System of variational inequalities with interconnected obstacles

R. Barkhudaryan (Yerevan State University, Yerevan, Armenia)
e-mail: barkhudaryan@ysu.am

We consider interconnected obstacle problems and develop a numerical approximation scheme for them. These problems are given as (weakly coupled) systems of variational inequalities and model optimal decision or switching under uncertainty. For example, these problems can be regarded as a system-version of the American-type option problem with multiple choices and switching between them. These systems can be written (for vector-valued functions $\mathbf{u} = (u_1, \cdots, u_m)$) as

$$\min(-L^i u_i - f_i, \min_{j \neq i}(u_i - u_j + \psi_{ij})) = 0, \qquad u_i|_{\partial\Omega} = g_i,$$

for $i = 1, \ldots, m$ and with the compatibility condition on $\partial\Omega$, $g_i - g_j + \psi_{ij} \geq 0$, $i \neq j \geq 0$. One has also to assume certain no-gain on the loop condition (see (1)). A solution to the preceding system is understood in the viscosity sense (see [1,2,3]).

For the sake of simplicity, we consider the case where $L^i \equiv L^j$. Additionally, for the optimal switching problem to be well defined, we impose a non-profit loop condition; that is, for any $x \in \Omega$ and any loop $i_0, i_1, \ldots i_p = i_0$, where $2 \leq p \leq n$, we assume that

$$\sum_{j=1}^{p} \psi_{i_{j-1},i_j} \geq 0. \tag{1}$$

We examine the above optimal switching problem from different perspectives focusing primarily on an iterative method and a monotone scheme. We begin by addressing the theory of viscosity solutions for these systems and show the equivalence between several definitions. Next, we consider a monotone scheme to study the existence of solutions and for numerical computations. Subsequently, we discuss an application in the theory of financial bubbles.

References

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