A general class of finite difference schemes arising in numerical analysis of Reaction-diffusion systems

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Abstract

This talk is devoted to the general class of finite difference schemes developed for a numerical approximation of solutions to a certain type of reaction–diffusion systems with m population densities. Let $\Omega \subset \mathbb{R}^2$ be a connected and bounded domain with smooth boundary and m be a fixed integer. We consider the steady-states of m competing species coexisting in the same area Ω. Let u_i denotes the population density of the i^{th} component with the internal dynamic prescribed by F_i.

We call the m-tuple $U = (u_1, \cdots, u_m) \in (H^1(\Omega))^m$, *a segregated state* if

$$u_i(x) \cdot u_j(x) = 0, \text{ a.e. for } \quad i \neq j, \ x \in \Omega.$$

The problem amounts to minimize

$$E(u_1, \cdots, u_m) = \int_\Omega \sum_{i=1}^{m} \left(\frac{1}{2} |\nabla u_i(x)|^2 + F_i(x, u_i(x)) \right) dx \tag{1}$$

over the set

$$S = \{(u_1, \ldots, u_m) \in (H^1(\Omega))^m : u_i \geq 0, u_i \cdot u_j = 0, u_i = \phi_i \quad \text{on} \quad \partial\Omega\},$$

where $\phi_i \in H^{\frac{1}{2}}(\partial\Omega)$, $\phi_i \cdot \phi_j = 0$, for $i \neq j$ and $\phi_i \geq 0$ on the boundary $\partial\Omega$.

First of all we construct finite difference schemes with standard discretization (uniform and a standard 5 point stencil approximation of Laplacian). These schemes themselves happened to be non-linear and implicit systems.

The main issues for these schemes were discussed in [3, 2, 1, 4], i.e. the schemes solution's existence, uniqueness and convergence.

References

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