Convergence Acceleration of Trigonometric Expansions

A. Poghosyan and R. Barkhudaryan (Institute of Mathematics of NAS, Armenia)
arnakp@gmail.com, rafayel@instmath.sci.am

We consider the problem of function reconstruction by its finite number of Fourier coefficients (continuous or discrete). In case of smooth, but non-periodic functions, the drawback of the classical trigonometric expansions is hidden in the Gibbs phenomenon. Different approaches are known for elimination of the phenomenon and convergence acceleration of trigonometric expansions.

First, we discuss a polynomial subtraction approach invented by Krylov [1] and independently considered by Lanczos [2]. The polynomial represents the discontinuities in the function and some of its first derivatives (jumps) and practical realization of the approach is connected with accurate approximation of the laters [3]. In a series of papers, the accuracy of jumps approximation and the convergence of corresponding trigonometric expansions we explored. It was proved that the expansions with approximate jumps are more accurate than with the exact ones. This interesting behavior of the Krylov-Lanczos-Eckhoff approach is known as auto-correction phenomenon ([4, 5, 6]).

Second, we discuss an approach based on rational correction functions ([7, 8, 9]). Rational corrections contain unknown parameters which determination is a crucial problem for realization of the rational approximations. One possible choice leads to Fourier-Pade approximations and interpolations. Another choice leads to optimal rational expansions.

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