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Iterative scheme for an non-local obstacle like problem

This work is based on a joined work with M. Juráš and M. Salehi [3].

Here we consider a non local free boundary problem formulated as a Hamilton-Jacobi equation:

$$\begin{cases} \min(-Lu(x) + f(x), u(x) - u(-x) - \psi(x)) = 0, & x \in \Omega, \\ u(x) = g(x), & x \in \partial\Omega, \end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}^n$ $(n \geq 2)$ is a bounded symmetric domain such that if $x \in \Omega$ then $-x \in \Omega$, $f \in C(\Omega)$, $g \in C(\partial\Omega)$ and $\psi \in C^2(\Omega)$.

As mentioned above we consider stationary case, i.e. the operator L is an elliptic operator of the form

$$Lu = a^{ij}(x)D_{ij}u + b^i(x)D_iu + c(x)u, \qquad a^{i,j} = a^{j,i},$$

where the coefficients $a^{i,j}$, b^i, c are assumed to be continues and the matrix $[a^{i,j}(x)]$ is positive definite for all $x \in \Omega$. Additionally we assume that the coefficients are "symmetric" in the domain Ω i.e. the operator applied to the function $u(-x)$ should be the same as operator applied to the function u at point $-x$.

It is easy to check that if u is a solution to equation (1), then $u(-x)$ is a solution to the reflected problem, i.e. all ingredients are reflected accordingly. Hence one has

$$u(x) \geq u(-x) + \psi(x) \geq u(x) + \psi(-x),$$

and in particular $\psi(x) + \psi(-x) \leq 0$ is forced as a condition for an existence theory.

Our aim here is to construct, through an increasing iterative scheme, a solution of the above problem. The scheme consists of a sequence of obstacle problems at each step that eventually converge to the solution.

R E F E R E N C E S

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2. *Berestycki H, Monneau R, Scheinkman JA.* A non-local free boundary problem arising in a theory of financial bubbles. // Philos Trans R Soc Lond Ser A Math Phys Eng Sci. 2014;372(2028):20130404, 36.

3. *Barkhudaryan R, Juráš M, Salehi, M* Iterative scheme for an elliptic non-local free boundary problem. // 2015, Applicable Analysis, pp. 1–13.