacle problems for elliptic operators correspond to value functions for perpetual American-style options on the underlying asset while solutions to obstacle problems for parabolic operators correspond to value functions for finite-horizon American-style options on the underlying asset. In addition, we report on our work on regularity of the free boundaries arising in these obstacle problems. This is joint work with Panagiota Daskalopoulos (Columbia) and Camelia Pop (Pennsylvania).

About uniform $\mathcal{O}$-supermartingale decomposition in nonstandard case

Karen Gasparian (Yerevan State University, Armenia)
kargasp@gmail.com

In this note we extend, under some additional conditions, the result of D. Kramkov [2] about uniform optional decomposition of supermartingales proved by S. Jacka [1] to the case when the filtration $\mathbb{F} = (\mathcal{F}_t)_{t\geqslant 0}$ given on a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$ doesn't meet the "usual" conditions.

Let us be given a set $\mathcal{P} = \{\mathbb{Q}\}$ of p.m's on $(\Omega, \mathcal{F})$ such that all $\mathbb{Q} \sim \mathbb{P}$. Denote by $\mathbb{Z}^{\mathbb{Q}} = \dfrac{d\mathbb{Q}}{d\mathbb{P}} > 0$. According to [3], there exists a unique optional $\mathcal{O}$ modification $\left(\mathbb{Z}_t^{\mathbb{Q}}\right)$ of a martingale $\left(\mathbb{E}\left(\mathbb{Z}^{\mathbb{Q}} \middle| \mathcal{F}_t\right)\right)$ (so-called $\mathcal{O}$-martingale) such that $\mathbb{Z}_T^{\mathbb{Q}} = \mathbb{E}\left(\mathbb{Z}^{\mathbb{Q}} \middle| \mathcal{F}_T\right)$ for all $\mathbb{F}$-stopping times T ($\left(\mathbb{Z}_t^{\mathbb{Q}}\right) \in \mathcal{M}(\mathbb{P})$).

We define the process ("stochastic logarithm")

$$\lambda_t^{\mathbb{Q}} = \int\limits_{]0,t]} \frac{d\left(\mathbb{Z}^{\mathbb{Q}}\right)_s^r}{\mathbb{Z}_{s-}^{\mathbb{Q}}} + \int\limits_{[0,t[} \frac{d\left(\mathbb{Z}^{\mathbb{Q}}\right)_{s+}^g}{\mathbb{Z}_s^{\mathbb{Q}}} \in \mathcal{M}_{loc}(\mathbb{P}) ,$$

where $\mathbb{Z}^{\mathbb{Q}} = \left(\mathbb{Z}^{\mathbb{Q}}\right)^r + \left(\mathbb{Z}^{\mathbb{Q}}\right)^g$ (see [4]).

Denote by Λ the set $\left\{\lambda^{\mathbb{Q}} : \mathbb{Q} \in \mathcal{P}\right\}$ and by Λ_{loc} the localizing class of Λ.

Theorem (cf. [1]). *If $\mathbb{P} \in \mathcal{P}$ and the space Λ_{loc} is closed under scalar multiplication, then any non-negative $\mathcal{O}$-supermartingale X w.r. $\mathbb{Q}$ for all $\mathbb{Q} \in \mathcal{P}$ admits a unique (up to indistinguishability) Doob-Meyer decomposition (see [5])*

$$X = M - A,$$

where $M \in \mathcal{M}_{loc}(\mathbb{Q})$ is a local $\mathcal{O}$-martingale for all $\mathbb{P} \in \mathcal{P}$ and $A \in \mathcal{A}_{loc}^{+} \cap \mathcal{P}_s$ is a locally integrable increasing strong predictable process with $A_0 = 0$.

Remark. In finance the process X is interpreted as a capital process of portfolio, M is an assets price of portfolio and A is a process of consumptions.

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Variational and quasi-variational mean-field games

Diogo Gomes (Instituto Superior Técnico, Portugal)
dgomes@math.ist.utl.pt

Mean field games were introduced by Lions and Lasry in a series of seminal papers, and independently, around the same time, by Caines and his co-workers. Mean field games model arise in the modelling systems with a large number of rational agents by dynamic non-cooperative games. These problems give rise to coupled systems of Hamilton-Jacobi-Focker-Plank equations, together with non-standard initial-terminal boundary conditions. Surprisingly, an important class of mean-field games can be regarded as an Euler-Lagrange equation of certain (non-coercive) functionals. We will present various techniques to establish a-priori estimates and existence of smooth solutions.

General intensity shape for optimal liquidation with limit orders

OLIVIER GUÉANT (Université Paris-Diderot, France)
olivier.gueant@ann.jussieu.fr

We address portfolio liquidation using a new angle. Instead of focusing only on the scheduling aspect like Almgren and Chriss, or only on the liquidity-consuming orders like Obizhaeva and Wang, we link the optimal trade-schedule to the price of the limit orders that have to be sent to the limit order book to optimally liquidate a portfolio. Most practitioners address these two issues separately: they compute an optimal trading curve and they then send orders to the markets to try to follow it. The results obtained in this paper can be interpreted and used in two ways: (i) we solve simultaneously the two problems and provide a strategy to liquidate a portfolio over a few hours, (ii) we provide a tactic to follow a trading curve over slices of a few minutes. As far as the model is concerned, the interactions of limit orders with the market are modeled via a point process pegged to a diffusive "fair price". Given the general shape of the intensity, we generalize the risk-neutral model of Bayraktar and Ludkovski and a model we developed earlier restricted to exponential intensity.

Portfolio Selection: A Stochastic Analysis

Chiang Kao (National Cheng Kung University, Taiwan)
ckao@mail.ncku.edu.tw

The conventional portfolio selection problem concerns maximizing the portfolio return at a given level of portfolio risk, or, equivalently, minimizing the portfolio risk at a given level of portfolio return, by allocating the available fund to a set of assets. Since the return rate of an asset usually is a random variable, expected values are conventionally used to make the mathematical model tractable. In this paper, the problem is formulated as a stochastic programming model by considering the asset returns as random variables. The efficient frontier in this case is a band expanded from both sides of the curve obtained from the conventional deterministic analysis. At a specific level of portfolio risk, various portfolio returns occur with different probabilities. It is shown that the portfolio return obtained from the deterministic analysis is less than or equal to the expected value of the random portfolio return obtained from the stochastic analysis. Similarly, the Sharpe ratio for compromising the return and risk is also a random variable in the stochastic analysis. It is found that the optimal way for allocating the fund is insensitive to the variation in asset returns. In addition to mathematical derivations, an example is used for illustration.

Backward martingale representation and the existence of a complete equilibrium

Dmitry Kramkov (Carnegie Mellon, USA)
kramkov@andrew.cmu.edu

Let $\mathbb{Q}$ and $\mathbb{P}$ be equivalent probability measures and let ψ be a J-dimensional vector of random variables such that $\frac{d\mathbb{Q}}{d\mathbb{P}}$ and ψ are defined in terms of a weak solution X to a d-dimensional stochastic differential equation. Motivated by the problem of *endogenous completeness* in financial economics we present conditions which guarantee that any local martingale under $\mathbb{Q}$ is a stochastic integral with respect to the J-dimensional martingale $S_t \triangleq \mathbb{E}^{\mathbb{Q}}[\psi|\mathcal{F}_t]$. While the drift $b = b(t,x)$ and the volatility $\sigma = \sigma(t,x)$ coefficients for X need to have only minimal regularity properties with respect to x, they are assumed to be analytic functions with respect to t. We provide a counter-example showing that this t-analyticity assumption for σ cannot be removed. The presentation is based on joint papers with Silviu Predoiu.

An Infinite-Dimensional Interest Rates Term Structure Model: Arbitrage-Free, Realistic and Practical

Victor Lapshin (Higher School of Economics, Moscow, Russia)
victor.lapshin@gmail.com

We present a new infinite-dimensional model of interest rates term structure within the Heath-Jarrow-Morton framework and its infinite-dimensional extension by Filipovic. Usual term structure models (e.g. Nelson-Siegel) don't allow for consistent stochastic dynamics: these models will cause arbitrage when modified to include any stochastic dynamics of their parameters. Usual finite dimensional stochastic models (e.g. Cox-Ingersoll-Ross or Hull-White) cannot offer a flexible enough snapshot yield curve. We "marry" the snapshot fitting possibilities of static models and the temporal variability of dynamic models at the price of going infinite-dimensional. The model is nevertheless fully practicable and applicable on the real data. We present evidence from the Russian bond market before and during the crisis.

Utility Maximization and Hedging in Incomplete Markets and related Backward Stochastic PDEs

Michael Mania (A. Razmadze Mathematical Institute, Georgia)
misha.mania@gmail.com

We study utility maximization problem and the problem of minimization of a hedging error using dynamic programming approach. We consider an incomplete financial market model, where the dynamics of asset prices are described by continuous semimartingale. Under some regularity assumptions we derive backward stochastic PDEs for value functions related to these problems and show that the strategy is optimal if and only if the corresponding wealth process satisfies a certain forward SDE. As examples the cases of power, exponential utilities and the mean-variance hedging problem are considered.

The talk is based on the common work with R. Tevzadze.

On PDE Models for (some) Socio-Economic Problems

Peter Markowich (University of Cambridge, UK)
P.A.Markowich@damtp.cam.ac.uk

We present and analyze

a) a free boundary model for price formation in economic markets

b) a kinetic Boltzmann type equation for opinion formation in human societies

c) a hyperbolic conservation law coupled to an eikonal mean field equation modeling human crowds.

Coupling and tracking of regime-switching martingales

Aleksandar Mijatović (Imperial College London, UK)
a.mijatovic@imperial.ac.uk

This talk will describe two explicit couplings of a pair of standard Brownian motions, which respectively yield the stochastic minimum of the coupling time and the tracking error of two regime-switching martingales. We will show that the optimal strategies for the two control problems, given by the respective stochastic correlations between the two Brownian motions, are explicit in the initial data, identical in size and opposite in sign.

This is joint work with Saul Jacka.

Adjoint expansions in local Levy models

Andrea Pascucci (Università di Bologna, Italy)
andrea.pascucci@unibo.it

We propose a novel method for the analytical approximation in local volatility models with Lévy jumps. The main result is an expansion of the characteristic function in a local Lévy model, which is worked out in the Fourier space by considering the adjoint formulation of the pricing problem. Combined with standard Fourier methods, our result provides efficient and accurate pricing formulas. In the case of Gaussian jumps, we also derive an explicit approximation of the transition density of the underlying process by a heat kernel expansion: the approximation is obtained in two ways, using PIDE techniques and working in the Fourier space. Numerical tests confirm the effectiveness of the method.

Degenerate-parabolic partial differential equations with unbounded coefficients, martingale problems, and a mimicking theorem for Ito processes

Camelia Pop (Rutgers University, USA)
apop@math.rutgers.edu

We solve four intertwined problems, motivated by mathematical finance, concerning degenerate-parabolic partial differential operators and degenerate diffusion processes. First, we consider a parabolic partial differential equation on a half-space whose coefficients are suitably Hölder continuous and allowed to grow linearly in the spatial variable and which becomes degenerate along the boundary of the half-space. We establish existence and uniqueness of solutions in weighted Hölder spaces which incorporate both the degeneracy at the boundary and the unboundedness

of the coefficients. Second, we show that the martingale problem associated with a degenerate elliptic differential operator with unbounded, locally Hölder continuous coefficients on a half-space is well-posed in the sense of Stroock and Varadhan. Third, we prove existence, uniqueness, and the strong Markov property for weak solutions to a stochastic differential equation with degenerate diffusion and unbounded coefficients with suitable Hölder continuity properties. Fourth, for an Itô process with degenerate diffusion and unbounded but appropriately regular coefficients, we prove existence of a strong Markov process, unique in the sense of probability law, whose one-dimensional marginal probability distributions match those of the given Itô process. This is joint work with Paul Feehan.

Option pricing model based on telegraph-like process

Nikita Ratanov (Universidad del Rosario, Colombia)
nikita.ratanov@urosario.edu.co

We consider the market models based on continuous time random motions with alternating constant velocities $c_0,\ c_1$ (so called "telegraph" process) and with jumps $h_0,\ h_1$ (possibly random) occurring at the switching times. While such markets may admit an arbitrage opportunity, the model under consideration is arbitrage-free and complete if directions of jumps in stock prices are in a certain correspondence with their velocity and interest rate behaviour.

In the framework of this model we capture bullish and bearish trends in a market evolution. Values $h_0,\ h_1$ describes sizes of possible crashes, jumps and spikes. Thus, we study a model that is both realistic and general enough to enable us to incorporate different trends and extreme events.

We construct financial market model based on the random processes with finite velocities which possess a simplicity of Black-Scholes model. Replicating strategies for European options are constructed in detail. Explicit formulae for option prices are obtained.

Some peculiarities as memory effects and a detailed description of volatility are discussed also.

Why are quadratic normal volatility models analytically tractable?

Johannes Ruf (University of Oxford, UK)
johannes.ruf@gmail.com

We discuss the class of "Quadratic Normal Volatility" (QNV) models, which have drawn much attention in the financial industry due to their analytic tractability and flexibility. We characterize these models as the ones that can be obtained from stopped Brownian motion by a simple transformation and a change of measure that only depends on the terminal value of the stopped Brownian motion. This explains the existence of explicit analytic formulas for option prices within QNV models in the academic literature. Furthermore, via a different transformation, we connect a certain class of QNV models to the dynamics of geometric Brownian motion.

Portfolio Optimisation under Transaction Costs

Walter Schachermayer (University of Vienna, Austria)
walter.schachermayer@univie.ac.at

We give an overview of some old and some new results on portfolio optimisation under transaction costs. The emphasis will be on an asymptotic point of view when proportional transaction costs tend to zero.

Homogenization and asymptotics with small transaction costs

Halil Mete Soner (ETH Zürich, Switzerland)
mete.soner@math.ethz.ch

The problem of investment and consumption in a market with transaction costs was formulated by Magill & Constantinides in 1976. Since then, starting with the classical paper of Davis & Norman an impressive understanding has been achieved. This problem of proportional transaction costs is a special case of a singular stochastic control problem in which the state process can have controlled discontinuities. The related partial differential equation for this class of optimal control problems is a quasi-variational inequality which contains a gradient constraint. Technically, the multi-dimensional setting presents intriguing free boundary problems.

It is well known that in practice the proportional transaction costs are small and in the limiting case of zero costs, one recovers the classical problem of Merton. Then, a natural approach to simplify the problem is to obtain an asymptotic expansion in terms of the small transaction costs. This was initiated in the pioneering paper of Constantinides. The first proof in this direction was obtained in the appendix of Shreve & Soner.

In this talk, we consider this classical problem of small proportional transaction costs and develop a unified approach to the problem of asymptotic analysis. We also relate the first order asymptotic expansion to an ergodic singular control problem.

This convergence result also provides a connection with homogenization. Indeed, the dynamic programming equation of the ergodic problem is the corrector equation in the homogenization terminology. This identification allows us to construct a rigorous proof similar to the ones in homogenization. Moreover, the ergodic problem is a singular one and its continuation region also describes the asymptotic shape of the no-trade region in the transaction cost problem.

The main proof technique is the viscosity approach of Evans to homogenization. This powerful method combined with the relaxed limits of Barles & Perthame provides the necessary tools. As well known, this approach has the advantage of using only a simple pointwise bound.

An SPDE governing the term structure of option prices

Josef Teichmann (ETH Zürich, Switzerland)
jteichma@math.ethz.ch

We show that parametrizations of volatility surfaces (and even more involved multivariate objects) by time-dependent Levy processes (as proposed by Carmona-Nadtochiy-Kallsen-Kruehner) lead to quite tractable term structure problems. An interesting SPDE of HJM-type arises and we discuss several solution concepts and solutions of it. Finally we show that affine process constitute the generic finite factor solution class of those SPDEs.

Numerical methods for evaluating financial options

Tatiana Vasilyeva (Volgograd State University, Russia)
tatiana_vas@mail.ru

Financial mathematics began to develop rapidly since 1973, when appeared the first article by F. Black and M. Scholes [1], marked by the Nobel Prize. This article [1] had issued in connection with the rapid development of stock trading in securities, especially options. In the paper [1], the authors obtained a differential equation for pricing of financial derivatives (options), depending on the share price. To date, the estimation theory of options has acquired a powerful mathematical tool, which includes the stochastic integro-differential equations and partial differential equations. Methods for solving these equations, taking into account the financial focus of active development of both traditional techniques developed earlier, as well as create new ones.

The financial instruments discussed in the report Call and Put options, which represent the right (but not the obligation) to buy or sell the risky asset at a fixed price within a prescribed period of time. The option price is $V(S,t)$ (here S-asset price, and t-time) and it is an unknown quantity in this problem because option is sold at the moment, when the price of an asset is known, and are executed after the specified time when the price of the asset changes.

Since financial markets are stochastic fluctuations, the natural means to model option prices are stochastic approaches. These methods are based on the formulation and modelling of stochastic differential equations, such as Monte Carlo methods [2, 3].

Binomial methods also take into account the probabilistic nature of the problem. In practice it is often sufficient only to determine the value of $V(S_0,0)$ option at the current spot price of S_0. In this case it may be too calculating the surface $V(S,t)$ throughout the area for information about $V(S_0,0)$. This is a relatively simple problem could be solved by the binomial method. The method is based on a tree-grid, at each point of which apply the binary rules.

Numerical solution of linear and nonlinear Black-Scholes models with respect to $V(S,t)$ by difference methods, the best of which was the method of Crank-Nicholson (symmetric difference scheme) [2, 3, 4], and finite element methods [2] will be presented at the report. The flexibility of the finite element method is particularly advantageous in the case of large-scale spaces for a standard option.

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Asian Options on Harmonic Average

Jan Vecer (Frankfurt School of Finance and Management, Germany)
J.Vecer@fs.de

The contracts written on the harmonic average of the underlying price are becoming increasingly popular in the foreign exchange market. If X denotes the foreign currency and Y denotes the domestic currency, the payoff of the contract is a function of an asset H defined as

$$H(T) = \left[\int_0^T [X_Y(t)]^{-1}\eta(t)dt\right]^{-1} Y(T) = \left[\frac{1}{\int_0^T Y_X(t)\eta(t)dt}\right] Y(T).$$

The harmonic average resembles a quanto option: the price $Y_X(t)$ is monitored with respect to the foreign currency X, but the payoff is settled in the domestic currency Y. Although the pricing problem is rather complex, it can be ultimately simplified to a PDE in one spatial variable after a numeraire change and using the time reversal argument.

Extended mean-field games, existence and uniqueness

Vardan Voskanyan and Diogo Gomes
(Instituto Superior Técnico, Portugal)
vartanvos@gmail.com, dgomes@math.ist.utl.pt

Mean field games is a new class of problems recently introduced by Lions and Lasry, and independently by Caines and his co-workers which attempts to understand the limiting behavior of systems involving large numbers of rational agents which play dynamic games under partial information and symmetry assumptions. We present a reformulation of this problem using random variables approach. This allows us to consider an extension of the original mean-field problem where the interaction a player and the mean field also takes into account the collective behavior of the players but not only its state.

Let $(\Omega, \mathcal{F}, P)$ be a probability space. Assume that the distribution of players is represented by a random variable $X(t) \in L^q(\Omega)$, which is differentiable and has derivative $\dot{X}(t) \in L^q(\Omega)$.

Suppose an individual player knows the distribution of players which is encoded on a trajectory $X(t) \in L^q(\Omega)$ for all times. His or her objective is to minimize a certain performance criterion, determined by a running cost $L : \mathbb{R}^d \times \mathbb{R}^d \times L^q(\Omega) \times L^q(\Omega) \to \mathbb{R}$, and a terminal cost $\psi : \mathbb{R}^d \times L^q(\Omega) \to \mathbb{R}$. We assume that $L(x, v, X, Z)$ depends only on the joint law of (X, Z), and that $\psi(x, X)$ depend only on the law of X. Define the Hamiltonian $H : \mathbb{R}^d \times \mathbb{R}^d \times L^q(\Omega) \times L^q(\Omega) \to \mathbb{R}$ as

$$H(p, x, X, Z) = \sup_{v \in \mathbb{R}^d} \{-v \cdot p - L(x, v, X, Z)\}.$$

Then the extended mean-field model is the following system of a Hamilton-Jacobi equation and an ODE in $L^q(\Omega)$:

$$\begin{cases} -u_t + H(D_x u, x, X, \dot{X}) = 0, \text{ in } [0, T] \times \mathbb{R}^d \\ \dot{X} = -D_p H(D_x u(X, s), X, X, \dot{X}), \text{ in } [0, T] \times \Omega \\ u(x, T) = \psi(x, X(T)), \quad X(0) = X_0. \end{cases}$$

Under certain conditions on X_0, ψ and L we prove that there exist a unique couple (u, X) where $u \in C([0, T] \times \mathbb{R}^d)$ is a viscosity solution to the Hamilton-Jacobi equation:

$$\begin{cases} -u_t + H(D_x u, x, X, \dot{X}) = 0, \text{ in } [0, T] \times \mathbb{R}^d \\ u(x, T) = \psi(x, X(T)), \end{cases}$$

and $X \in C^{1,1}([0, T]; L^q(\Omega))$ is a classical solution to the ODE:

$$\begin{cases} \dot{X} = -D_p H(D_x u(X, s), X, X, \dot{X}), \text{ in } [0, T] \times \Omega \\ X(0) = X_0. \end{cases}$$

Spatial Decay Estimates for Parabolic Integro-Differential Equations

KAREN YERESSIAN (Technische Universität Darmstadt, Germany)
yeressian@mathematik.tu-darmstadt.de

To compute the value of many financial derivatives numerically using PDE or PIDE methods one should compute the solution to a parabolic equation set in an unbounded domain. One common approach is to approximate the solution with solution to an equation set in a bounded domain [MPS], [DRSS]. Then one is interested in the error estimate for this approximation.

To obtain these error estimates one may study the spatial asymptotic behavior of solutions. In the last years financial models based on Lévy stochastic processes have been considered by many researchers. In the talk spatial decay estimates for solutions to parabolic integro-differential equations which arise from Lévy stochastic processes are presented. We prove spatial decay estimates in the form of weighted norms. Our approach allows general decay estimates with few assumptions.

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Calibration of Stochastic Volatility Models by Convex Regularization

Jorge P. Zubelli (IMPA, Brasil)
zubelli@gmail.com

Local volatility models are extensively used and well-recognized for hedging and pricing in financial markets. They are frequently used, for instance, in the evaluation of exotic options so as to avoid arbitrage opportunities with respect to other instruments. The PDE (inverse) problem consists in recovering the time and space varying diffusion coefficient in a parabolic equation from limited data. It is known that this corresponds to an ill-posed problem.

The ill-posed character of local volatility surface calibration from market prices requires the use of regularization techniques either implicitly or explicitly. Such regularization techniques have been widely studied for a while and are still a topic of intense research. We have employed convex regularization tools and recent inverse problem advances to deal with the local volatility calibration problem.

We describe a theoretical approach to calibrate the local volatility surface from quoted European option prices, by introducing in convex regularization techniques, a priori information not only based on the historical variance of the underlying stock price or on the implicit volatility but also on hedging strategies under a Bayesian framework. The novelty of the strategy includes a carefully selected basis together with a pricing mechanism that optimally interpolates computed prices.

We investigate theoretical as well as practical consequences of our methods and illustrate our results both with real and with simulated data. This is joint work with V. Albani (IMPA), A. De Cezaro (FURG), and O. Scherzer (Vienna).