Stochastic and Analytic Methods in Mathematical Physics

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Direct and Inverse Problems in the Theory of Description of Random Fields

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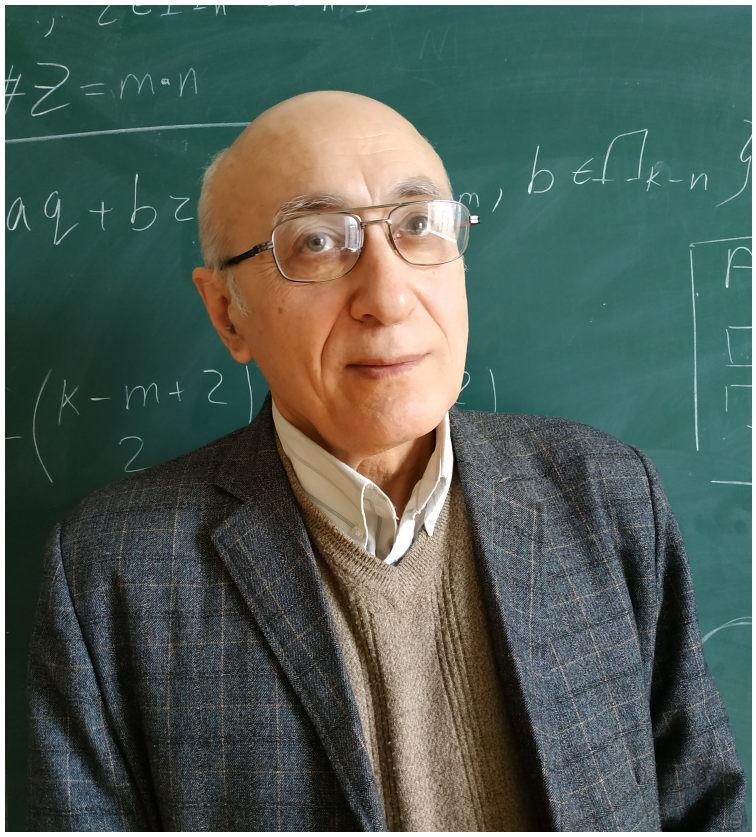
Roland Dobrushin
(20.07.1929 – 12.11.1995)



Victor Arzumanian
(02.06.1948 – 08.04.2019)

Random field and its systems of finite-dimensional distributions

Let P be a random field on $\mathbb{Z}^d$, $d \geq 1$ with finite phase space X

P is an *infinite-dimensional* measure

$\Downarrow$

$\mathscr{P}(P)$ is a system of *finite-dimensional* distributions generated by the random field P

Direct problem is the problem of construction of a system $\mathscr{P}(P)$ of finite-dimensional distributions generated by P, which restores P

$$P \to \mathscr{P}(P)$$

Inverse problem is the problem of the existence of a random field P such that $\mathscr{P}(P) = \mathscr{P}$

$$\mathscr{P} \to P(\mathscr{P})$$

If $\mathscr{P}$ is a unique solution of the inverse problem we say that $\mathscr{P}$ determines a random field

Kolmogorov's system

$\mathscr{P}(P) = \{P_V, V \in W\}$, where

$$P_V(A) = (P)_V(A) = P(AX^{\mathbb{Z}^d \setminus V}), \qquad V \in W$$

W is the set of all finite subsets of $\mathbb{Z}^d$

- $\mathscr{P}(P)$ restores the random field P

Characteristic property of the elements of $\mathscr{P}(P)$ is

$$(P_V)_I = P_I, \qquad I \subset V \in W$$

- Consistent system $\mathscr{P} = \{P_V, V \in W\}$ determines a random field P such that $\mathscr{P}(P) = \mathscr{P}$

In the talk

• For a given positive random field P, we consider the general system $Q(P)$ of its conditional distributions and its subsystems:

system of conditional distributions with conditions in finite volumes $Q^{fin}(P)$

Palm-type system $Q^{\mathbb{1}}(P)$

Dobrushin-type systems $Q^{D}(P)$

systems of one-point conditional distributions $Q_1^{fin}(P)$, $Q_1^{inf}(P)$

• Direct and inverse problem for these systems

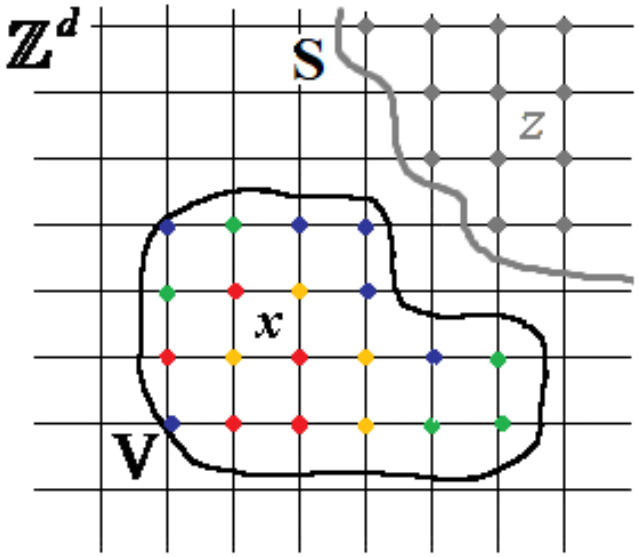
• Gibbs form is universal for conditional distributions

• Direct and inverse problems for Gibbs random fields

Direct problem

For a given positive random field P, **general conditional distribution** is the set

$$Q(P) = \{q_V^z, z \in X^S, S \subset \mathbb{Z}^d \backslash V, V \in W\}$$



where for finite S and all $z \in X^S$

$$q_V^z(x) = \frac{P_{V \cup S}(xz)}{P_S(z)}$$

for not finite S and P-almost all configurations $z \in X^S$

$$q_V^z(x) = \lim_{\Lambda \uparrow S} \frac{P_{V \cup \Lambda}(xz_\Lambda)}{P_\Lambda(z_\Lambda)}$$

Note: $Q(P) = Q^{fin}(P) \bigcup Q^{inf}(P)$

Theorem. *Let P be a random field. Then its general conditional distribution $Q(P)$ restores P.*

Theorem. *Let P be a random field. Then its conditional distribution $Q^{fin}(P)$ with conditions in finite volumes restores P.*

• System $Q_1^{fin}(P)$ of one-point conditional distributions with condition in finite volumes

$$q_t^z(x) = \frac{P_{\{t\}\cup S}(xz)}{P_S(z)}, \qquad V = \{t\}, S \in W, t \notin S$$

Theorem. *Let P be a random field. Then its one-point conditional distribution $Q_1^{fin}(P)$ with conditions in finite volumes restores P.*

• System $Q^{\mathbb{1}}(P)$ of Palm-type conditional distribution with condition at a point

$$q_V^z(x) = \frac{P_{V\cup\{t\}}(xz)}{P_t(z)}, \qquad V \in W, S = \{t\}, t \notin V$$

Theorem. *Let P be a random field. Then its Palm-type conditional distribution $Q^{\mathbb{1}}(P)$ restores P.*

Conditional distribution with infinite conditions $Q^{inf}(P)$

$$q^z_V(x) = \lim_{\Lambda\uparrow S} \frac{P_{V\cup\Lambda}(xz_\Lambda)}{P_\Lambda(z_\Lambda)}, \qquad V \in W, S \notin W, V \cap S = \varnothing$$

Dobrushin-type conditional distribution $Q^D(P)$

$$q^z_V(x) = \lim_{\Lambda\uparrow\mathbb{Z}^d\setminus V} \frac{P_{V\cup\Lambda}(xz_\Lambda)}{P_\Lambda(z_\Lambda)}, \qquad V \in W$$

Dobrushin-type one-point conditional distributions $Q^D_1(P)$

$$q^z_t(x) = \lim_{\Lambda\uparrow\mathbb{Z}^d\setminus\{t\}} \frac{P_{\{t\}\cup\Lambda}(xz_\Lambda)}{P_\Lambda(z_\Lambda)}, \qquad t \in \mathbb{Z}^d$$

Generally speaking, such systems **do not** restore the r.f.

Theorem. *Let P be a random field such that its Dobrushin-type (one-point) conditional distribution $Q^D(P)$ ($Q^D_1(P)$) quasilocal and satisfy Dobrusin's uniqueness condition. Then $Q^D(P)$ ($Q^D_1(P)$) restores P.*

Inverse problem

General specification is the system

$$Q = \{q_V^z, z \in X^S, S \subset \mathbb{Z}^d \backslash V, V \in W\}$$

of finite-dimensional distributions q_V^z on X^V parameterized by boundary conditions z, which elements satisfy the following conditions:

1. for all $I, V \in W$, $S \subset \mathbb{Z}^d \backslash V \backslash I$ and $x \in X^V$, $y \in X^I$, $z \in X^S$

$$q_{V \cup I}^z(xy) = q_V^z(x) q_I^{zx}(y);$$

2. for all $V \in W$, $S \subset \mathbb{Z}^d \backslash V$ and any sequence of finite sets $\Lambda = \{\Lambda_n\}_{n \geq 1}$ such that $\Lambda_n \uparrow S$

$$q_V^{zS}(x) = \lim_{\Lambda \uparrow S} q_V^{z\Lambda}(x), \qquad x \in X^V, z \in X^{\mathbb{Z}^d}.$$

Note: $Q = Q^{fin} \bigcup Q^{inf}$

Theorem. *General specification determines a random field, that is, there exists a unique random field P which general conditional distribution coincides with the given general specification.*

For all $V \in W$ and $x \in X^V$ put

$$P_V(x) = \frac{q_V^y(x)}{q_I^x(y)} \left(\sum_{\alpha \in X^V} \frac{q_V^y(\alpha)}{q_I^\alpha(y)} \right)^{-1}$$

- P_V does not depend on $y \in X^I$ and $I \in W(\mathbb{Z}^d \backslash V)$;
- P_V is a probability distribution on X^V;
- $\{P_V, V \in W\}$ is consistent in Kolmogorov's sense $\Rightarrow \exists!\, P$;
- $Q(P) = Q$

For a positive general specification, the consistency condition

$$q^z_{V\cup I}(xy) = q^z_V(x)q^{zx}_I(y) = q^z_I(y)q^{zy}_V(x)$$

has **equivalent form** ($x \in X^V$, $y, v \in X^I$)

$$\frac{q^z_{V\cup I}(xy)}{q^z_{V\cup I}(uy)} = \frac{q^{zy}_V(x)}{q^{zy}_V(u)}$$

Indeed,

$$\left.\begin{array}{l} q^z_{V\cup I}(xy) = q^z_I(y)q^{zy}_V(x) \\ q^z_{V\cup I}(uy) = q^z_I(y)q^{zy}_V(u) \end{array}\right\} \Rightarrow \frac{q^z_{V\cup I}(xy)}{q^z_{V\cup I}(uy)} = \frac{q^{zy}_V(x)}{q^{zy}_V(u)}$$

On the over hand, since

$$q^z_{V\cup I}(xy)q^{zy}_V(u) = q^z_{V\cup I}(uy)q^{zy}_V(x),$$

taking sum over all $u \in X^V$, we obtain

$$q^z_{V\cup I}(xy) = q^z_I(y)q^{zy}_V(x)$$

Specification with finite boundary conditions is the system

$$Q^{fin} = \{q_V^z, z \in X^S, V, S \in W\}$$

of finite-dimensional distributions parameterized by boundary conditions in finite volumes which elements satisfy the following condition: for all pairwise disjoint $I, V, S \in W$ and all $x \in X^V$, $y \in X^I$, $z \in X^S$

$$q_{V \cup I}^z(xy) = q_V^z(x) q_I^{zx}(y)$$

Theorem. *Specification with finite boundary conditions determines a random field.*

Theorem. *Specification with finite boundary conditions Q^{fin} determines a general specification Q such that $Q^{fin} \subset Q$.*

Since for elements of Q^{fin} we have

$$q^y_{t\cup V}(xu) = q^y_t(x)q^{xy}_V(u), \qquad q^x_{s\cup V}(yu) = q^x_s(y)q^{xy}_V(u),$$

elements with Palm-type boundary conditions satisfy

$$\frac{q^y_{t\cup V}(xu)}{q^x_{s\cup V}(yu)} = \frac{q^y_t(x)}{q^x_s(y)}$$

Palm-type specification is a consistent system

$$Q^{\mathbb{1}} = \left\{q^z_V, z \in X^{\{t\}}, t \in \mathbb{Z}^d\backslash V, V \in W\right\}$$

of finite-dimentional distributions parameterized by conditions at points.

Theorem. *Let the elements of Palm specification $Q^{\mathbb{1}}$ be such that for any $I, V \in W$, $I \cap V = \varnothing$ and $z \in X^{\{t\}}$, $t \in \mathbb{Z}^d\backslash V$*

$$\sum_{y\in X^I} q^z_{V\cup I}(xy) = q^z_V(x), \qquad x \in X^V.$$

Then there exists a unique random field P such that $Q^{\mathbb{1}}(P) = Q^{\mathbb{1}}$ and a unique general specification Q such that $Q^{\mathbb{1}} \subset Q$

$$P_V(x) = p_t(x_t) q^{x_t}_{V\setminus\{t\}}(x_{V\setminus\{t\}}), \qquad x \in X^V, V \in W,$$

where

$$p_t(x) = \frac{q^y_t(x)}{q^x_s(y)} \left(\sum_{\alpha \in X^{\{t\}}} \frac{q^y_t(\alpha)}{q^\alpha_s(y)} \right)^{-1},$$

$y \in X^{\{s\}}$, $s \in \mathbb{Z}^d \setminus V$ and $t \in V$ are arbitrary

$$q^z_V(x) = \frac{q^{z_t}_{V\cup(S\setminus\{t\})}\left(x z_{S\setminus\{t\}}\right)}{q^{z_t}_{S\setminus\{t\}}\left(z_{S\setminus\{t\}}\right)}, \qquad V, S \in W$$

where $t \in S$ is an arbitrary point

Since for for elements of Q with $z \in X^{\mathbb{Z}^d \setminus (V \cup I)}$ we have

$$q^z_{V \cup I}(xy) = q^z_I(y) q^{zy}_V(x),$$

taking sum over all $x \in X^V$ we obtain

$$q^z_I(y) = \sum_{u \in X^V} q^z_{V \cup I}(uy)$$

and hence

$$q^z_{V \cup I}(xy) = q^{zy}_V(x) \sum_{u \in X^V} q^z_{V \cup I}(uy)$$

On the over hand, this condition is equivalent to

$$\frac{q^z_{V \cup I}(xy)}{q^z_{V \cup I}(uy)} = \frac{q^{zy}_V(x)}{q^{zy}_V(u)}$$

Dobrushin-type specification is the system

$$Q^D = \{q^z_V, z \in X^S, S = \mathbb{Z}^d \backslash V, V \in W\}$$

of finite-dimensional distributions parameterized by boundary conditions "everywhere outside" which elements satisfy the consistency condition

$$\frac{q^z_{V \cup I}(xy)}{q^z_{V \cup I}(uy)} = \frac{q^{zy}_V(x)}{q^{zy}_V(u)}$$

for all $z \in X^{\mathbb{Z}^d \backslash (V \cup I)}$

Theorem. *Quasilocal Dobrushin-type specification Q^D wich elements satisfy Dobrudhin's uniqueness condition determines a random field P.*

General 1–specification is the system

$$Q_1 = \{q_t^z, z \in X^S, S \subset \mathbb{Z}^d \backslash \{t\}, t \in \mathbb{Z}^d\}$$

of one-point distributions parameterizad by boundary conditions and satisfying the following consistency conditions:

1. for all $t, s \in \mathbb{Z}^d$, $S \subset \mathbb{Z}^d \backslash \{t, s\} \cup \varnothing$ and $z \in X^S$, $x, u \in X^{\{t\}}$, $y, v \in X^{\{s\}}$

$$q_t^{zy}(x) q_s^{zx}(v) q_t^{zv}(u) q_s^{zu}(y) = q_s^{zx}(y) q_t^{zy}(u) q_s^{zu}(v) q_t^{zv}(x);$$

2. for any $t \in \mathbb{Z}^d$, $S \subset \mathbb{Z}^d \backslash \{t\}$ and any sequence of finite sets $\Lambda = \{\Lambda_n\}_{n \geq 1}$ such that $\Lambda_n \uparrow S$

$$q_t^z(x) = \lim_{\Lambda \uparrow S} q_t^{z\Lambda}(x), \qquad x \in X^{\{t\}}, z \in X^S.$$

1–specification with finite boundary conditions Q_1^{fin}

Dobrushin-type 1–specification Q_1^D

Theorem. *General 1–specification Q_1 determines a r.f. P: there exists a unique r.f. P such that $Q_1(P) = Q_1$*

Theorem. *1–specification Q_1^{fin} with finite boundary conditions determines a r.f. P:*
there exists a unique r.f. P such that $Q_1^{fin}(P) = Q_1^{fin}$

Theorem. *Quasilocal Dobrushin-type 1–specification Q_1^D satisfying Dobushin's uniqueness condition determines a r.f. P: there exists a unique r.f. P such that $Q_1^D(P) = Q_1^D$*

• General 1–specification Q_1 determines the unique general specification Q such that $Q_1 \subset Q$. There is an exact formula for elements of Q in terms of elements of Q_1:
For example, for all $t, s \in \mathbb{Z}^d$, $S \subset \mathbb{Z}^d \backslash \{t, s\}$ and $x \in X^{\{t\}}$, $y \in X^{\{s\}}$

$$q^z_{\{t,s\}}(xy) = \frac{q^{zv}_t(x)}{q^{zv}_t(u)} \cdot \frac{q^{zx}_s(y)}{q^{zx}_s(v)} \cdot C$$

where C is the normalizing constant and configurations $u \in X^{\{t\}}$, $v \in X^{\{s\}}$ are arbitrary.

• Specification Q inherits properties of Q_1
– Q is Markovian if and only if Q_1 is such

• Q_1 and constructed Q determine the same random field

Dachian, Nahapetian 2001, 2004 for $Q_1^D \longleftrightarrow Q^D$

- General specification Q and its subsystems

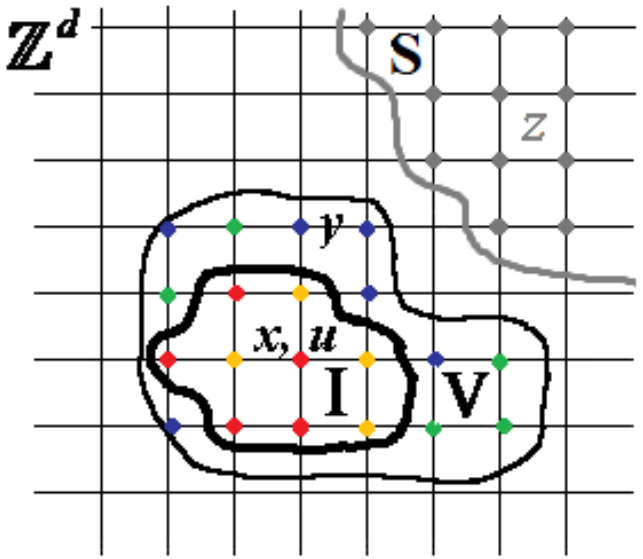
$$
\begin{array}{ccccccc}
Q_1 & & Q_1^{fin} & & & & \\
\updownarrow & & \updownarrow & & & & \\
Q & = & Q^{fin} & \cup & Q^{inf} & & \\
 & & \updownarrow & & \downarrow & & \\
 & & Q^{\mathbb{1}} & & Q^D & \longleftrightarrow & Q_1^D
\end{array}
$$

- Elements of specification Q have a Gibbs form in terms of a transition energy field

For Q^D (Q_1^D): Dachian, Nahapetian 2019 and Khachatryan, Nahapetian 2019

General transition energy field

is a system $\Delta = \{\Delta_V^z(\cdot,\cdot), z \in X^S, S \subset \mathbb{Z}^d \backslash V, V \in W\}$ of functions on pairs of configurations from X^V parameterized by boundary conditions z in volume S outside V and satisfying the following conditions:



1. energy conservation law

$$\Delta_V^z(x,u) = \Delta_V^z(x,w) + \Delta_V^z(w,u)$$

2. consistency conditions

$$\Delta_V^z(xy,uy) = \Delta_I^{zy}(x,u)$$

and

$$\lim_{\Lambda \uparrow S} \Delta_V^{z\Lambda}(x,u) = \Delta_V^z(x,u)$$

Δ_V^z is the energy spent by the system to change the configuration in volume V under boundary condition z in volume S

Relationship between energy and probability

Theorem. *A set of functions* $Q = \{q_V^z, z \in X^S, S \subset \mathbb{Z}^d \backslash V, V \in W\}$ *is a specification if and only if its elements have a Gibbs form*

$$q_V^z(x) = \frac{\exp\{\Delta_V^z(x,u)\}}{\sum\limits_{\alpha \in X^V} \exp\{\Delta_V^z(\alpha,u)\}}, \qquad x \in X^V,$$

where $u \in X^V$ *is an arbitrary configuration and* Δ_V^z *is the element of (uniquely determined) the transition energy field* $\mathbf{\Delta} = \{\Delta_V^z, z \in X^S, S \subset \mathbb{Z}^d \backslash V, V \in W\}$.

The condition

$$\Delta^z_{V\cup I}(xy,uy)=\Delta^{zy}_V(x,u)$$

is equivalent to the following one

$$\Delta^z_{V\cup I}(xy,uv)=\Delta^{zy}_V(x,u)+\Delta^{zu}_I(y,v)$$

Hence

$$\Delta^z_V(x,u)=$$

$$=\Delta_{t_1}^{zx_{t_2}x_{t_3}..x_{t_{|V|}}}(x_{t_1},u_{t_1})+\Delta_{t_2}^{zu_{t_1}x_{t_3}...x_{t_{|V|}}}(x_{t_2},u_{t_2})+...$$

$$+\Delta_{t_{|V|}}^{zu_{t_1}u_{t_2}...u_{t_{|V|}-1}}(x_{t_{|V|}},u_{t_{|V|}}),$$

where $t_1,t_2,...,t_{t_{|V|}}$ is some numeration of points V

General one-point transition energy field

is a set $\mathbf{\Delta}_1 = \left\{\Delta_t^z(\cdot,\cdot), z \in X^S, S \subset \mathbb{Z}^d \backslash \{t\}, t \in \mathbb{Z}^d\right\}$ of functions at points t of the lattice on pairs of configurations from $X^{\{t\}}$ parameterized by boundary conditions z in volume S outside t and satisfying the following conditions

1. energy conservation law

$$\Delta_t^z(x,u) = \Delta_t^z(x,y) + \Delta_t^z(y,u);$$

2. consistency conditions

$$\Delta_t^{zy}(x,u) + \Delta_s^{zu}(y,v) = \Delta_s^{zx}(y,v) + \Delta_t^{zv}(x,u).$$

and

$$\lim_{\Lambda \uparrow S} \Delta_t^{z\Lambda}(x,u) = \Delta_t^z(x,u)$$

• One-point transition energy field $\mathbf{\Delta}_1$ determines the transition energy field $\mathbf{\Delta}$ such thet $\mathbf{\Delta}_1 \subset \mathbf{\Delta}$

Theorem. *A set of functions* $Q_1 = \{q_t^z, z \in X^S, S \subset \mathbb{Z}^d \backslash \{t\}, t \in \mathbb{Z}^d\}$ *is a specification if and only if its elements have a Gibbs form*

$$q_t^z(x) = \frac{\exp\{\Delta_t^z(x,u)\}}{\sum\limits_{\alpha \in X} \exp\{\Delta_t^z(\alpha,u)\}}, \qquad x \in X^{\{t\}},$$

where $u \in X^{\{t\}}$ *is an arbitrary configuration and* Δ_t^z *is the element of (uniquely determined) the one-point transition energy field* $\mathbf{\Delta}_1 = \left\{\Delta_t^z, z \in X^S, S \subset \mathbb{Z}^d \backslash \{t\}, t \in \mathbb{Z}^d\right\}$.

$$\begin{array}{ccc} Q & \longleftrightarrow & Q_1 \\ \updownarrow & & \updownarrow \\ \mathbf{\Delta} & \longleftrightarrow & \mathbf{\Delta}_1 \end{array}$$

Direct and inverse problems in the theory of Gibbs random fields

The standard form of the theory of random processes:

1. general definition and statements in terms of $\mathscr{P}(P)$ — the direct problem
2. construction of models by a given consistent system $\mathscr{P}$ — the inverse problem

The theory of Gibbs random fields:

1. definition of a Gibbs r.f. as a solution of the inverse problem for a consistent system of finite-dimensional distributions parameterized by infinite boundary conditions — Gibbs specification with a given potential $\Rightarrow$ a set of models of Gibbs r.f. with a given potential

2. a purely probabilistic definition of a Gibbs random field in terms of its finite-dimensional distributions $\Rightarrow$ a general theory of Gibbs r.f. and direct application of probabilistic methods to the mathematical problems of statistical physics

Dachian, Nahapetian 2009, 2019

Random field P is a **Gibbs random field** if it is positive, the limits

$$q_t^z(x) = \lim_{\Lambda \uparrow \mathbb{Z}^d \setminus \{t\}} \frac{P_{\{t\}\cup\Lambda}(xz_\Lambda)}{P_\Lambda(z_\Lambda)}, \qquad x \in X^{\{t\}},$$

exist for all $t \in \mathbb{Z}^d$ and $z \in X^{\mathbb{Z}^d\setminus\{t\}}$, are strictly positive, and the convergence is **uniform** with respect to z.

• *Canonical 1–specification* $\mathcal{Q}_1^D = \{q_t^z, z \in X^S, S = \mathbb{Z}^d\setminus\{t\}, t \in \mathbb{Z}^d\}$

Random field P is a **Gibbs random field** if it is positive, the limits

$$\Delta_t^z(x,u) = \lim_{\Lambda \uparrow \mathbb{Z}^d \setminus \{t\}} \ln \frac{P_{\{t\}\cup\Lambda}(xz_\Lambda)}{P_{\{t\}\cup\Lambda}(uz_\Lambda)}, \qquad x,u \in X^{\{t\}},$$

exist for all $t \in \mathbb{Z}^d$ and $z \in X^{\mathbb{Z}^d\setminus\{t\}}$, and the convergence is **uniform** with respect to z.

Gibbs form of a Gaussian random field

Let P be a Gaussian random field with mean values $m = \{m_t, t \in \mathbb{Z}^d\}$ and covariance matrix $C = \{c_{ts}, t, s \in \mathbb{Z}^d\}$; $A = \{a_{ts}, t, s \in \mathbb{Z}^d\} = C^{-1}$

Theorem. *Gaussian random field P is Gibbsian if and only if the sequence of sums $\sum_{s\in\Lambda} a_{ts}(z_s - m_s)$ converges for all $t \in \mathbb{Z}^d$ and $z \in \mathbb{R}^{\mathbb{Z}^d}$ as $\Lambda \uparrow \mathbb{Z}^d\backslash\{t\}$, and the convergence is uniform with respect to z.*

Conditional densities of the Gaussian Gibbs random field have the Gibbs form

$$f_t^z(x) = \frac{\exp\{\Delta_t^z(x,u)\}}{\int\limits_{\mathbb{R}} \exp\{\Delta_t^z(\alpha,u)\}d\alpha}, \qquad x, u \in \mathbb{R}^{\{t\}}, z \in \mathbb{R}^{\mathbb{Z}^d\backslash\{t\}}, t \in \mathbb{Z}^d,$$

with transition energy

$$\Delta_t^z(x,u) = \frac{a_{tt}}{2}\left((u-m_t)^2 - (x-m_t)^2\right) + (u-x)\sum_{r\in\mathbb{Z}^d\backslash\{t\}} a_{tr}(z_r - m_r)$$

Thank You

for the Attention!

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Hamiltonian

$$\Delta_t^z(x,u) = \Delta_t^z(x,w) + \Delta_t^z(w,u) \Rightarrow \Delta_t^z(x,u) = H_t^z(u) - H_t^z(x)$$

Hamiltonian (potential energy) is a set $H = \left\{H_t^{zS}, S = \mathbb{Z}^d \backslash \{t\}\right\}$ of such functions that for al $t, s \in \mathbb{Z}^d$, $x, u \in X^{\{t\}}$, $y, v \in X^{\{s\}}$ and $z \in X^{\mathbb{Z}^d \backslash \{t,s\}}$

$$H_t^{zy}(x) + H_s^{zx}(v) + H_t^{zv}(u) + H_s^{zu}(y) = H_s^{zx}(y) + H_t^{zy}(u) + H_s^{zu}(v) + H_t^{zv}(x)$$

H_t^z is called a *one-point Hamiltonian*

Theorem. *A set of functions* $\overline{Q}_1 = \{q_t^{zs}, S = \mathbb{Z}^d \backslash \{t\}\}$ *is a specification of Dobrushin's type if and only if its elements have a Gibbs form*

$$q_t^z(x) = \frac{\exp\{-H_t^z(x)\}}{\sum\limits_{\alpha \in X} \exp\{-H_t^z(\alpha)\}}, \qquad x \in X^{\{t\}},$$

where H_t^z *is the one-point Hamiltonian.*