

# Linear and convex programming problems in smart grid management

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## Abstract

We propose a model of power consumption game in smart grid. It aims to formulate the problem of consumption-price management from user point of view and interests. Based on the user prescribed consumption constraints coming from own business or domestic processes, s/he deals with a linear or convex programming task.

## The Model

The underlying idea of the model is to interpret the flexible generation of power prices in smart grid as a discrete memoryless source. It implies, in particular, that the existing prices  $p_1 \leq p_2 \leq \dots \leq p_k$  have different probabilities to appear in the system, which are produced independently of each other and sum up to 1. Moreover, those probabilities may vary from one day interval (of  $s$  variants) to another.

In this context, the task of user is to automatically manage the consumption strategies over the day to maintain necessary thresholds of power usage and at the same time to reach the lowest bill.

Formally, the consumer has  $s$  probability distributions

$$V_i \triangleq (v_i(p_1), \dots, v_i(p_k)), \quad \sum_{j=1}^k v_i(p_j) = 1, \quad i = \overline{1, s},$$

on the prices, versus the day interval s/he performs the consumption.

Let the user knows that in order to fulfill his/her needs a consumption plan should be configured for each of the day intervals and that is possible to make in terms of average power usages. In other words, the following plans ( $\bar{c}_i$ ) have to be realized

$$\sum_{j=1}^k v_i(p_j) c_{ij} = \bar{c}_i, \quad i = \overline{1, s} \quad (1)$$

where  $c_{ij} \geq b_{ij} \geq 0$  is the consumption coefficient for the interval  $i$  applied to the price  $p_j$  which comes with the probability  $v_i(p_j)$ , and naturally is lower bounded by a nonnegative number  $b_{ij}$ .

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However, in view of (1) the user wants to minimize the money that will be paid out:

$$\sum_{i=1}^s \sum_{j=1}^k v_i(p_j) p_j c_{ij} \rightarrow \min. \quad (2)$$

Equations (1) and (2) together make a consumption-price tradeoff problem which is an easily resolvable linear programming task. The solutions turn to be consumption peaks at the lowest prices for each of the intervals and minimum  $b_{ij}$  everywhere else. But this is not a valid solution in many applications. That is why other important constraints come into the model here, namely, special restrictions in design of the consumption strategies  $c_{ij}$ . For example,

$$\max_j c_{ij} - \min_j c_{ij} \leq \Delta c_i, \quad i = \overline{1, s}, \quad (3)$$

where  $\Delta c_i$  are some positive numbers that indicate the allowable consumption fluctuations. The optimization (2) subject to (1) and (3) still makes a linear programming problem. The constraints (3) mitigate the initial solution in terms of peaks.

We notice some relevant papers [1] and [2].

## Extensions

The convex programming comes in when the user replaces the restrictions in (3) with ones on convex functions:

$$g_i(c_{i1}, \dots, c_{ik}) \leq 0, \quad i = \overline{1, s}. \quad (4)$$

For instance, the following constraints on mean square consumption energies can be considered:

$$\sum_{j=1}^k c_{ij}^2 \leq E_i > 0, \quad i = \overline{1, s}. \quad (5)$$

Now the equations (1), (2), and (4)/(5) constitute a convex programming problem.

## Vision

With the suggested model and the problems posed, our vision of the future home and business users is that they are equipped with configurable module-optimizers which automatically compute and apply the above described best consumption strategies to optimize their budgets for the power.

## References

- [1] A.H. Moshenian-Rad, V. W.S. Wong, J. Jatskevich, and R. Schober, "Optimal and autonomous incentive-based energy consumption scheduling algorithm for smart grid", *IEEE PES Innovative Smart Grid Technologies Conference 2010*.
- [2] H. Li and R.C. Qiu, "Need-based communication for smart grid: when to inquire power price?", arXiv:1003.2138v1 [cs.IT] 10 Mar 2010.