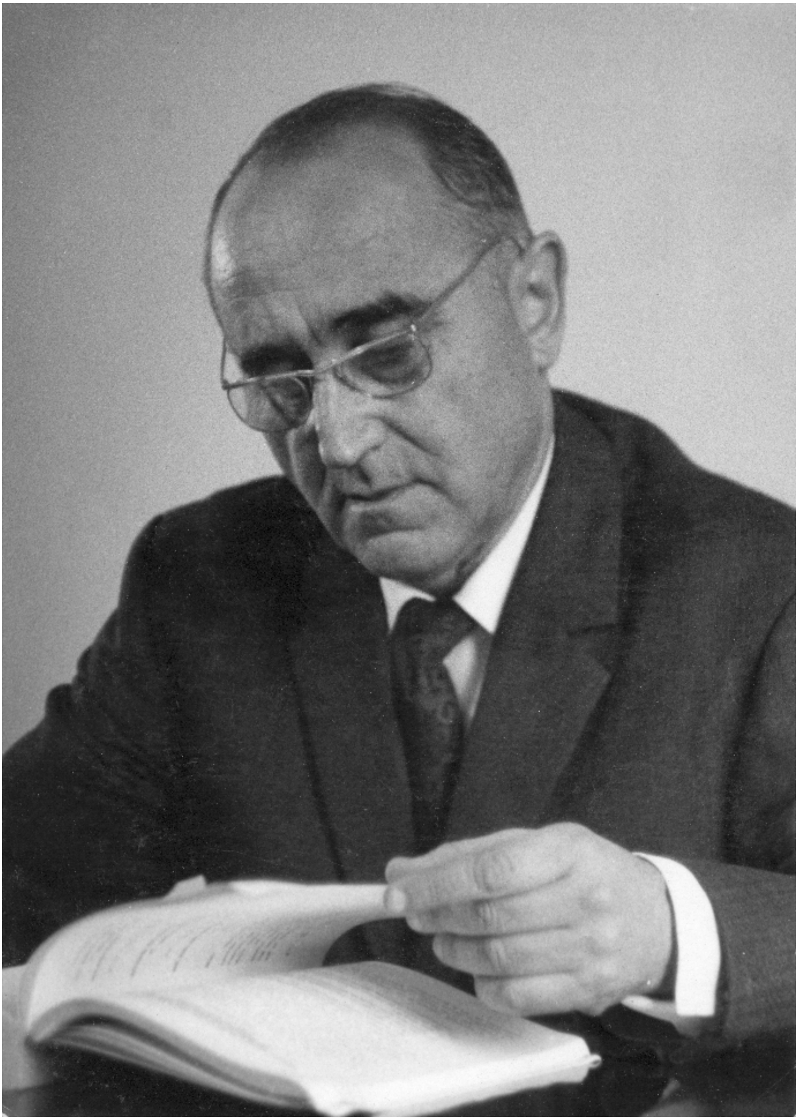

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ABSTRACTS

Contents

On hyperidentities defined by the ternary associativity

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An algebra with binary and ternary operations is called {2,3}-algebra. A {2,3}-algebra $(Q;U)$ is called:

a) functionally non-trivial if the sets of its binary and ternary operations are non-singleton;

b) q-algebra if there exists a ternary quasigroup operation in U;

c) invertible algebra if its every operation is a quasigroup operation.

In the talk we characterize the hyperidentities defined by the equality

$$((x,y,z),u)=(x,(y,z,u)),$$

satisfying in the functionally non-trivial invertible {2,3}-algebras and q-algebras.

Self-distributed and Non-associative Algebraic Structures in Cryptology

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A way to increase the robustness of a cryptographic algorithm toward unauthorized inversion can be obtained through application of non-commutative or non-associative algebraic structures. An algebraic structure, written $< X, Y, ..., \circ, *, ..., R_1, R_2, ..., x, y >$, is an n-tuple, where elements $x \in X$, $y \in Y$ are distinct, domains and ranges of functions, also n-ary operations $\circ$ and $*$ are cartesian products of the sets, and binary relations R_1 and R_2 are defined on the sets. Non-associative algebra assumes a vector space over a field, which defines the operation of multiplication interacting with the addition operation by the ordinary distribution law. The operation of multiplication, meanwhile, is not necessarily commutative or associative. For example, a quasigroup, unlike finite groups, does not possess associativity, neither has an identity element. Obviously, handling such structures without possessing knowledge on their construction will require an exponential number of looking ups in order to identify underlying components. In the case of non-associative algebraic models, the number of parentheses can be huge with each placement of parentheses dictating a unique type of computing [1]. Still, solely quasigroup based cryptosystems are vulnerable against specific attacks.

With this regard, a self-distributive algebra $(A, *)$ that satisfies the identity $x * (y * z) = (x * y) * (x * z)$ for every $x, y, z \in A$, and $*$ distributes over itself, can stand for a non-associative version of the more well-known non-abelian finite groups. Given the fact that two different algebraic structures share similar characteristics, where one structure is defined on a set X and a similar structure is defined on a set Y, one can establish a mapping from X into Y that preserves some characteristics of the underlying domain structures. Given $< X, \circ >$ and $< Y, * >$ being algebraic structures with operations $\circ$ and $*$, respectively, the function $f : X \to Y$ will be a homomorphism from $< X, \circ >$ to $< Y, * >$, if for every $x_1, x_2 \in X$, $f(x_1) * f(x_2) = f(x_1 \circ x_2)$.

A complete work will be dedicated to construct a special kind of homomorphism from self-distributed algebraic structures to quasigroups aimed at reinforcement of cryptographic algorithms.

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Mathematical Description of Subdiffusion of Particles in Polyelectrolyte Films

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In this paper, we mathematically describe an experiment in which the kinetics of diffusion of charged particles in a polyelectrolyte film has been investigated. It was shown experimentally that at small times the diffusion of particles is fast, and at long times it is slow. The mathematical model of the diffusion kinetics, based on the classical diffusion equation, which belongs to the class of partial differential equations of parabolic type, cannot describe the experimentally observed kinetics. If we modify the classical equation by replacing the whole time derivative with the fractional derivative, we can use the obtained equation to describe the experimentally observed diffusion kinetics. Such a modification of the equation can be physically justified by the fact that the systems, that we study, have a special memory property, and as it is well known, this circumstance makes it possible to replace the whole time derivative by a fractional derivative in the equations. From a comparison of the theoretical kinetic curve with the experimental, the diffusion coefficient and the order of the fractional derivative were obtained.

Blaschke C^*-Algebras

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The report proposed a construction of a C^*-algebra, which is the direct limit of Toeplitz algebras, corresponding to a family of Blaschke products. This algebra can be considered as a specific generalization of the semigroup reduced C^*-algebra. The C^*-algebra Blaschke is associated with each non-single-point representing measure with a support on the Shilov boundary of the uniform Blaschke algebra. The main result is that the constructed algebra does not depend on the choice of measure, that is, it represents a categorical object. Some other properties connecting the Blaschke algebra and the Blaschke C^*-algebra are also presented.

The work is a natural continuation of the earlier studies of S. Grigorian and T. Tonev.

On harmonic projection operators on mixed norm spaces over the real ball

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Over the unit ball in $\mathbb{R}^n$ $(n \geq 2)$, we study Bergman type projection operators $T_{\beta,\lambda}$ with a special harmonic Poisson-Bergman type kernel. We prove that operators $T_{\beta,\lambda}$ continuously map mixed norm spaces $L(p,q,\alpha)$ into itself for certain values of parameters. For a particular choice of parameters, the operators continuously project mixed norm spaces $L(p,q,\alpha)$ onto their harmonic subspaces.

Fuel-Time Optimal Control of the Electromechanical System

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An electromechanical system of the second order is considered, which approximately describes the dynamics of an individual link of the arm of a multi-link manipulator, if each link is controlled by the voltage supplied by an independent drive motor, and the dynamic interference of various degrees of freedom is sufficiently small [1]. For the manipulator model under consideration, in [1] the problems of constructing optimal control under different quality criteria (travel time, consumed energy, positioning accuracy) were studied, which ensures the movement of the system from an arbitrary initial state to a given final state of rest, including under additional constraints. In [2, 3], various systems were considered that represent models of mechanical and electromechanical systems containing an electric motor, with various restrictions on control, including mixed ones. There, problems of constructing a limited control that lead the system from an arbitrary initial state to the given terminal state, including a state of rest, in a finite time, has been investigated. In this article, as a criterion of optimality, is considered a functional that takes into account both the momentum transferred to the mechanical part of the system by the control voltage by means of the electric drive reducer and the time of the control process. The problem of optimal control with such a functional is related to the class of problems of minimizing fuel consumption or resources [4, 5]. Using the method of the maximum principle of the Pontryagin [6], an optimal control in the form of a synthesis is constructed which provides the movement of the considered system from an arbitrary initial state to a given final state of rest and minimizes the value of the combined functional. At the same time, two curves of switching of optimal control are constructed, which in the phase plane of the considered system form the disjoint domains. Depending on which domain the initial state is in, the movement of the system to the final state of rest - to the origin - occurs in the optimal control mode of different structure: with one or with two moments or without moments of switching. Formulas are obtained for calculating the moments of switching of optimal control and the optimal time of the control process.

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On a Dirichlet Problem for One Improperly Elliptic Equation

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Let $D = \{z : |z| < 1\}$ be the unit disk with boundary $\Gamma = \partial D$. We consider the sixth order equation

$$\sum_{k=0}^{6} A_k \frac{\partial^6 U}{\partial x^k \partial y^{6-k}} = 0, \quad (x, y) \in D, \tag{1}$$

where A_k are such complex constants ($A_0 \neq 0$), that the roots λ_j, $j = 1, \ldots, 6$ of the characteristic equation

$$\sum_{k=0}^{6} A_k \lambda^{6-k} = 0 \tag{2}$$

satisfy the condition $\Im\lambda_k > 0$, $k = 1, 2, \ldots, 6$, that is the equation (1) is improperly elliptic. We suppose also, that i is a double root of this equation. We seek the solution U of the equation (1), which belongs to the class $C^6(D) \bigcap C^{(2,\alpha)}(D \bigcup \Gamma)$ and on the boundary Γ satisfies the Dirichlet boundary conditions

$$\left.\frac{\partial^k U}{\partial r^k}\right|_{\Gamma} = f_k(x, y), \quad k = 0, 1, 2 \quad (x, y) \in \Gamma, \tag{3}$$

Here f_k $(k = 0, 1, 2)$ are prescribed functions on Γ, $\frac{\partial}{\partial r}$ is a derivative with respect to r ($z = re^{i\theta}$). Class of the functions f_k must be determined separately.

The problem (1), (3) for improperly elliptic equation (1) is not correct, (see [1]). In the papers [2,3] there were investigated some cases of properly elliptic equation (1).

In the present talk we suppose, that $\lambda_j \neq i$, $j = 1, 2, 3, 4$ four roots of the characteristic equation (2), what not equal to i, satisfy the conditions: 1) $\lambda_1 = \lambda_2 = \lambda_3 \neq \lambda_4$; 2) $\lambda_1 = \lambda_2 \neq \lambda_j$, $j = 3, 4$ and $\lambda_3 \neq \lambda_4$; 3) and when all λ_j $j = 1, 2, 3, 4$ are simple roots. It was shown that for the normal solvability of the problem the boundary functions must be analytic in some annulus $\rho < |z| < 1$, where ρ determined by the roots of the equation (2) in explicit form. For all three cases it was obtained new formulas for defect numbers of the problem (1), (3), and in some cases it is possible to determine exact value of these defect numbers. The solutions of the homogeneous problem (1), (3) and conditions, provided solvability of the in-homogeneous problem (1), (3) are determined in explicit form.

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On a non-local parabolic free boundary problem arising in a theory of financial bubbles

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In this talk, we study a non-local free boundary problem arising in financial bubbles. The model equation, studied here, is the following free boundary problem formulated as a Hamilton-Jacobi equation:

$$\min(Lu, u(t,x) - u(t,-x) - \psi(t,x)) = 0, \quad (t,x) \in \mathbb{R}^+ \times \Omega, \tag{1}$$

where $\Omega \subset \mathbb{R}$ is a symmetric bounded domain such that if $x \in \Omega$ then $-x \in \Omega$ and $\psi \in C^2(\mathbb{R}^+ \times \Omega)$, and the operator L is the following parabolic operator

$$Lu = u_t + a^{ij}(x)D_{ij}u + b^i(x)D_iu + c(x)u, \qquad a^{i,j} = a^{j,i}.$$

Here the coefficients $a^{i,j}$, b^i, c are assumed to be continues and the matrix $[a^{i,j}(x)]$ is positive definite for all $x \in \Omega$. Additionally we assume that the coefficients are "symmetric" in the domain Ω.

We discuss iterative method for numerical results, which consists of a sequence of parabolic obstacle problems at each step to be solved, that in turn gives the next obstacle function in the iteration. The convergence of the proposed algorithm is proved. We study the finite difference scheme for this algorithm and obtain its convergence.

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On Some Trends and Principles Related to Arbitrary Meromorphic or Analytic Functions in a Given Domain

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The first results (principles) related to arbitrary meromorphic, particularly analytic functions in a given domain were established by Cauchy (19-th century), while the next results arisen a century later in Ahlfors theory of covering surfaces (1935).

In this survey we present some other (diverse type) results of the same generality which were obtained since 1970s.

The majority of these results occurs in three trends in theory of meromorphic functions: Gamma-lines, proximity property, and universal version of value distribution theory.

Each of these trends complements the classical Nevanlinna value distribution theory or Ahlfors theory and also reveals some new type of phenomena.

Content: list of sections.

(The results in each section relate to arbitrary meromorphic or analytic functions in a given domain.)

1. Two principles related to derivatives.
2. Results related to level sets and Gamma-lines.
3. Three simple consequences related to a-points.
4. Ahlfors fundamental theorems in terms of windings and a new interpretation of deficient values.
5. Universal version of value distribution.

This work was supported by Marie Curie (IIF) award.

Application Fourier's Method for Studying the Resonance Oscillations in Waveguides

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A Fourier medhod for solving the problem of resonanse oscillations in the flat finite waveguides is presented. It is suggested that the plane waveguide consists of two parts. In a rectangular Cartesian coordinate system, the first part with the index (1) occupies the region

$$-a_1 \leqslant x < 0,\ 0 \leqslant y < 0,\ -\infty < z < \infty,$$

and the second part with the index (2) occupies the region

$$0 \leqslant x < a_2,\ 0 \leqslant y < 0,\ -\infty < z < \infty.$$

It is considered shear elastic vibrations (antiplane deformation):

$$u = 0,\ v = 0,\ w = w(t, x, y).$$

Equations of wave propagation for parts of the waveguide have the form:

$$c^2 \Delta w_i = \frac{\partial^2 w_i}{\partial t^2};\ i = 1, 2.$$

The boundary conditions are:

$$\frac{\partial w_1(t, x, 0)}{\partial y} = 0, \qquad w_2(t, x, 0) = 0,$$

$$w_2(t, x, b) = 0, \qquad \frac{\partial w_2(t, x, b)}{\partial y} = 0,$$

$$\frac{\partial w_1(t, -a_1, y)}{\partial x} = 0, \qquad w_2(t, a_2, y) = 0.$$

At the junction of the waveguides the conditions are:

$$w_1(t, 0, y) = w_2(t, 0, y), \qquad \frac{\partial w_1(t, 0, y)}{\partial x} = \frac{\partial w_2(t, 0, y)}{\partial x}.$$

Systems With Polynomial Potential. Existence of Bounded and Periodic Soliton Solutions. Numerical Realization of Soliton Solutions

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For equations of mathematical physics, which are the Euler-Lagrange equation of the corresponding variational problems, an important class of solutions are soliton solutions. In a number of models, such solutions are well approximated by soliton solutions for finite-difference analogues of the initial equations, which, in place of a continuous environment, describe the interaction of clumps of a environment placed at the vertices of the lattice. Emerging systems belong to the class of infinite-dimensional dynamical systems. The most widely considered classes of such problems are infinite systems with Frenkel-Kontorova potentials (periodic and slowly growing potentials) and Fermi-Pasta-Ulam (potentials of exponential growth).

In the framework of the developed approach, the study of soliton solutions (solutions of the traveling wave type) is based on the existence (in case of the quasilinear right-hand side of the functional differential equation) of a one-to-one correspondence between soliton solutions for infinite-dimensional dynamical systems and solutions of induced functional differential equations of pointwise type (FDEPT).

Another class of problems is related to the study of soliton solutions for equations of mathematical physics without using finite-difference analogs. At the same time, the potentials can have lags in time. In this case, the induced equation for traveling waves also turns out to be a FDEPT.

For both classes of problems, the existence and uniqueness theorem for a solution of an induced FDEPT guarantees the existence and uniqueness of a soliton solution with given initial values for systems with quasilinear potential. Moreover, for systems with a quasilinear potential, one can formulate the conditions for the existence of a bounded and periodic solution. The formulated conditions, as well as the estimate of the radius of the ball in which the solution is contained, are given in terms of the characteristics of the right-hand side of the induced FDEPT. It is very important that such conditions do not use information about the spectral properties of the linearized equation (equations in variations) that essentially simplifies their verification. A system with a polynomial potential can be redefined by changing the potential outside a given ball, so that the emerging potential turns out to be quasilinear. If a guaranteed periodic soliton solution for such an overdetermined system lies in the ball outside which the potential is redefined, then we obtain the conditions for the existence of a bounded and periodic soliton solution for the initial system with a polynomial potential. Another important task is the numerical realization of bounded and periodic soliton solutions for systems with a polynomial potential, which has been successfully solved.

On Associative Identities With Functional Variables

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In this talk the Belousov theorem on linearity of invertible algebras with the $\forall\exists(\forall)$-identity is extended over the other $\forall\exists(\forall)$-identities of associativity. As a consequence we obtain the equivalency of the considered $\forall\exists(\forall)$-identities of associativity and non-trivial hyperidentities of associativity in systems of groups.

On the minimal circle of a flat compact

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Let $S \subset \mathbb{C}$ be bounded. Then there exists a circle containing S. The English mathematician James Joseph Sylvester in 1857 proposed to calculate the smallest circle containing S. This problem may be considered as a facility location problem (the 1-center problem) in which the location of a new facility must be chosen to provide service to a number of customers, minimizing the farthest distance that any customer must travel to reach the new facility. Without loss of generality it may be assumed that S is closed and convex. For simplest case of a segment the minimal circle's diameter coincides with that segment. For a triangle there are two possibilities. For an obtuse triangle the diameter of the minimal circle coincides with the greater side of the triangle and for acute triangle the minimal circle coincides with the circumscribed circumference of the triangle.

The algorithm of solution of this problem proposed in this talk is based on the following facts [2].

The minimal circle is unique.

The minimal circle of a set S can be determined by at most three extreme points [1] of S which lie on the boundary of the circle. If it is determined by only two points, then the line segment joining those two points must be a diameter of the minimal circle. If it is determined by three points, then the triangle consisting of those three points is not obtuse.

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Riemannian geometry as a tool for solving a number of fundamental problems of Hamiltonian mechanics varieties

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Is classical mechanics reversible? This is one of the fundamental unresolved problems of modern physics, the solution of which can significantly affect the axiomatic of quantum mechanics. However, this question is of paramount importance for mathematics, since it can substantially change our understanding of the differential equations describing dynamical systems. Recall that the dynamical systems theory it is a mathematical theory that draws on analysis, geometry, and topology - areas which in turn had their origins in Newtonian mechanics. As is known, the classical three-body problem with the use of 10 integrals of motion in a symplectic space reduces to a system of the eighth order, which is solved as a Cauchy problem. A. Poincaré showed that although the system of Hamilton equations is time reversible, nevertheless, it manifests chaotic behavior in phase space on large sections of the phase space, which at first glance contradicts the formulation of the Cauchy problem. Such systems were later called classical dynamical systems or Poincaré systems. We formulated the general classical three-body problem as a problem of geodesic trajectories flows on the Riemannian manifold. It is shown that a curved space with local coordinate system allows detecting new hidden symmetries of the internal motion of a dynamical system and reduces the three-body problem to the system of 6th order. It is proved that the equivalence of the initial Newtonian three-body problem and the developed representation provides coordinate transformations in combination with the underdetermined system of algebraic equations. The latter makes the system of geodesic equations relative to the evolutionary parameter (internal time) irreversible, which reveals the true causes of the emergence of chaos in symplectic space of the Hamiltonian system. Equations of deviation of close geodesic trajectories describing the characteristic properties of a dynamical system are obtained, depending on the evolutionary parameter. Various systems of stochastic differential equations (SDE) are derived to describe the motion of a three-body system taking into account the influence of external regular and stochastic forces. Using the system of SDE, a partial differential equation of the second order for the joint probability distribution of the momentum and coordinate of dynamical system in the phase space is obtained.

Weakly hyperassociative semigroups

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The present talk is devoted to the necessary and sufficient conditions of semigroups, which polynomially satisfy the following hyperidentities (1)-(3).

$$F\left(F\left(x,x,x\right),x,x\right)=F\left(x,x,F\left(x,x,x\right)\right), \tag{1}$$

$$F\left(F\left(x,x,x\right),x,x\right)=F\left(x,x,F\left(x,y,x\right)\right), \tag{2}$$

$$F\left(F\left(x,x,x\right),x,y\right)=F\left(x,x,F\left(x,x,y\right)\right). \tag{3}$$

The Notion of $\beta\delta$-Reduction Has the Church-Rosser Property for Canonical Notion of Δ-Reduction

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The definitions used in this paper can be found in [1–3]. Let M be a partially ordered set, which has a least element $\perp$, which corresponds to the indeterminate value, and each element of M is comparable only with $\perp$ and itself. The set of types (denoted by Types), the set of variables of type α (denoted by V_α) and the set of terms of type α (denoted by Λ_α) are defined in [2]. $V = \bigcup\limits_{\alpha \in Types} V_\alpha$, $\Lambda = \bigcup\limits_{\alpha \in Types} \Lambda_\alpha$. We assume that M is a recursive set and considered terms use variables of any order and constants of order ≤ 1, where constants of order 1 are strongly computable, monotonic functions with indeterminate values of arguments, see [1].

A term of the form $\lambda x_1 ... x_k[\tau[x_1, ..., x_k]](t_1, ..., t_k)$, where $x_i \in V_\alpha, i \neq j \Rightarrow x_i \not\equiv x_j, \tau \in \Lambda, t_i \in \Lambda_{\alpha_i}, \alpha_i \in Types, i, j = 1, ..., k, k \geq 1$, is called a β-redex, its convolution is the term $\tau[t_1, ..., t_k]$. A δ-redex has a form $f(t_1, ..., t_k)$, where $f \in [M^k \to M]$, $t_i \in \Lambda_M, i = 1, ..., k, k \geq 1$, its convolution is either $m \in M$ and in this case $f(t_1, ..., t_k) \sim m$ or a subterm t_i and in this case $f(t_1, ..., t_k) \sim t_i, i = 1, ..., k$, see [2]. A term containing no $\beta\delta$-redexes is called $\beta\delta$-normal form.

Definition. Let C be a recursive set of strongly computable, monotonic functions with indeterminate values of arguments. The following canonical notion of δ-reduction is called main canonical notion of δ-reduction if for every $f \in C$, $f : M^k \to M$, $k \geq 1$ we have:

If $f(m_1, ..., m_k) = m$, where $m, m_1, ..., m_k \in M$ and $m \neq \perp$, then $(f(\mu_1, ..., \mu_k), m) \in \delta$, where $\mu_i = m_i$ if $m_i \neq \perp$, and $\mu_i \equiv t_i, t_i \in \Lambda_M$ if $m_i = \perp, i = 1, ..., k, k \geq 1$.

If $f(m_1, ..., m_k) = \perp$, where $m_1, ..., m_k \in M$, then $(f(m_1, ..., m_k), \perp) \in \delta$.

Theorem. For the main canonical notion of δ-reduction the notion of $\beta\delta$-reduction has the Church-Rosser property.

Corollary. For the main canonical notion of δ-reduction every typed λ-term has unique $\beta\delta$-normal form.

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Universal Functions

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Functions' existences, being universal in different senses,(the universality of the functions was manifested by the argument translations, the difference quotients, high order derivatives and Taylor series respectively),were considered in many papers.

In this talk we will construct functions such that their universality is manifested in the classical systems through their Fourier series.

The purpose to describe the structure of functions that are universal for $L^p(0,1)$-spaces, $0 \leq p < 1$,with respect to the signs of Fourier-Walsh coefficients.

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On Inductive Limits for Systems of C^*-algebras

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Consider a covariant functor from the category of an arbitrary partially ordered set into the category of C^*-algebras and their star homomorphisms. In this case one has inductive systems of algebras over maximal directed subsets. For a functor whose values are Toeplitz algebras, we show that each such an inductive limit is isomorphic to the reduced semigroup C^*-algebra defined by a semigroup of rationals. We endow an index set for a family of maximal directed subsets with a topology and study its properties. We establish a connection between this topology and properties of inductive limits.

The Direct and Inverse Problems for the Family of Sturm-Liouville Operators

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We study the direct and inverse problems for the family of Sturm-Liouville operators generated by a fixed potential q and a family of separated boundary conditions. We prove that the union of the spectra of all these operators can be represented as a smooth surface (as the values of a real analytic function of two variables), which has specific properties. We call this function "the eigenvalues' function of the family of Sturm-Liouville operators (EVF)". From the properties of this function we select those, which are sufficient for a function of two variables to be the EVF of a family of Sturm-Liouville operators.

Boundary Value Problem With Infinite Index

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The generalized Riemann boundary value problem for analytic functions is investigated in the weighted space. It is supposed that the weight function has infinitely many zeroes on the unit circumference. It is proved that the homogeneous problem has an infinite number of linearly independent solutions and under some additional conditions on the order of zeroes of the weight function these solutions determined in explicit form.

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On boundary properties of products B_α $(-1 < \alpha \leqslant 0)$

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Classes N_α and products B_α, $-1 < \alpha < +\infty$ were introduced by M. M. Djrbashyan as generalization of R. Nevanlina's class meromorphic functions in the unit circle and the Blaschke product respectively.

The connections of boundary properties and density of zeros of Blaschke product are well studied. V. S. Zakaryan and I. V. Hovhannisyan have shown that the boundary values of any convergent product B_α $(-1 < \alpha < 0)$ belong to the Lipschit's class $Lip(-\alpha, 1)$.

Main results of this report are the followings:

Theorem 1. *Let $-1 < \alpha < 0$, $0 < \beta < 1$, $1 < p < +\infty$ and the sequence of zeros $\{a_n\}$ satisfy the Frostman type condition:*

$$\sum_{n=1}^{\infty} \left(\frac{1 - |a_n|}{|e^{i\theta} - a_n|} \right)^{1+\alpha} < +\infty, \qquad \textit{for all } \theta \in [0; 2\pi],$$

then the conditions that the boundary values $B_\alpha(e^{i\varphi}; \{a_n\})$, $B(e^{i\varphi}; \{a_n\})$, $\dfrac{B_\alpha(e^{i\varphi}; \{a_n\})}{B(e^{i\varphi}; \{a_n\})}$, $\dfrac{B(e^{i\varphi}; \{a_n\})}{B_\alpha(e^{i\varphi}; \{a_n\})}$ belong to the class $Lip(\beta, p)$ are equivalent.

Theorem 2. *There doesn't exist a convergent product B_α $(-1 < \alpha < 0)$ whose boundary values belong to the class $Lip(1, 1)$.*

Theorem 3. *Let $-1 < \alpha < 0$, $1 < p < +\infty$, and the sequence $\{a_n\}$ satisfy the Frostman's type condition for all $\theta \in [0; 2\pi]$. Then the following conditions are equivalent:*

1. *The sequence $\{a_n\}$ consists of the finite number of union of sequences satisfying Neuman's condition:*

$$\sup_{n \geqslant 1} \frac{1 - |u_{n+1}|}{1 - |u_n|} < 1, \qquad (|u_n| \uparrow 1).$$

2. *$B_\alpha(e^{i\varphi}; \{a_n\}) \in Lip\left(\frac{1}{p}, p\right)$ for some $p \in (1; +\infty)$.*

Note that the analogue results for Blaschke product of theorem 2 are proven by Kim H. O., and of theorem 3 – by J. E. Verbitsky.

On Logarithmic Bounds of Directional Maximal Operators

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Let

$$M_v f(x) = \sup_{t>0} (2t)^{-1} \int_{-t}^{t} |f(x - tv)|\, dt,$$

be the maximal function performed in direction v in $\mathbb{R}^2$. For a set of directions V denote $M_V f = \max_{v\in V} M_v f$. The classical result of Nagel, Stein and Wainger [9] states that the operator M_V is bounded on L^2 if V is a lacunary set of directions. On the other hand it was proved by Katz that

Theorem 1 ([6]). *For any finite set of unit vectors V, we have*

$$\|M_V\|_{L^2\to L^2} \lesssim \log_+ \#V. \tag{1}$$

The Hilbert transform analogous of such operator is defined as follows. Set $H_v f(x) = \int_{\mathbb{R}} f(x - tv)\frac{dt}{t}$ to be the Hilbert transform in direction v. Given finite set of unit vectors V define the operator

$$H_V f(x) = \max_{v\in V} |H_v f(x)|.$$

It is a well known consequence of the Rademacher-Menshov theorem that we have

Theorem 2. *For any finite set of unit vectors V we have*

$$\big\|H_V\big\|_{L^2\to L^2} \lesssim \log_+ \#V. \tag{2}$$

There was no any nontrivial bound for directional Hilbert transforms for a long time. It was shown in [3] that the norm bound is necessarily logarithmic in cardinality $\#V$ of the set V, in strong contrast to the result of Nagel, Stein and Wainger [9]. Namely, we have

Theorem 3 ([3]). *For any finite set V of unit vectors it holds*

$$\big\|H_V\big\|_{L^2\to L^2} \gtrsim \sqrt{\log_+ \#V}. \tag{3}$$

This theorem shows that even for infinite lacunary sets of directions the operator H_V is unbounded on L^2 in contrast to the results of [9]. Nevertheless, Di Plinio-Parissis proved that

Theorem 4 ([2])**.** *For any finite lacunary set V of unit vectors it holds*

$$\|H_V\|_{L^2 \to L^2} \lesssim \sqrt{\log_+ \#V}. \tag{4}$$

Thus we see that in the case of finite lacunary set of directions it holds a better bound than (2) and it is sharp by (3)

Many extensions of these results have been considered. Herein, we prove results, which allow for much rougher examples than singular integrals in a choice of directions. Let $K_a(x)$, $a \in \mathbb{R}$, be a family of Calderón-Zygmund kernels with uniformly bounded Fourier transforms, $\|\hat{K}_a\|_\infty < M$, such that $K_a(x)$ as a function in two variables a and x is measurable on $\mathbb{R}^2$. For a unit vector v in $\mathbb{R}^2$ with a perpendicular vector $v^\perp$ we consider an operator T_v written by

$$T_v f(x) = \int_{\mathbb{R}} K_{x \cdot v^\perp}(t) f(x - tv) dt, \quad x \in \mathbb{R}^2, \tag{5}$$

for compactly supported smooth functions f on $\mathbb{R}^2$. Notice that on the v-directer lines $x \cdot v^\perp = l$ the operator T_v defines one dimensional Calderón-Zygmund operators, and those can be different as the line varies. For a finite collection of unit vectors V denote

$$T_V f(x) = \max_{v \in V} |T_v f(x)|. \tag{6}$$

Among the others as a corollary to our main result we derive the following new result.

Theorem 5 ([5])**.** *If the family of Calderón-Zygmund kernels $K_a(x)$ satisfies the above conditions, then for any finite collection of unit vectors V, we have*

$$\|T_V\|_{L^2 \to L^2} \lesssim (\log_+ |V|)^2.$$

No prior result we are aware of has permitted a variable choice of operator, as the line varies. The method of proof is by way of *sparse operators*. Namely we use the recent pointwise domination of singular integrals by a positive operator [1, 4, 7, 8] to reduce the corollary above to a setting, where the operators are *positive*. These positive operators, called sparse operators are 'bigger than the maximal function by logarithmic terms', and so the proofs of the sparse operator bounds imply the corollary above.

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Weighted $\overline{\partial}$-Integral Representations for Weighted L^p-Classes of C^1-Functions in the Matrix Disc

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Let $m, n \geq 1$ be arbitrary natural numbers. Denote by M_{mn} the space of all complex $m \times n-$ matrices. For arbitrary matrix $\eta \in M_{mn}$ denote by $\eta^* \in M_{nm}$ its Hermitian conjugate matrix. Further, $I^m (m \geq 1)$ is the unit $m \times m-$matrix from M_{mm}. The Lebesgue measure in M_{mn} can be written in the following natural way:

$$d\mu_{mn}(\eta) = \prod_{k=1}^{m} \prod_{j=1}^{n} dm(\eta_{kj}), \quad \eta = (\eta_{kj})_{1 \leq k \leq m, 1 \leq j \leq n}. \tag{1}$$

The domain

$$R_{mn} = \{\eta \in M_{mn} : I^m - \eta \cdot \eta^* \quad is \quad positive \quad definite\} \tag{2}$$

is called a matrix unit disc (Cartan classical domain of type I). Note that this domain can be defined by the condition $||\eta|| < 1$ where $|| \bullet ||$ is the spectral (operator) matrix norm.

For a function $f \in C^1(R_{mn})$ put

$$(\overline{\partial} f)(\eta) = \left(\frac{\partial f(\eta)}{\partial \overline{\eta_{kj}}} \right)_{1 \leq k \leq m, 1 \leq j \leq n}. \tag{3}$$

For $\forall z \in R_{mn}$, φ_z is the rational automorphism of the domain R_{mn} with the following properties:

$$\varphi_z(z) = 0, \quad \varphi_z(0) = z; \tag{4}$$

$$\varphi_z(\varphi_z(\eta)) \equiv \eta \quad \Leftrightarrow \quad (\varphi_z)^{-1}(\eta) \equiv \varphi_z(\eta) \quad (\forall \eta \in \overline{R_{mn}}). \tag{5}$$

For complex number β with $Re\beta > -\frac{1}{\nu}, \quad \nu = \min\{m; n\}$, put

$$\Phi_\beta(\eta) = \int_1^{1/||\eta||^2} t^{mn-1} \cdot [\det(I^m - t \cdot \eta\eta^*)]^\beta \, dt, \quad \forall \eta \in R_{mn} \backslash \{0\}. \tag{6}$$

Theorem. Let $m, n \geq 1$ are natural numbers and $\nu = \min\{m; n\}$. Assume that $1 \leq p < \infty, \alpha > -\frac{1}{\nu}, Re\beta \geq \alpha$ and a function $f \in C^1(R_{mn})$ satisfies the following two conditions:

$$\int_{R_{mn}} |f(\eta)|^p \cdot [\det(I^m - \eta \cdot \eta^*)]^\alpha d\mu_{mn}(\eta) < +\infty; \tag{7}$$

$$\int_{R_{mn}} |(\overline{\partial} f)(\eta)|^p \cdot (1 - ||\eta||^2)^{\alpha+1} d\mu_{mn}(\eta) < +\infty, \quad if \quad \alpha \geq 0; \tag{8}$$

$$\int_{R_{mn}} |(\overline{\partial} f)(\eta)|^p \cdot (1 - ||\eta||^2)^{\nu\alpha+1} d\mu_{mn}(\eta) < +\infty, \quad if \quad -\frac{1}{\nu} < \alpha \leq 0. \tag{9}$$

Then the following integral representation is valid ($z \in R_{mn}$):

$$f(z) = c_{mn}(\beta) \cdot \int_{R_{mn}} \frac{f(\eta) \cdot [det(I^m - \eta\eta^*)]^\beta}{[det(I^m - z\eta^*)]^{m+n+\beta}} d\mu_{mn}(\eta) -$$

$$-c_{mn}(\beta) \cdot \int_{R_{mn}} < (\overline{\partial} f)(\eta), (I^m - \eta z^*)(I^m - zz^*)^{-1}(\eta - z) > \times$$

$$\times \frac{[det(I^m - \eta z^*)]^\beta}{[det(I^m - zz^*)]^\beta \cdot [det(I^m - z\eta^*)]^{m+n}} \times$$

$$\times \underbrace{\int_1^{1/||\varphi_z(\eta)||^2} t^{mn-1} \cdot [\det(I^m - t \cdot \varphi_z(\eta)\varphi_z(\eta)^*)]^\beta \, dt}_{\Phi_\beta(\varphi_z(\eta))} \, d\mu_{mn}(\eta). \tag{10}$$

Interassociativity via Hyperidentities of Associativity

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In this talk we study interassociativity of semigroups through the non-trivial hyperidentities of associativity.

On an example of the C^*-algebra generated by a multi-valued mapping

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Let a pair of selfmappings is given $\varphi, \psi : X \longrightarrow X$. Using these two we can construct the multi-valued mapping $\varphi \times \psi^* : X \times X \longrightarrow X \times P(X)$, where ψ^* is the "inverse" mapping to ψ and $P(X)$ is the set of all finite subsets of X. I will discuss some properties of the algebras generated by these mappings. In particular, I will consider the case when every $x \in X$ under the both φ and ψ has the constant number of preimages (n and m). I will show that in this case the algebra is generated by bicyclic semigroup and isomorphic to the algebra generated by the mapping $\varphi \times \varphi^* : \mathbb{N} \times \mathbb{N} \longrightarrow \mathbb{N} \times \mathbb{N} \cup \emptyset$ when φ is a shift.

On Generalized Holomorphic and Special Functions in Hypercomplex Function Theory

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Considering the foundation of Quaternionic Analysis by R. Fueter and his collaborators in the beginning of the 1930s as starting point of Hypercomplex Function Theory, the interest in multivariate analysis using Clifford algebras grew significantly since the 70-ties of the last century. A great amount of papers referring different classes of Special Functions in this context have appeared, particularly by means of algebraic methods, either Lie algebras or through Lie groups and symmetric spaces. In our talk we will rely on the generalization of the classical approach to Special Functions applying hypercomplex derivatives and several hypercomplex variables. In this context special attention will be payed to the role of Special Functions as inter-mediator between continuous and discrete mathematics.

On Integral Logarithmic Means of Blaschke Products for a Half-Plane

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Let the sequence of complex numbers $\{w_k\}_1^\infty$ lies in the lower half-plane $G = \{w : Im\,(w) < 0\}$ and satisfies the condition

$$\sum_{k=1}^{\infty} |v_k| < +\infty. \tag{1}$$

Then the infinite product of Blaschke

$$B\,(w) = \prod_{k=1}^{\infty} \frac{w - w_k}{w - \overline{w}_k}$$

converges in the half-plane G, defining there an analytic function with zeros $\{w_k\}_1^\infty$.

We define an integral logarithmic mean of order $q,\ 1 \le q < +\infty$ of Blaschke products for the half-plane by the formula

$$m_q\,(v, B) = \left(\int_{-\infty}^{+\infty} |\log |B\,(u + iv)||^q\,du \right)^{\frac{1}{q}}, \qquad -\infty < v < 0.$$

Let's denote by $n(v)$ the number of zeros of the function B in the half-plane
$\{w : Im\,(w) < v\}$.

Applying developed by one of the authors "method of Fourier transforms for meromorphic functions" [1], [2], in this paper we obtain estimates for $m_q\,(v, B)$ by the function $n\,(v)$.

In the case of a circle for $q = 2$ the problem was posed by A. Zygmund. In 1969 this problem was solved by the method of Fourier series for meromorphic functions by G.R. MacLane and L.A. Rubel [3]. In [4]

V.V. Eiko and A.A. Kondratyuk investigated this problem in the general case, when $1 \leq q < +\infty$.

In the case of a half-plane in [5], by the method of Fourier transforms for meromorphic functions, is solved the problem of the connection of the boundedness of $m_2(v, \pi_\alpha)$ to the distribution of zeros of the products π_α of A.M. Dzhrbashyan [6]. The function π_α coincides with B for $\alpha = 0$.

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On One Version of the Fixed Point Theorem

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In this talk we present a new version of the classical fixed-point theorem, as well as present same application.

The classical Banach fixed-point theorem, known also as contraction mapping theorem, can be formulated as follow: If (X, ϱ) is a complete metric space and

$$A : (X, \varrho) \to (X, \varrho)$$

is a contraction operator, i.e., there is $0 \leq \alpha < 1$ such that

$$\varrho(Ax_1, Ax_2) \leq \alpha\varrho(x_1, x_2),$$

then the operator A has a unique fixed point. Precisely, there exists a unique $x_0 \in X$, such that $Ax_0 = x_0$.

In this note we introduce a new version of this principle for factor spaces. Let X be a Banach space and X_1 be a closed subspace of X. The factor space X/X_1 consists of classes $[x] = \{x + y; y \in X_1\}$, generated by element $x \in X$ is a Banach space with the norm defined as (see [1])

$$||[x]||_{X/X_1} = inf\{||x + y||; y \in X_1\}.$$

Now let us consider an operator A acting from X to X/X_1 . Following the common sense we introduce the two definitions below.

Definition 1: Operator A is called contraction if there is $0 \leq \alpha < 1$ that

$$||Ax_1 - Ax_2||_{X/X_1} \leq \alpha||x_1 - x_2||_X$$

for every $x_1, x_2 \in X$.

Definition 2: $x_0 \in X$ is called a fixed-point of the operator A if $Ax_0 = [x_0]$. Our result is the following version of the fixed point theorem.

Theorem: Let X be a Banach space and X_1 be a closed subspace of X. If

$$A : X \to X/X_1.$$

is a contraction mapping, then it has a fixed point.

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On One Property of the Franklin System in $C[0,1]$ and $L^1[0,1]$

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A basis $\{e_n\}_{n=0}^{\infty}$ of Banach space X is said to be *boundedly complete* if for every sequence $\{a_n\}_{n=0}^{\infty}$ of scalars with $\sup\limits_{n\in\mathbb{N}}\left\|\sum\limits_{k=0}^{n} a_k e_k\right\| < +\infty$, the series $\sum\limits_{n=0}^{\infty} a_n e_n$ converges. If a space possesses a boundedly complete basis, then the space is isomorphic to a dual space. In particular $C[0,1]$ and $L^1[0,1]$ do not have boundedly complete bases. Trying to find a weaker property, which may accrue for basis in nondual spaces, J.R. Holub introduced in [1] the following concept:

Definition. A semi-normalized basis $\{e_n\}_{n=0}^{\infty}$ of a Banach space X is said to be *monotonically boundedly complete* if whenever $\{a_n\}_{n=0}^{\infty}$ is a sequence of scalars which decreases monotonically to zero and for which $\sup\limits_{n\in\mathbb{N}}\left\|\sum\limits_{k=0}^{n} a_k e_k\right\| < +\infty$, then $\sum\limits_{n=0}^{\infty} a_n e_n$ converges.

He proved, in particular, that the Schauder's basis in $C[0,1]$ is monotonically boundedly complete and asked whether the Haar basis in $L^1[0,1]$ and Franklin basis in $C[0,1]$ are monotonically boundedly complete as well. In [2] V. Kadets proved the monotonically boundedly completeness for Haar basis. Using some results obtained in [3] – [7], we proved the following theorems

Theorem 1. *The normalized Franklin basis for $C[0,1]$ is monotonically boundedly complete.*

Theorem 2. *The normalized Franklin basis for $L^1[0,1]$ is monotonically boundedly complete.*

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Normally Flat Semisymmetric Submanifolds With Zero Index of Nullity

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Let M be a smooth Riemannian manifold with metric g and Riemannian connection ∇. The curvature tensor R is defined by the equation $R(X,Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z$, where X, Y, Z are the arbitrary tangent vector fields on M. The Ricci tensor $R_1$of type (1,1) is defined as follows: if $(e_1, \cdots, e_m)$ is the local basis of orthonormal tangent vector fields on M, then for any X we put $R_1(X) = \sum\limits_{i=1}^{m} R(X, e_i)\, e_i$. For $R_1 = 0$ a manifold M is called a Ricci-flat, and for $R_1 = \lambda I$ it is called an Einstein. The curvature operators $R(X,Y)$ act as differentiations of a tensor algebra, for example,

$$\begin{aligned}(R(X,Y)\,R)(U,V)W =& R(X,Y)(R(U,V)\,W) - R(R(X,Y)\,U,V)\,W - \\ & - R(U,R(X,Y)V)W - R(U,V)R(X,Y)W.\end{aligned}$$

If $R(X,Y)\ R = 0$, then the manifold M is called semisymmetric. This property is intrinsic and is preserved under isometric immersions. The nullity index of the manifold M at a point $x \in M$ is the dimension of the subspace $T_x^{(0)} = \{X \in T_x(M) : \ R(X,\, Y) = 0 \ \forall Y \in T_x(M)\}$.

The following assertions are true.

Theorem 1. *In Euclidean space E_n, the normally flat semisymmetric Einstein submanifold M $(\dim M \geq 3)$ with non-zero Einstein constant is a submanifold of constant non-zero sectional curvature. If a normally flat semisymmetric submanifold M $(\dim M \geq 3)$ is Ricci-flat, then it is locally Euclidean.*

Theorem 2. *In Euclidean space E_n a normally flat semisymmetric submanifold M with zero index of nullity has a parallel Ricci tensor and is a direct product of submanifolds of constant sectional curvature.*

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Algebras with hyperidentities of lattice varieties

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A super-Boolean algebra is an algebra with hyperidentities of the variety of Boolean algebras. A super-De Morgan algebra is an algebra satisfying hyperidentities of the variety of De Morgan algebras. In this talk we give:

1) a characterization of super-Boolean algebras with two binary and one unary operations;

2) a characterization of super-De Morgan algebras with two binary and one unary operations;

3) a characterization of subdirectly irreducible super-Boolean algebras;

4) a characterization of subdirectly irreducible super-De Morgan algebras;

5) a characterization of free n-generated super-Boolean algebras with two binary, one unary and two nullary operations;

6) a characterization of free n-generated super-De Morgan algebras with two binary and one unary operations.

A number of open problems are formulated.

On idempotent and hyperassociative algebras

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This talk is devoted to idempotent and hyperassociative algebras for which the condition of transitivity of commutativity holds. It is shown that such algebras with one additional condition are the rectangular structures of semilattices.

On a Class of Infinite Systems of Linear Algebraic Equations, Arising in Statistical Physics

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An infinite system of linear algebraic equations, concerning the problem of description of Gibbs specifications, is considered. The conditions of existence of the system's solutions, as well as the condition of the uniqueness of the solution, are given. Also, some properties of the solution which are important from the statistical physics' point of view are studied.

Some Properties of a Conform Mapping of Riemann Spaces

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It is known that if a curve is given in the parametric form $x^i = x^i(t)$ in the Riemann space V^n, then the vector $\dfrac{dx^i}{dt}$ is a tangent to the curve and the length of a curve arc is found by the formula $S(\alpha,\beta) = \int\limits_{\alpha}^{\beta} \sqrt{g_{ab}\dfrac{dx^a}{dt} \cdot \dfrac{dx^b}{dt}}\, dt$. It is also known that conform mapping between the Riemann spaces V^n and $\tilde{V}^n$ is one to one, differentiable and preserving the angle between the two curves. If λ^a and μ^a are the tangent vectors to the curves $x^i = x^i(t)$ and $y^i = y^i(t)$ in V^n, then the angle α between the curves is: $cos\,\alpha = \dfrac{g_{ab}\lambda^a\mu^b}{\sqrt{g_{ab}\lambda^a\lambda^b}\sqrt{g_{ab}\mu^a\mu^b}}$, and the angle $\tilde{\alpha}$ between the images of these curves is: $cos\,\tilde{\alpha} = \dfrac{\tilde{g}_{ab}\lambda^a\mu^b}{\sqrt{\tilde{g}_{ab}\lambda^a\lambda^b}\sqrt{\tilde{g}_{ab}\mu^a\mu^b}}$.

The necessary and sufficient conditions of conformity of the mapping between the Riemann spaces V^n and $\tilde{V}^n$ are given.

On an Over-Convergence Phenomenon for Fourier Series

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This talk is devoted to the acceleration of the convergence of the partial sums for classical Fourier series. The proposed acceleration methods are based on explicit formulas. It is shown in the main result, that the use of a finite number of Fourier coefficients makes it possible exact approximation of a given function from an infinite-dimensional set of quasi-polynomials. In this sense, we call the corresponding essentially nonlinear algorithms as over-convergent.

Numerical results demonstrate the effectiveness of the proposed algorithms.

M. Djrbashyan as the Founder of the Theory of Differential Equations With Fractional Derivatives

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This talk presents a brief history of the creation of the works of professor M. Djrbashyan, related to differential equations in fractional derivatives. In the review, along with the formulations of mathematical results, it is noted his research methods, as well as methods of attracting young scientists to the science.

On Λ-Definability of McCarthy Basis Functions, as Functions With Indeterminate Values of Arguments

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The McCarthy basis functions, as functions with indeterminate values of arguments. Let $N = \{0, 1, 2, ...\}$ be the set of natural numbers, each natural number $n \in N$ will be called an atom. Let us define the set of symbolic expressions, which we denote by S-expressions [1].

1. $n \in N \to n \in S$-expressions,
2. $m_1, ..., m_k \in S$-expressions, $k \geq 0 \Rightarrow (m_1 \ ... \ m_k) \in S$-expressions and is called a list.

If $k = 0$, then the list ( ) will be called the empty list. We denote the empty list by the atom 0. Thus the empty list will be both an atom and a list. Let $M = S$-expressions $\cup \{\perp\}$, where $\perp$ is the element which corresponds to indeterminate value. A mapping $\phi : M^k \to M, k \geq 1$, is said to be function with indeterminate values of arguments. The McCarthy basis functions are as follows: *car*, *cdr*, *null*, *atom*, *not*:$M \to M$, *and*, *or*, *eq*, *cons*:$M^2 \to M$, if:$M^3 \to M$.

Untyped λ-terms. Let us fix a countable set of variables V. The set Λ of terms is defined as follows: 1) If $x \in V$, then $x \in \Lambda$, 2) If $t_1, t_2 \in \Lambda$, then $(t_1 t_2) \in \Lambda$, 3) If $x \in V$ and $t \in \Lambda$, then $(\lambda x t) \in \Lambda$. A head normal form and β-equality ($=_\beta$) are defined in a standard way (see [2]).

λ-definability of the McCarthy basis functions. We introduce notations for some terms: $I \equiv \lambda x.x$, $T \equiv \lambda xy.x$, $F \equiv \lambda xy.y$, $\Omega \equiv (\lambda x.xx)(\lambda x.xx)$, $[\ ] \equiv I$, $[t_1, ..., t_k] \equiv \lambda x.xt_1[t_2, ..., t_k]$, where $x, y \in V, t_i \in \Lambda, i = 1, ..., k, k \geq 1$. Let $M = S$-expressions $\cup \{\perp\}$. To each $m \in M$ we associate the term m' as follows: $0' \equiv I$, $(n+1)' \equiv \lambda x.xFn'$, where $n \in N$; $(m_1...m_k)' \equiv [m'_1, ..., m'_k]$, where $m_i \in S$-expressions, $i = 1, ..., k, k \geq 0$; $\perp' \equiv \Omega$.

Definition. We say that the term $\Phi \in \Lambda$ λ-defines (see [3]) the function $\phi : M^k \to M$ $(k \geq 1)$ as a function with indeterminate values of arguments, if for all $m_1, ..., m_k \in M$ we have:

$\phi(m_1, ..., m_k) = m$ and $m \neq \perp \Rightarrow \Phi m'_1...m'_k =_\beta m'$,

$\phi(m_1, ..., m_k) = \perp \Rightarrow \Phi m'_1 ... m'_k$ does not have a head normal form.

Theorem. For functions $car, cdr, cons, null, atom, if, eq, not, and, or$ there exist terms, which λ-define them.

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The existence of rainbow triangles in weak geomeynamics

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Alexander Kharazishvili showed in 2015 that, if all points of the standard Euclidean plane are colored blue, red, and green, with each color being used, then there are continuum many rainbow acute, obtuse, right, and isosceles triangles. A triangle is said to be *rainbow* if its vertices are of different colors. If we take a purely geometric view, then the cardinality part of the theorems does not belong to the realm of geometry, nor it is expressible in first-order logic. What remains is the very interesting statement that there exist rainbow acute, obtuse, right, and isosceles triangles. We will look for minimal assumptions under which one can prove the first three of these statements. We show that in any ordered plane with a symmetric orthogonality relation which allows for a meaningful definition of acute and obtuse angles, in which all points are colored with three colors, such that each color is used at least once, there must exist both an acute triangle whose vertices have all three colors and an obtuse triangle with the same property. We also show that, in a geometry endowed with an orthogonaility relation, in which there is a reflection in every line, in which all right angles are bisectable, which satisfies Bachmann's *Lotschnittaxiom* (the perpendiculars raised on the sides of a right angle intersect), in which all points are colored with three colors, such that each color is used at least once, there exists a right triangle with all vertices of different colors.

Duality in the spaces of functions harmonic in the unit ball of $\mathbb{R}^n$

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We study the Banach spaces $h_\infty(\varphi)$, $h_0(\varphi)$, and $h^1(\eta)$ of harmonic functions over the unit ball in $\mathbb{R}^n$. These spaces depend on a weight function φ and a weight measure η.

In [1] the duality problem was solved, which consists in finding a weight measure η such that $h^1(\eta)$ is an intermediate space dual to $h_0(\varphi)$ and a predual to $h_\infty(\varphi)$, i.e. $h^1(\eta) \sim h_0(\varphi)^*$ and $h^1(\eta)^* \sim h_\infty(\eta)$.

In [1] the case $n = 2$ is considered, when the harmonic function is a real part of the function, holomorphic in B_2. Therefore u has an expansion in a series on degrees z and $\overline{z}$. It allows to apply the methods of complex analysis.

In [2] we consider duality problem in the case of harmonic functions in the unit ball of $\mathbb{R}^n$, $n > 2$. The multidimensional case has the specifics in the sense that we can not speak about connection between harmonic and holomorphic functions, and instead of degrees z and $\overline{z}$ we deal with spherical harmonics.

For a given function φ from a sufficiently broad class of functions, we solve the duality problem.

Note that in an earlier work [3] the duality problem was solved in the case when φ is normal.

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On the Convergence of Quasi-periodic Approximation

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We consider the classical problem of function reconstruction by its finite number of Fourier coefficients or discrete Fourier coefficients. It is well known that approximation of a periodic and smooth function by the truncated Fourier series or trigonometric interpolation is highly effective. However, the convergence is poor if function is discontinuous or non-periodic. The oscillations caused by the Gibbs phenomenon degrade the quality of approximations and interpolations on the entire interval.

We explore the convergence acceleration of the truncated Fourier series for smooth, but non-periodic, functions by an approach known as quasi-periodic extensions. This approach was accomplished successfully in a series of papers for trigonometric interpolation (see [1]-[6]). The idea was in extension of the function outside of the interval of interpolation $[-1, 1]$ to $[a, b] \supset [-1, 1]$ and apply trigonometric functions with larger periodicity $b - a > 2$. The unknown values of the function on the extended domain could be reconstructed by solving a system of linear equations.

We consider this approach for Fourier approximations. The main problem is calculation of the Fourier coefficients on the extended interval $[a, b]$. We consider application of various quadrature formulas for integral calculation by assuming that the unknown values on the extended interval are some free parameters to be defined. The latest could be found as the solution of a system of linear equations arising from the assumption that some of the newly calculated Fourier coefficients should be vanished. For a special choice of the grid points in the quadratures, the matrix of the system can be inverted explicitly. It allows convergence estimation and derivation of the exact constants of the asymptotic errors for comparison with the classical approximations.

Results of numerical experiments confirm theoretical estimates.

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On a identity in the classes of entire functions and some their applications

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Let $w \in A_p$ be a Mackenhoupt weight, i. e. $w(x) \geq 0$ is a function mesurable on R, satisfying condition

$$\left(\int_j w(x)dx\right)\left(\int_j w(x)^{-\frac{1}{p-1}}dx\right) \leq C \cdot |J|^p, (p > 1)$$

where $J \subset R$ is an arbitrary internal, $|J|$ is its the length and C is a constant.

Denote by $W_\sigma^p(wdx)(\sigma > 0)$ the space of entire functions $f(z)$ of exponential type with the norm

$$||f||^p = \int_R |f(x)|^p w(x)dx < +\infty$$

Occurs

Theorem. 1°. The space $W_\sigma^p(wdx)$ entire functions f, which satisfy to conditions

$$f(z)e^{\pm i\sigma z} \in H_p^{\pm}(wdx),$$

where $H_p^{\pm}(wdx)$ are the weighted Hardi space in halfplanes $Imz > 0$ and $Imz < 0$ respectively.

2°. For $f \in W_\sigma^p(wdx)$ the identity hold

$$f(z) = \int_R f(t)\frac{\sin\sigma(t-z)}{\sigma(t-z)}dt, z \in C$$

Artin Dynamical System and Riemann Zeta Functions

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We consider the quantisation of the Artin dynamical system [1] defined on the fundamental region of the Lobachevsky plane. This fundamental region of the modular group SL(2,Z) has finite volume and infinite extension in the vertical axis that correspond to a cusp. In classical regime the geodesic flow in this fundamental region represents one of the most chaotic dynamical systems, has mixing of all orders, Lebesgue spectrum and non-zero Kolmogorov entropy. The classical correlation functions decay exponentially with an exponent proportional to the entropy [2]. Here we calculated quantum mechanical two- and four-point correlation functions [3].

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On Fourier Transforms of Functions of Bounded Type in Tubular Domain

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Suppose $\mathbb{C}^n$ is the n-dimensional complex space,

$\mathbb{C}^n_+ = \{z = (z_1, ..., z_n)\,, Im\ z_j > 0, j = 1, ..., n\}$, $\mathrm{H}\big(\mathbb{C}^n_+\big)$ - is the set of all analytic functions in $\mathbb{C}^n_+, H^\infty\left(\mathbb{C}^n_+\right) = H\left(\mathbb{C}^n_+\right) \cap L^\infty\left(\mathbb{C}^n_+\right)$. Denote the set of all analytic functions bounded type i.e,

$$N\left(\mathbb{C}^n_+\right) = \left\{f : f\left(z\right) = \frac{h_1\left(z\right)}{h_2\left(z\right)}, h_j \in H^\infty\left(\mathbb{C}^n_+\right), \right.$$
$$\left. j = 1, 2, h_2\left(z\right) \neq 0, z \in \mathbb{C}^n_+ \right\}.$$

In the one - dimensional case, the class $N\left(\mathbb{C}^n_+\right)$ coincides with the well-known Nevanlinna class,i.e., of functions f for which $log\,|f|$ has an n-harmonic majorant in $\mathbb{C}^n_+$. In the multidimensional case, the class $N\left(\mathbb{C}^n_+\right)$ fundamentally different from Nevanlinna class. As is know , if f belongs to the Smirnov class $N^+\left(\mathbb{C}^n_+\right)$ and its boundary value on R^n belongs L^1 then f belongs to the Hardy class $H^1\left(\mathbb{C}^n_+\right)$ and, therefore, the Fourier transform $\widehat{f}$ of this function vanishes on R^n/R^n_+. The simple example shows this assertion does not holds for $N\left(\mathbb{C}^n_+\right)$. We prove that if $\widehat{f}\left(x\right) \to 0, x \in R^n_-$ sufficiently rapidly as $|x| \to +\infty$ then the function $\widehat{f}$ identically vanishes on R^n/R^n_+. Moreover,a necessary and sufficient condition on the rate of decrease for this assertion to hold was found. We also give several interesting,in our view, applications to the theories of Hardy classes and quasi-analytic classes of functions.

Mathematics for Artificial Consciousness

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This work is dedicated to the study of cognitive systems, and tries to find out whether the artificial intelligence, when is combined with self-organized and collaborative computing agents, can create some sort of artificial consciousness. The methodology for building agents specifies how a deductive agent can be decomposed into the construction of a set of component modules and how these modules should be made to interact. The total set of modules and their interactions has to provide an answer to the question of how the sensor data and the current internal state of the agent determine the actions and future internal state of the agent.

Model of agent machinery and abstract architecture is presented by the following mathematical structure, MS. written $MS =< E, Ac, Node, Ag >$, where E is the set of environment states, Ac is the set of actions, Node is a vertex on the semantic graph, and Ag is a mapping $E^* \to Ac$, respectively. Deductive logic, DL, for the MS is constructed on a strict logical argument, which in this case presents a set of specified rules of inference.

In this closed system, the database of classified entities is specified using formulae of first-order predicate logic, where integration of discrete information is based on how attitudes are formed and changed through the integration (mixing, combining, or simply adding) of new information with existing cognitions or thoughts, each relevant piece of information having two qualities: value and weight. The proposed structure points out to the need for construction of a value/weight type of a cellular automaton defined by a 4-tuple $< Z, S, N, f >$, where Z is a finite lattice, S is finite set of cell values, N is the finite neighborhood, and f is the local transition function defined by the transition table. Specifically, nodes get connected by input data, and connections get validated applied a relevant predicate calculus. Cognitive machinery reconfigures an activation network based on weights.

Let A be the set of entities which present environmental symbols or discrete pieces of information, and A^* be the free monoid generated by A, i.e. A^* contains all of the finite sequences of elements of A for a finite cellular automaton.

Definition: A finite state cellular automaton is said to *recognize* the subset $W \subseteq A^*$ and implements a deductive logic DL, when the machine is started in a specific initial state s_0, the machine produces a 1 output for each sequence in W, and otherwise produces a 0 output.

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An Extremal Property of Delaunay Triangulation and Its Applications in Mathematical Physics

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The popular method for numerical solution of some problems of mathematical physics is the finite element method. This method needs a mesh of triangles. The convergence rate of iteration process of numerical solution of the problem by the finite elements method depends on geometrical configuration of the mesh.

We prove the following extremal property: Let M be a triangular mesh with set of knots K. Denote by $S(M)$ the sum of cotangents of interior angles of all triangles from M.

Theorem 1. *For any fixed set of knots K, the sum $S(M)$ as a function on mesh M reaches his minimum for Delaunay triangulation.*

Using this extremal property, the theorem is obtained, that for any fixed knots set, for numerical solution of Maxwell equation of magnetic field the optimal mesh is Delaunay triangulation.

Degenerate First Order Differential-Operator Equations

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We consider the following boundary value problem for degenerate differential-operator equation of first order

$$Lu \equiv t^{\alpha} u' + Au = f, \quad u(0) - \mu u(b) = 0, \tag{1}$$

where $t \in (0, b)$, $\alpha \geq 0, \mu \in \mathbb{C}$, $A : H \to H$ is linear operator in separable Hilbert space H, $f \in L_{2,\beta}((0, b), H)$, i.e.,

$$\|f\|_{\beta}^{2} = \int_{0}^{b} t^{\beta} \|f(t)\|_{H}^{2} \, dt < \infty.$$

Suppose that the operator $A : H \to H$ has a complete system of eigenfunctions $\{\varphi_k\}_{k=1}^{\infty}$, forming a Riesz basis in H, i.e., $A\varphi_k = a_k \varphi_k$, $k \in \mathbb{N}$ and for every $x \in H$ we have

$$x = \sum_{k=1}^{\infty} x_k \varphi_k$$

and there are some positive constants c_1 and c_2 such that

$$c_1 \sum_{k=1}^{\infty} |x_k|^2 \leq \|x\|_H^2 \leq c_2 \sum_{k=1}^{\infty} |x_k|^2.$$

We prove that under some conditions on the operator A and the number μ the corresponding boundary value problem (1) has unique generalized solution $u \in L_{2,\beta}((0, b), H)$ for $2\alpha + \beta < 1$, $\alpha \geq 0, \beta \geq 0$ and for every $f \in L_{2,\beta}((0, b), H)$.

Boundedness of the cluster sets of harmonic functions

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In this paper, we use widespread notations and definitions. We denote a unit circle, unit circumference and hypercycle respectively by D, Γ and $L(\xi, \varphi)$, ending at a point $\xi = e^{i\theta}$ and forming an angle with a radius at this point φ, $-\frac{\pi}{2} < \varphi < \frac{\pi}{2}$. Let $H(\xi, \varphi_1, \varphi_2)$ be a subdomain of a circle D, bounded with hypercycles $L(\xi, \varphi_1)$ and $L(\xi, \varphi_2)$. Let's suppose that $f(z)$ is a harmonic function, defined in D. It is common to be denote Riemann Liouvilles integral operator by $D^{-\alpha}$ where $\alpha > 0$. We denote cluster set of a function $f(z)$ at a point ξ regarding to set $S \subset D$ by $C(f, \xi, S)$, for which point $\xi \in \Gamma$ is a limit point. The following assertion is correct.

Theorem: *Let $f(z)$ is a normal harmonic function in D and ξ is an arbitrary point Γ.*

If cluster set $C(r^{-\alpha} D^{-\alpha}(f), \xi, L(\xi, \varphi_1))$ and $C(r^{-\alpha} D^{-\alpha}(f), \xi, L(\xi, \varphi_2))$ are bounded above (or below) with a value α at fixed values $\varphi_1, \varphi_2 \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, then a cluster set $C(r^{-\alpha} D^{-\alpha}(f), \xi, H(\xi, \varphi_1, \varphi_2))$ is bounded above (or below) with a number α, where $0 < \alpha < +\infty$.

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Description of Linear Continuous Functionals on the Space $A^*(\omega)$

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$A^*(\omega)$ holomorphic class in D – unit disc function $f(z)$ is studied for which

$$\|f(x)\|_{A^*(\omega)} = \int\limits_D \omega(1-|z|)\log^+|f(z)|\, dm_2(z) < +\infty,$$

where $\omega(r)$ – is a monotonous, non-negative function from $L^1(0;1)$ class. It is worth stating that in case of $\omega(r) = (1-r^2)^\alpha$, $(\alpha > -1)$, these classes were first presented and thoroughly studied in the research works of M. M. Djrbashyan [1]. Supposing that $\omega(r)$ meets the condition

$$\sup\left|\frac{\omega'(r)(1-r)}{\omega(r)}\right| = q_\omega < +\infty.$$

In this case it is assumed that $0 < q_\omega < 1$ has its place, when $\omega(r)$ is growing monotonously. The main result is the following theorem:

Theorem: *If* $\Phi \in A^*(\omega)$, *there is a holomorphic function in* D

$$q(z) = \sum_0^\infty b_k * z^k, \qquad b_k = 0\left(\exp\left[-\frac{C}{\omega\left(\nu\left(\frac{1}{k}\right)\right)\left(1-\nu\left(\frac{1}{k}\right)\right)^2}\right]\right) \tag{1}$$

$$\Phi(f) = \lim_{r\to 1-0}\frac{1}{2\pi}\int\limits_{-\pi}^{\pi} f(re^{i\varphi})\, g(e^{-i\varphi})\, d\varphi = \sum_{k=0}^\infty a_k b_k \tag{2}$$

For each $f(z) = \sum\limits_{k=0}^\infty a_k z^k \in A^*(\omega)$, *where the row* (2) *absolutely coincides.*

Conversely, if coefficients of the function $g(z) = \sum\limits_{k=0}^\infty b_k z^k$ *meet the condition* (1), *the function* $g(z)$ *is holomorphic in* D *and defines the functional* $\Phi(f)$, *which is linear and continuous in* $A^*(\omega)$.

When the main theorem is proved other additional validations are also asserted:

Lemma 1: *Let*

$$f(z)=\sum_{k=0}^{\infty}a_kz^k\in A^*(\omega)\quad and\quad f_r(z)=f(rz)=\sum_{k=0}^{\infty}a_kr^kz^k,\quad 0<r<1,$$

then

$$\lim_{r\to 1-0}\rho(f_r,f)=0.$$

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